

RESEARCH ARTICLE

Improving the Starting Performance of Induction Machines through the Exploitation of Skin Effect: A Natural Modeling (abc) Approach

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ABSTRACT

This article presents a comparative analysis of the starting performance of a squirrel-cage induction machine based on rotor slot topology. The study focuses on enhancing electromagnetic torque and reducing the inrush current when starting with high inertia. Unlike conventional approaches, the modeling is performed in the natural reference frame (*abc*) to avoid the simplifying assumptions associated with the Park transformation and to better represent the machine's physical reality. Four deep-bar geometries are examined: rectangular, trapezoidal, inverse trapezoidal, and double-cage. The skin effect, which drives the variation of the parameters r_r and I_{fr} , is incorporated through the coefficients k_r and k_s , calculated via a staircase discretisation method. Simulation results highlight the effectiveness of each structure in reducing starting time while limiting stator electrical stress, thereby providing precise selection criteria for designing rotors adapted to severe starting cycles.

Keywords

Squirrel-cage induction machine;
Natural reference frame;
Skin effect;
High-inertia starting;
Different deep rotor slots

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I. INTRODUCTION

The start-up of induction motors (IMs), particularly when driving high-inertia loads, is a critical phase that generates significant thermal and mechanical stresses. During this transient state, the stator currents are high, and the electromagnetic torque must be sufficient to overcome the resistance torque. These stresses can reduce the machine's service life and degrade its overall performance, hence the need to optimize the starting characteristics [1]. Among the influencing parameters, the rotor geometry plays a decisive role. In particular, the shape of the rotor's deep slots determines the current distribution in the bars and allows the skin effect to be exploited in order to modulate the apparent rotor resistance as a function of frequency [2]. This property provides a means of passively controlling the trade-off between high starting torque and limited inrush current. In this context, this article presents a detailed analysis of the influence of the rotor slot shape on dynamic start-up performance. The study focuses on evaluating the electromagnetic torque and stator current for different bar geometries. Modelling is carried out in the natural three-phase coordinate system (*abc*), which forms the basis of the approach. Unlike the conventional transformation to the rotating two-phase coordinate system (*dqo*), the (*abc*) model allows the machine's actual physical quantities to be retained [3][4]. It also allows space harmonics and any asymmetries to be introduced directly, thereby enabling a better description of the electromagnetic behavior [3]. Admittedly, this approach has a higher computational cost, but it is particularly well suited to the analysis of long transients, during which the rotor frequency varies over a wide range. A comparative study was therefore carried out on four rotor bar configurations, each utilising the skin effect in a different way: (I) a rectangular geometry, used as a reference;

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(II) a trapezoidal geometry, allowing the rotor resistance to be varied gradually; (III) an inverted trapezoidal geometry designed to concentrate the current at the surface; (IV) a double-cage structure, allowing start-up performance to be separated from that at rated speed [5]. The effect of frequency on the rotor parameters is taken into account through the use of correction factors k_r for resistance and k_x for inductance. These coefficients are used to quantify the frequency dependence of electrical quantities in rotors, linked to the skin effect in the rotor conductors. Their evaluation is based, firstly, on a basic analytical form associated with a step function model, which accounts for the non-uniform distribution of current in the bars that make up the rotor. To ensure the reliability of the coefficients thus obtained and to provide a better representation of the electromagnetic phenomenon, this is validated by a numerical approach based on the finite element method (FEM), as presented in [2, 5]. The study focuses on a squirrel-cage induction motor (15 kW; 100 N·m). The case analysed involves a direct start at rated voltage, driving a high-inertia load ($J = 2 \text{ kg}\cdot\text{m}^2$), to investigate the influence of rotor slot geometries on dynamic performance during start-up. In this context, three main objectives are considered: (I) reducing the transient time to achieve better steady-state performance; (II) limiting voltage drops on the grid caused by high inrush currents; and (III) preserving the thermal integrity of the windings by controlling the temperature rises associated with overcurrent conditions. The rotor's geometric design is therefore a key factor in improving the performance of the entire machine. A well-designed rotor makes it possible to combine high starting torque with limited stator current, whilst reducing thermal stresses [6]. The optimised design helps to limit premature ageing of the insulation systems, which is usually caused by repeated and severe starts, resulting in more reliable and efficient operation over the long term [7-8].

II. MATHEMATICAL MODELING IN THE NATURAL COORDINATE SYSTEM (ABC)

The modelling of the MAS is based on deriving the equations for stator and rotor stresses, in which the squirrel-cage rotor winding is equivalent to the three-phase stator winding. To ensure the accuracy of the dynamic model, the skin effect is taken into account.

1. The modified squirrel cage

Fig. 1 shows a simplified cage winding (a) and its phasor diagram (b), illustrating the busbar and ring currents. The busbars are numbered from 1 to Q_r , with positive directions indicated by arrows. The parameters of a busbar are r_{bar} and l_{bar} , while r_{ring} and l_{ring} represent those of the ring sections between busbars. The total impedance of the winding is derived from these. A phase diagram of the busbar currents is plotted at the v harmonic, with a phase shift of $v\alpha_u p$. The angle α_u is defined as mechanical and related to the electrical angle $p\alpha_u$. Finally, a polygon is constructed to represent the bus bar currents

At each connection point between the bar and the ring, Kirchhoff's first law is satisfied, as shown in Fig. 1(a):

$$I_{ring,x} = I_{bar,x} + I_{ring,x-1} \quad (01)$$

Furthermore, the phase angle between the mutual circulating currents is equal to $v\alpha_u p$, while the phase shift between the currents in the two segments of the ring is αp . Each bar constitutes a single winding of a rotor phase, with a number of turns given by $N_r = 1/2$. The current flowing through a bar thus corresponds to the rotor phase current. According to Fig. 1(b), the rms value of the induced current in a bar is denoted by I_{bar} .

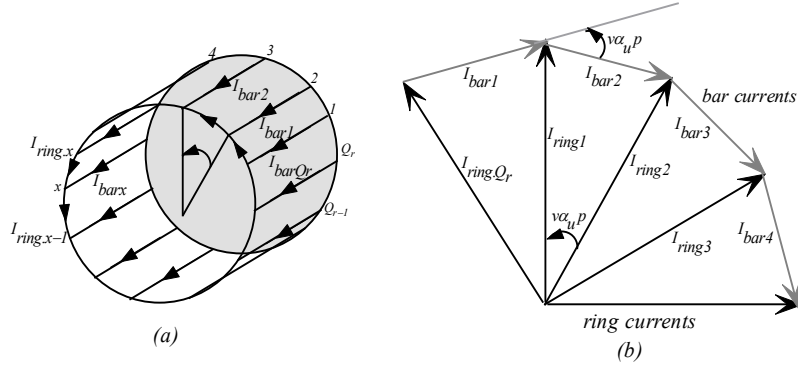


Figure 1. (a) Squirrel-cage winding,
(b) Phasor sector for the busbar currents and a portion of the current vector diagram [1,3]

Consequently, the current in the ring is determined as follows

$$I_{ring} = \frac{I_{barv}}{2 \cdot \sin\left(\frac{p\pi}{Q_r}\right)} \quad (02)$$

The expressions for the resistance and leakage inductance of a rotor bar, including the contribution of the short-circuit rings, are given as follows:

$$r_r = r_{bar} + \frac{r_{ring}}{2 \cdot \sin^2\left(\frac{p\pi}{Q_r}\right)} \quad (03)$$

$$l_r = l_{bar} + \frac{l_{ring}}{2 \cdot \sin^2\left(\frac{p\pi}{Q_r}\right)} \quad (04)$$

The parameters r_r and l_r characterize a rotor bar including the effect of the short-circuit rings, whereas r_{ring} and l_{ring} describe the rings themselves. The accompanying figure compares the rotor cage before and after this effect is taken into account. The mathematical model of the machine is then established in the abc coordinate system without considering spatial harmonics.

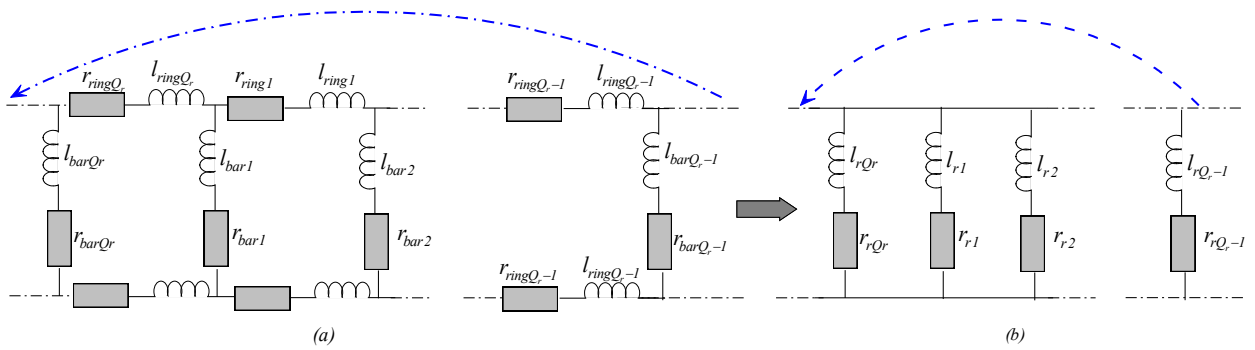


Figure 2. (a): Squirrel cage before adding the ring proportions to the rotor bars;
(b): Squirrel cage after adding the ring proportions to the rotor bars [3]

2. Mathematical model of the tree phase induction Machine

This section presents the mathematical model of the machine in the natural coordinate system abc , neglecting spatial harmonics. This method provides a more realistic and simpler representation of the machine than the dqo transformations, while facilitating the analysis of its overall behavior [9,10].

a. Electrical equations

The three-phase asynchronous motor under study has three stator phases and 38 rotor phases corresponding to the rotor bars. The electrical equations describing its operation are expressed in the natural *abc* coordinate system as follows:

$$\begin{cases} [v_s] = [r_s][i_s] + \frac{d\psi_s}{dt} \\ [v_r] = [r_r][i_r] + \frac{d\psi_r}{dt} \end{cases} \quad (05)$$

With:

$$\begin{aligned} [v_s] &= [v_{as} \ v_{bs} \ v_{cs}]^t; [i_s] = [i_{as} \ i_{bs} \ i_{cs}]^t; [\psi_s] = [\psi_{as} \ \psi_{bs} \ \psi_{cs}]^t; \\ [v_r] &= [v_{r1} \ v_{r2} \ v_{r3} \ \dots \ v_{rj} \ \dots \ v_{rQ_r}]^t = 0_{Q_r}; [i_r] = [i_{r1} \ i_{r2} \ i_{r3} \ \dots \ i_{rj} \ \dots \ i_{rQ_r}]^t; \\ [\psi_r] &= [\psi_{r1} \ \psi_{r2} \ \psi_{r3} \ \dots \ \psi_{rj} \ \dots \ \psi_{rQ_r}]^t; [r_s] = [r_{as} \ r_{bs} \ r_{cs}][I_3]; [r_r] = [r_{r1} \ r_{r2} \ \dots \ r_{rj} \ \dots \ r_{rQ_r}][I_{Q_r}]. \end{aligned}$$

b. Magnetic equations

The expressions of the stator and rotor flows are given by the following matrix form:

$$\begin{cases} [\psi_s] = [L_{s,s}][i_s] + [M_{s,r}][i_r] \\ [\psi_r] = [L_{r,r}][i_r] + [M_{r,s}][i_s] \end{cases} \quad (06)$$

With:

$$[L_{s,s}] = l_{fs}[I_3] + l_{ms} \cos\left((n-m)\frac{2\pi}{3}\right) \quad (7.a)$$

$$[L_{r,r}] = l_r k_x(f_r)[I_{Q_r}] + l_{mb} \cos\left((i-j)\frac{2\pi}{Q_r}\right) \quad (7.b)$$

Where: $[I_{Q_r}]$: The identity matrix of size; $n = 1, 2, 3$: Row index; $m = 1, 2, 3$: Column index; $i = 1, 2, 3, \dots, Q_r$: Row index; $j = 1, 2, 3, \dots, Q_r$: Column index.

$[L_{s,s}]$; $[L_{r,r}]$: Matrices of stator and rotor inductors, respectively; l_{fs} and l_r : Stator and rotor leakage inductors, respectively; l_{ms} and l_{mb} : Maximum stator and rotor magnetization inductors, respectively;

The matrix $[M_{s,r}]$ represent the coupling between the stator tree phase ($k \in \{0, 1, 2\}$) and the rotor bars ($j \in \{0, 1, 2, \dots, Q_r\}$).

$$[M_{s,r}] = m_{sr} \cos\left(\theta_r + \frac{(2j-1)\pi}{Q_r} - k\frac{2\pi}{3}\right) \quad (08)$$

With: $m_{sr} = \sqrt{l_{ms}l_{mb}}$: Maximum mutual inductance between stator phase and corresponding rotor phase;

$[M_{r,s}] = [M_{s,r}]^t$: Coupling inductance matrices between star and rotor and vice versa;

After the development of the two equations (5) and (7), we obtain the following system of equation:

$$\begin{cases} [v_s] = [r_s][i_s] + \frac{d}{dt}\{[L_{s,s}][i_s] + [M_{s,r}][i_r]\} \\ [v_r] = [r_r][i_r] + \frac{d}{dt}\{[M_{r,s}][i_s] + [L_{r,r}][i_r]\} \end{cases} \quad (09)$$

$$\begin{cases} [v_s] = [r_s][i_s] + [L_{s,s}]\frac{d}{dt}[i_s] + [M_{s,r}]\frac{d}{dt}[i_r] + \left\{\frac{d}{dt}[M_{s,r}]\right\}[i_r] \\ [v_r] = [r_r][i_r] + [M_{r,s}]\frac{d}{dt}[i_s] + \left\{\frac{d}{dt}[M_{r,s}]\right\}[i_s] + \left\{\frac{d}{dt}[L_{r,r}]\right\}[i_r] + [L_{r,r}]\frac{d}{dt}[i_r] \end{cases} \quad (10)$$

$$\begin{cases} [v_s] = [r_s][i_s] + [L_{s,s}]\frac{d}{dt}[i_s] + [M_{s,r}]\frac{d}{dt}[i_r] + \frac{d\theta_r}{d\theta_r}\left\{\frac{d}{dt}[M_{s,r}]\right\}[i_r] \\ [v_r] = [r_r][i_r] + [M_{r,s}]\frac{d}{dt}[i_s] + \frac{d\theta_r}{d\theta_r}\left\{\frac{d}{dt}[M_{r,s}]\right\}[i_s] + \frac{df_r}{df_r}\left\{\frac{d}{dt}[L_{r,r}]\right\}[i_r] + [L_{r,r}]\frac{d}{dt}[i_r] \end{cases} \quad (11)$$

With : $\frac{d}{dt}[M_{s,r}] = \frac{d\theta_r}{d\theta_r}\frac{d}{dt}[M_{s,r}] = \frac{d\theta_r}{dt}\frac{d}{d\theta_r}[M_{s,r}] = \omega_r\frac{d}{d\theta_r}[M_{s,r}]$; $\frac{d}{dt}[M_{r,s}] = \omega_r\frac{d}{d\theta_r}[M_{r,s}]$;

$$\frac{d}{dt}[L_{r,r}] = \frac{d}{dt}\{l_r(f_r)[I_{Qr}] + l_{mb}\cos\left((i-j)\frac{2\pi}{Q_r}\right)\} = \frac{df_r}{df_r}\frac{d}{dt}\{l_r(f_r)[I_{Qr}]\} = \frac{df_r}{dt}\frac{d}{df_r}\{l_r(f_r)[I_{Qr}]\}; [v_r] = 0_{Qr}$$

$$\begin{cases} \frac{d}{dt}[i_s] = [L_{s,s}]^{-1}\left\{[v_s] - [r_s][i_s] - [M_{s,r}]\frac{d}{dt}[i_r] - \omega_r\left\{\frac{d}{d\theta_r}[M_{s,r}]\right\}[i_r]\right\} \\ \frac{d}{dt}[i_r] = [L_{r,r}]^{-1}\left\{[v_r] - [r_r(f_r)][i_r] - [M_{r,s}]\frac{d}{dt}[i_s] - \omega_r\left\{\frac{d}{d\theta_r}[M_{r,s}]\right\}[i_s] - \frac{df_r}{dt}\frac{d}{df_r}\{l_r(f_r)\}[i_r]\right\} \end{cases} \quad (12)$$

c. Electromagnetic torque equation

The expression of the electromagnetic torque is reduced to:

$$T_{em} = p[i_s]^t \frac{d}{d\theta_r}[M_{s,r}][i_r] \quad (13)$$

Where: p: Number of pair poles.

d. Mechanical equation

The general equation of the speed rotation of the machine is written:

$$J \frac{d\Omega_r}{dt} = T_{em} - T_r - K_f \Omega_r \quad (14)$$

With: J: Total inertia; K_f : Viscous friction; Ω_r : Rotor mechanical speed.

III. MODELLING OF THE SKIN EFFECT AND ROTOR GEOMETRIES

1. Topologies studied

This section describes the modelling and simulation of various rotor bar geometries in order to assess the impact of the skin effect on the electromagnetic performance of the MAS under study. Figure 3 shows the four main configurations analyses, derived from industrial MAS: rectangular, trapezoidal, inverted trapezoidal and double cage [5]. To ensure a rigorous comparison and isolate the influence of shape, all geometries share a reference height of $h = 30$ mm and equivalent cross-sections. Only the double-cage structure has a slightly greater overall height (34 mm) to accommodate the isthmus separating the two start-up and operating cages. These configurations are subjected to a comparative analysis to quantify their influence on dynamic behavior, particularly during start-up phases.

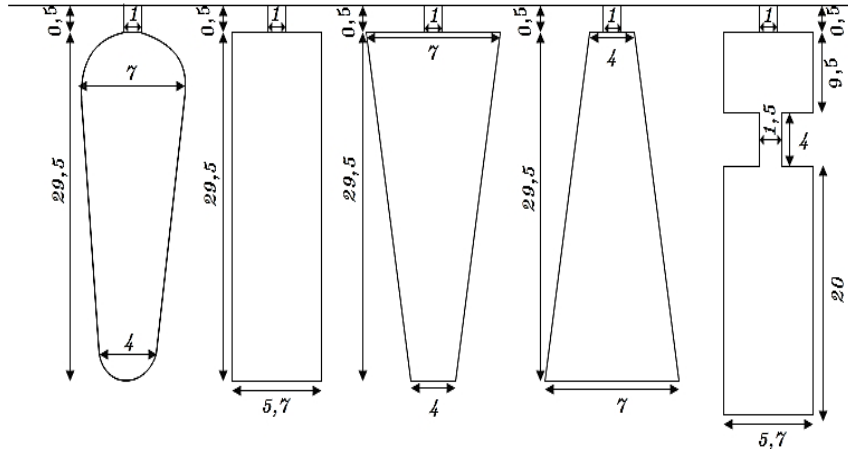


Figure 3. Geometries of the notches studied [2]

2. Sub-conductor model (R-L circuit theory)

The method used is based on discretizing the rotor bar into several elementary layers, following a so-called ‘staircase’ model, as shown in Figures 4 and 5. This approach allows the geometry of the bar to be accurately represented and ensures high precision, with a negligible relative error compared to more computationally intensive numerical methods [6]. For each elementary conductor i , the impedance is defined by its resistance and leakage reactance according to the following expressions:

$$\begin{cases} x_i = 2\pi f_r l_i = 2\pi f_r \mu_0 \frac{\Delta h_i \ell}{b_i} \\ r_i = \rho \frac{\ell}{\Delta h_i b_i} \end{cases} \quad (15)$$

Where: f_r : Rotor frequency; ρ : Conductor resistivity; l : Length of the magnetic circuit.

The equivalent impedance of the bar (Z_{eq}), including the short-circuit ring sections, is obtained by connecting these elements in series in a ladder network. This discretization method is suitable for all complex shapes (oval, bottle-shaped, etc.) [7]. In this study, a division into 100 elementary layers was chosen to ensure optimal convergence of the results.

$$Z_{eq} = r_a + j(x_a + x_1) + \frac{1}{\frac{1}{r_1} + \frac{1}{jx_2 + \frac{1}{\frac{1}{r_2} + \frac{1}{jx_3 + \frac{1}{\frac{1}{r_3} + \frac{1}{jx_4 + \frac{1}{\frac{1}{r_4} + \dots + jx_n + r_n}}}}}}} \quad (16)$$

Where: r_a ; x_a : the resistance and leakage reactance of the short-circuit ring sections, referenced to a rotor bar, respectively.

In this method, the correction coefficients k_r and k_x (introduced during the analytical modelling) are derived from the equivalent impedance Z_{eq} [1]. The coefficient k_r , which reflects the increase in resistance, is defined as the ratio of the real part of Z_{eq} to the DC resistance of the bar (r_0). Conversely, the coefficient k_x , which reflects the decrease in stray conductivity, corresponds to the ratio of the imaginary part of Z_{eq} to the leakage reactance of the slot at very low rotor frequencies.

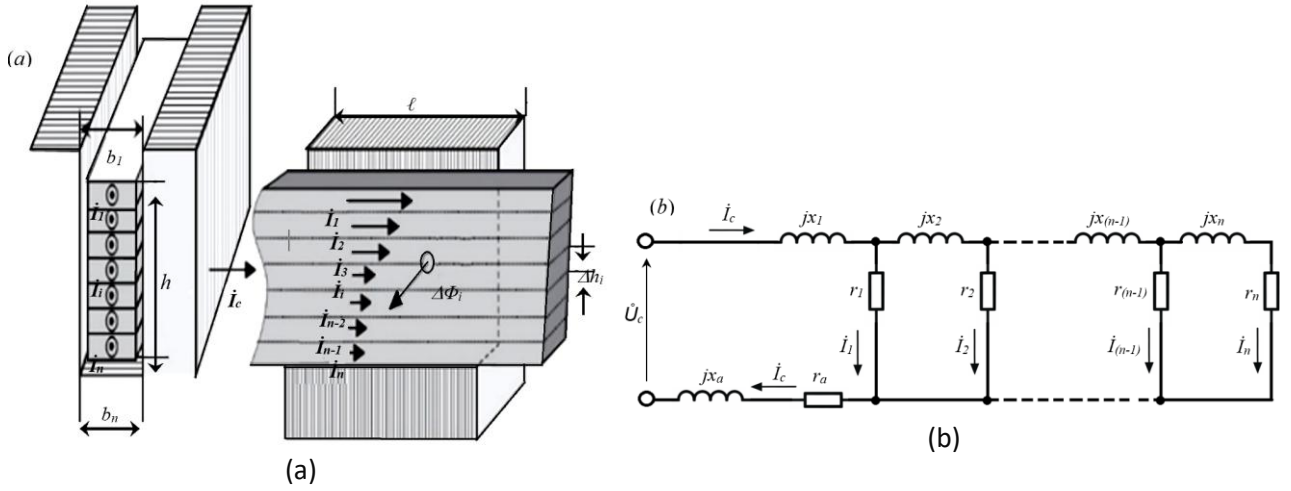


Figure 4. Ladder network modelling of a squirrel-cage bar: (a) discretisation of the bar into elementary conductors and (b) equivalent ladder circuit incorporating the end-ring impedances relative to the rotor bars [1]

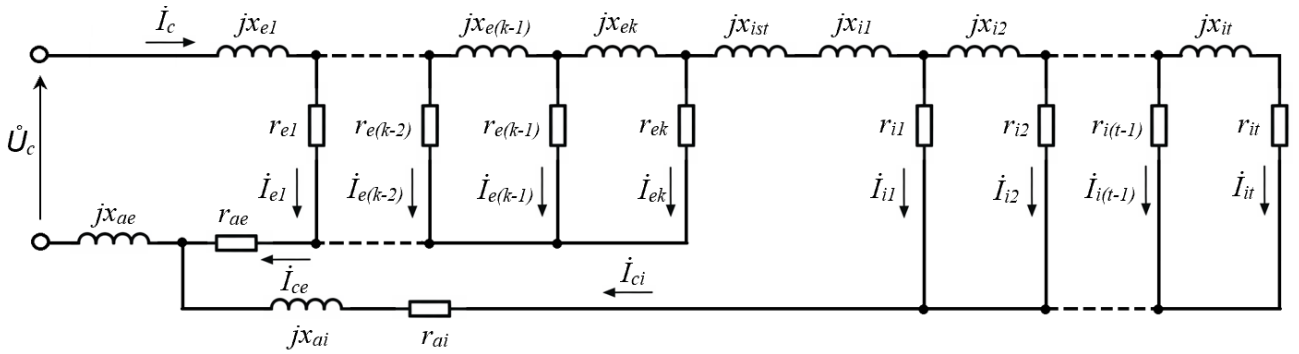


Figure 5. Multi-section equivalent circuit of a double-cage rotor with separate short-circuit rings

IV. SIMULATION RESULTS AND DISCUSSION

The influence of the rotor slot geometry on frequency-dependent parameters is first investigated using the correction coefficients k_r and k_x which are shown in Fig. 6 and Fig. 7. The coefficient k_r , which determines the apparent rotor resistance, increases with frequency due to the skin effect. This increase is highly sensitive to the geometry of the slots. Double-cage and inverted trapezoidal-slot configurations thus exhibit the highest values ($k_r > 3$), indicating a significant increase in effective resistance during start-up (at $f_r = 50\text{Hz}$). In contrast, the rectangular and trapezoidal configurations exhibit more moderate variations ($2.4 > k_r > 2$). At zero frequency, all configurations converge towards $k_r = 1$, confirming identical behavior in steady-state operation. This result highlights the ability of certain geometries to improve starting torque without increasing Joule losses during nominal operation. Conversely, the leakage reactance coefficient k_x decreases as the frequency increases (Fig. 7). This is due to the concentration of current in the region near the slot openings, which reduces the leakage flux and thus the rotor inductance itself. The double-cage structure shows the greatest decrease ($k_x \approx 0.45$) whilst the other geometries stabilise at higher values ($k_x \approx 0.65$). At low frequency ($f_r \approx 0$), all the curves coincide ($k_x = 1$). At high frequency, the decrease in reactance facilitates the current and improves starting.

The transient electromagnetic behavior is illustrated in Fig. 8, which shows the evolution of the electromagnetic torque over time for the various rotor configurations of the motors under study. The results show a significant increase in starting torque achieved by increasing the apparent rotor resistance through the skin effect, and an improvement in speed-up compared to a motor without the skin effect. The double-

cage structure supports higher torque and a faster dynamic response, confirming the benefits of combining high starting resistance with low resistance at steady state.

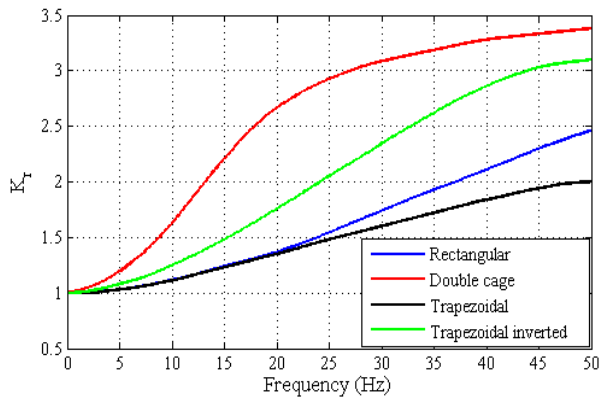


Figure 6. Variation of k_r as a function of frequency

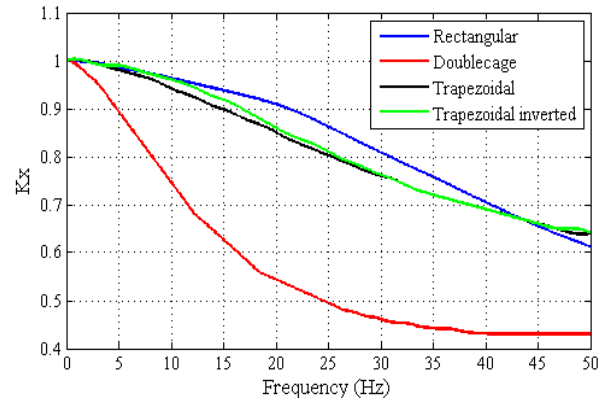


Figure 7. Variation of k_x as a function of frequency

Fig. 9 shows the variation in rotational speed over time. Although all configurations reach a similar operating speed (around 157 rad/s), the acceleration time varies significantly from one motor to another. The absence of skin effect results in slow speed dynamics (steady state reached at 3.5 s), whilst geometries that take the skin effect into account improve these dynamics with stabilisation (around 2.2 s) in the case of the rectangular and direct trapezoidal slots. The inverted trapezoidal slot accelerates the machine further (reaching speed in around 1.8 to 2 s), thanks to better current distribution and higher torque. The double-cage structure stands out for its fastest acceleration, reaching rated speed in between 1.3 and 1.5 seconds; this offers the best performance in terms of acceleration speed.

Finally, Fig. 10 illustrates the variation of the stator current over time for the various rotor structures studied. Although all configurations exhibit a high inrush current initially, the rate of decay depends heavily on the rotor structure. The slot without skin effect exhibits excessive current consumption at start-up and takes a long time to decrease (around 3.7 s). The rectangular and trapezoidal geometries improve the response, stabilising at around 2.2 s. The inverted trapezoidal configuration offers an even faster reduction in current. Meanwhile, the double-cage configuration ensures the fastest stabilisation of the current and effectively limits its peak, thereby reducing the electrical stresses on the power supply network. Overall, the results show that the geometry of the rotor slots, through its influence on the skin effect, plays a decisive role in starting performance. The double-cage structure appears to be the most effective solution, offering an optimal compromise between high starting torque, reduced inrush current and rapid transient response.

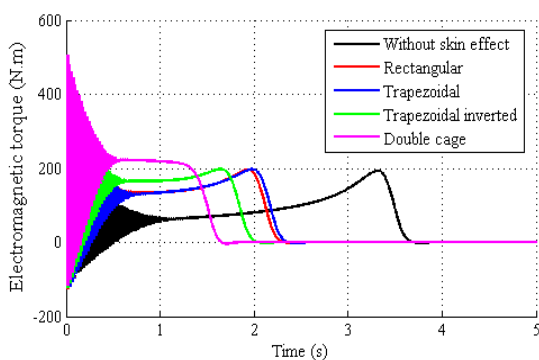


Figure 8. Machine torque

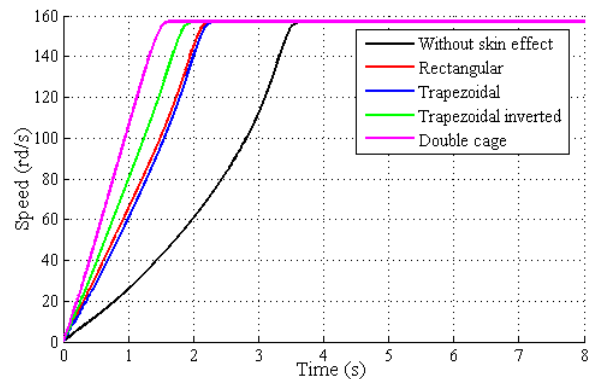


Figure 9. Machine speed

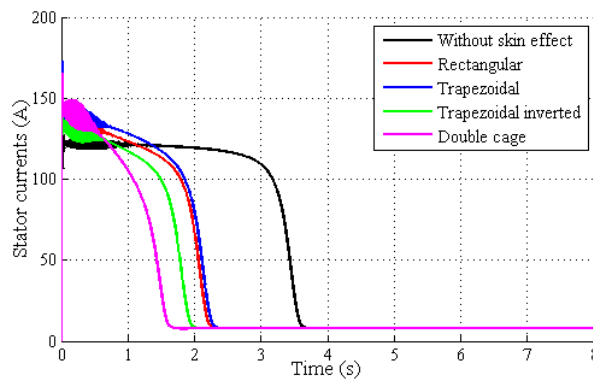


Figure 10. Stator Currents

V. CONCLUSION

This article has demonstrated the importance of the rotor's geometric design in improving the dynamic performance of the MAS. By using rigorous matrix modelling in the natural coordinate system (abc), we were able to accurately quantify the influence of deep bars on transient behavior. A comparison between rectangular, trapezoidal and double-cage shapes shows that adjusting the k_r and k_x coefficients allows for an ideal compromise between high starting torque, moderate inrush current and reduced start-up time. This approach offers an excellent balance between the accuracy of FEM and the computational efficiency of an analytical model, allowing the manufacturer to replace simplifying assumptions with an approach closer to the actual machine, which is essential for the diagnosis and design of modern industrial drives.

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