



New Approach to Achieve Best Control for Certain Class of Multi-variable Systems

Riad BENDIB^{1*}, ELakam MECHHOUD ², Hamid BENTARZI.³

¹ Laboratory of automatic, Petrochemical Department , SKIKDA 20 August 1955 University, Algeria

² Laboratory of automatic, ISTA institute, SKIKDA 20 August 1955 University, Algeria

³ Laboratory of Signals and Systems IGEE BOUMERDES university Algeria

Corresponding Author Email: r.bendib@univ-skikda.dz

ABSTRACT

Received: May, 26th, 2026

Accepted: July, 19th, 2026

Published: July, 23rd, 2026

Keywords:

MIMO, interactions, decoupling, DRMC, multi-loop Distillation column, Feed plate,

This study addresses the challenge of improving the dynamic behavior of a binary distillation column, which operates as a 2X2 Multi-Input Multi-Output (MIMO) system. We propose modifying the design of these complex systems to minimize interactions between the existing control loops. Specifically, this study focuses on the location of the feed plate—a design aspect that is generally finalized before control issues are even considered. To evaluate the degree of interaction between the loops, the dynamic magnitude criterion (DRMC) is utilized. The results demonstrate a significant difference in interaction levels depending on the feed plate's position. Consequently, selecting a location with minimal loop interaction can substantially optimize the column's overall performance.

1. INTRODUCTION

The conventional industrial approach to managing multivariable control problems relies on multi-loop control systems [1-5]. These systems consist of P conventional PI or PID controllers [11,15-17], where P represents the number of both controlled and manipulated variables. Multi-loop systems remain highly favored in industry because they are well understood and, in most cases, deliver a satisfactory level of performance when properly designed.

This design process can be broadly divided into two stages: (a) selecting the controlled and manipulated variables to be paired on a one-to-one basis, and (b) selecting the control law and tuning the controllers to achieve an acceptable level of performance.

The problem associated with control loop interactions can be stated as follows, adhering to Shinskey's definition (1979) [13]: if the steady-state or dynamic gain of a given controlled variable—in response to a specific manipulated variable—changes when other previously open loops are closed, then interaction exists within the system. If the controller in question was tuned while all other loops were in manual mode, that tuning becomes incorrect once the other loops are switched to automatic because their feedback loops alter the effective gain of that particular loop. Depending on the degree of interaction, this can lead to system instability or, at the very least, degraded performance.

Hence, one of the major criteria employed in the first design stage is the assessment of these interactions; specifically, controlled and manipulated variables must be paired to ensure that the resulting control loops exhibit weak interaction. Distillation columns (Figure 1) serve as prime examples of such complex MIMO systems [8]. Furthermore, distillation is a highly energy-intensive process, responsible for roughly 95% of all fluid separation in the chemical industry and accounting for an estimated 3% of global energy consumption [8-9]. To address these challenges, this paper presents a new approach to minimizing the degree of loop interaction in a binary distillation column by implementing targeted design modifications.

2. DYNAMIC MODELING OF DISTILLATION COLUMN

2.1 Mathematical description

The dynamic model of the distillation column [10–12] was developed by solving the overall mass and component balances for each tray. This model incorporates additional algebraic equations to account for steady-state energy balances, vapor-liquid equilibrium (VLE) relationships, and the Francis weir formula for liquid flow in the reboiler. To derive the analytical expressions, the following simplifying assumptions were made::

- Ideal equilibrium stages
- Constant relative volatility and molar flows.

Fundamental Process Equations

Overall Material Balance for Stage i

$$dM_i/dt = L_{i+1} - L_i + V_{i-1} - V_i \quad (1)$$

Light-Component Mass Balance per Stage

$$\frac{d(M_i x_i)}{dt} = L_{i+1} x_{i+1} + V_{i-1} y_{i-1} - L_i x_i - V_i y_i \quad (2)$$

Algebraic equations : The vapor composition y_i is related to the liquid composition x_i on the same stage through the vapor-liquid equilibrium (VLE) relationship::

$$y_i = (\alpha x_i) / (1 + (\alpha - 1) x_i) \quad (3)$$

Where α is the relative volatility.

Feed stage $i=NF$, assuming that the feed enters and mixes immediately with the liquid holdup on the feed stage

$$dM_i/dt = L_{i+1} - L_i + V_{i-1} - V_i + F \quad (4)$$

$$\frac{d(M_i x_i)}{dt} = L_{i+1} x_{i+1} + V_{i-1} y_{i-1} - L_i x_i - V_i y_i + F x_F \quad (5)$$

Total condenser: $i=NT$ ($M_{NT}=M_D$, $L_{NT}=L_T$)

$$dM_i/dt = V_{i-1} - L_i - D \quad (6)$$

$$\frac{d(M_i x_i)}{dt} = V_{i-1} y_{i-1} - L_i x_i - V_i y_i - D x_i \quad (7)$$

Reboiler : $i=1$ ($M_i=M_B$, $V_i=V_B=V$)

$$dM_i/dt = L_{i+1} - V_i - B \quad (8)$$

$$\frac{d(M_i x_i)}{dt} = L_{i+1} x_{i+1} - V_i y_i - B x_i \quad (9)$$

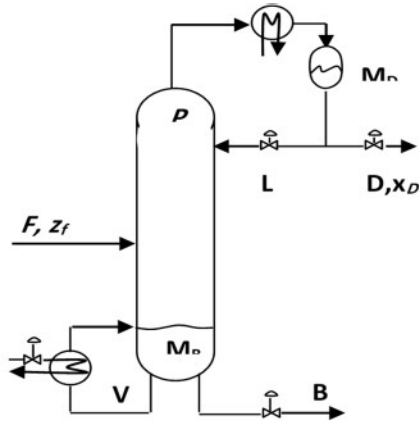


Figure.1 Distillation Column

2.2 The linear model

Linearization of the above equations around an operating state gives a linear model given in state space representation by:

$$\dot{x} = Ax + Bu \quad (10a)$$

$$y = Cx \quad (10b)$$

x , state vector $x^T = (x_d, x_7, \dots, x_1, x_b, P_c, V_s)$

u , input vector, $u^T = (L_r, P_f, F, z_f, P_{ss}, X_v)$

y , output vector and $y^T = (x_7, x_b)$

A, B and C are given in appendix

3. THE OPTIMAL FEED PLATE LOCATION

The feed-stage location is that location which, with a given set of other operating specifications, will result in the widest separation between x_D (the top product) and x_B (the bottom product) for a given number of stages. Or, if the number of stages is not specified, the optimum feed location is the one that requires the lowest number of stages to accomplish a specified separation between x_D and x_B . The optimum feed location can be estimated in the design stage using graphical or analytical methods.

4. ANALYSIS OF INTERACTIONS IN MIMO SYSTEM

Several methods are used to quantify the degree of interaction between the loops of a multivariable system. One of the most widely recognized methods is the Relative Gain Array (RGA), proposed by Bristol in 1966 [5,12]. The RGA provides a clear overview of interaction levels and offers a reliable indication of the optimal input-output pairing for multi-loop control approaches. However, its primary drawback is that it assumes ideal controllers by focusing strictly on steady-state behavior [5]. To address this limitation, the next section introduces the Dynamic Resiliency Magnitude Criterion (DRMC)—a powerful tool for assessing dynamic interactions that will be utilized throughout the remainder of this work.

4.1 The Dynamic Relative Magnitude Criterion

The DRMC consists of a matrix of frequency response plots, specifically mapping magnitude against log-frequency for each of its elements [4]. These elements are organized into an array of diagonal and off-diagonal components for graphical interpretation. Much like the Relative Gain Array (RGA) [5–7], the diagonal elements of the DRMC correlate the open-loop and closed-loop behavior of the fully controlled system.

4.1.1 The construction of DRMC elements

■ The diagonal elements

The diagonal elements are computed by considering the ratio of the transfer function where no controller is envisaged to the transfer function where all loops are closed (in automatic) except the loop i

$$\delta_{ii}(s) = \frac{\left(\frac{y_i(s)}{u_i(s)} \right)_{\text{all loops are open}}}{\left(\frac{y_i(s)}{u_i(s)} \right)_{\text{all loops are closed except loop } i}} \quad (11)$$

■ The off-diagonal elements

The off-diagonal elements are computed by calculating the ratio of the transfer function between output (i) and a set point for input (j) such that $i \neq j$ to the transfer function between the output (i) and its own set point. i.e.

$$\delta_{ij}(s) = \frac{\left(\frac{y_i(s)}{u_{jsp}(s)} \right)_{i \neq j}}{\left(\frac{y_i(s)}{u_{ksp}(s)} \right)_{k \text{ is const}}} \quad (12)$$

A.2 The interpretation of DRMC elements

The Dynamic Relative Magnitude Criterion (DRMC) explicitly illustrates how individual control loops respond to their own set-points via diagonal elements, and to external set-points via off-diagonal elements. By definition, system interaction caused by closed control loops remains weak for variable pairs whose relative magnitude approaches unity at the loop resonant frequencies. As the magnitude of the diagonal elements—representing the relationship between controlled variables y_i and manipulated variables u_i —departs from unity, a higher degree of interaction is to be expected. Hence these diagonal elements quantify how a single loop responds to its own set-point changes, they do not indicate the direction or magnitude of perturbations originating from other loops. Consequently, a single loop might exhibit ideal local performance yet still experience significant disturbances from a set-point change in an adjacent loop. To address this, the off-diagonal elements δ_{ij} are introduced to quantify the direction and magnitude of disturbances propagating between control loops. Specifically, δ_{ij} expresses the relative excitation of the j th loop compared to the response of the i th loop when a set-point change is introduced to the i th loop. Within the system's operating frequency range (i.e., the loop resonant frequencies), δ_{ij} must be much smaller than unity to ensure effective interaction or disturbance rejection. For a completely decoupled, non-interacting system, the diagonal matrix Δ_{de} equal or very close to unity,

5. SIMULATION AND RESULTS

Figures (2),(3),(4),(5), and (6) show the DRMC diagonal and off-diagonal elements for the cases where the feed plate is plate N° 4, N°5, N°7, N°3, and N°1 respectively. The resonant frequencies for the two loops for the above cases are listed in Table 1

Table 1. Resonant frequencies

The feed plate location	ω_{r1}	ω_{r2}
Plate N°4	0.221	0.06
Plate N°5	0.07	0.03
Plate N°7	0.3	0.04
Plate N°3	0.01	0.01
Plate N°1	0.01	0.01

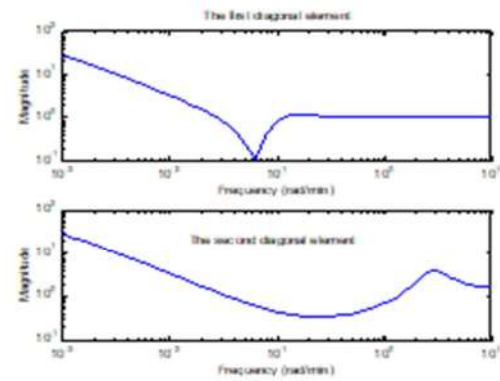


Figure .2a DRMC Diagonal elements stage No=4

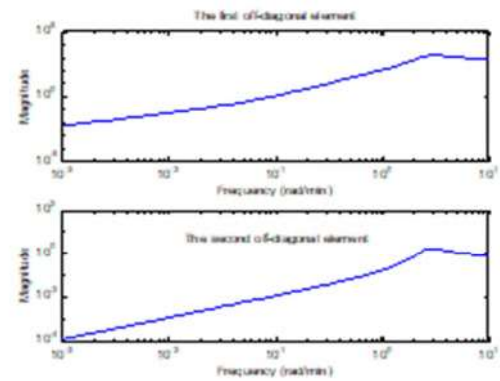


Figure .2b DRMC Off Diagonal elements stage No=4

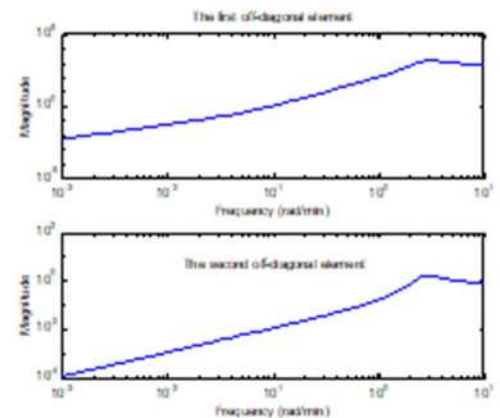


Figure .3a DRMC Diagonal elements stage No=5

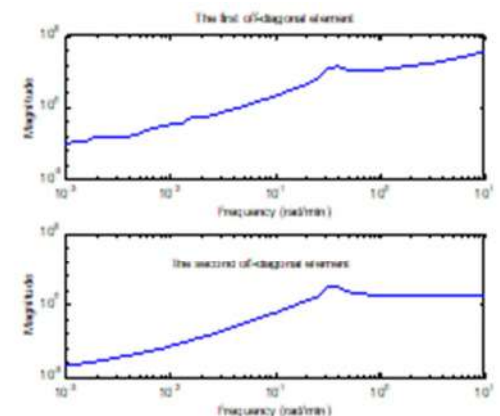


Figure .3b DRMC Off-Diagonal elements stage No=5

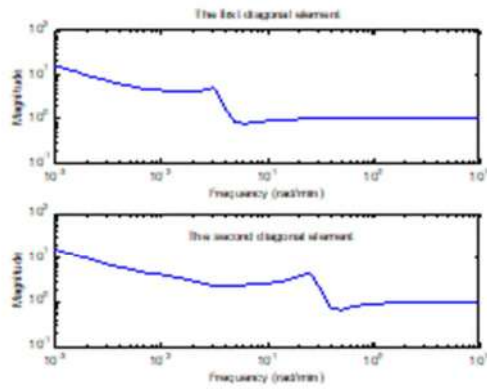


Figure 4a DRMC Diagonal elements stage No=7

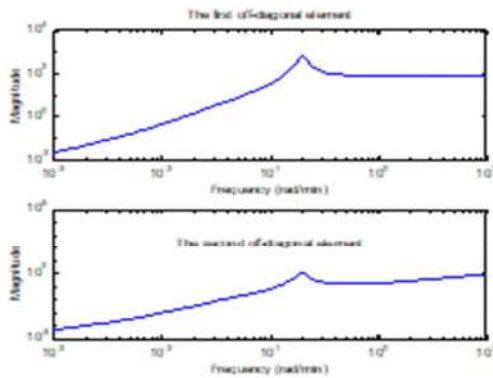


Figure 4a DRMC Off Diagonal elements stage No=7

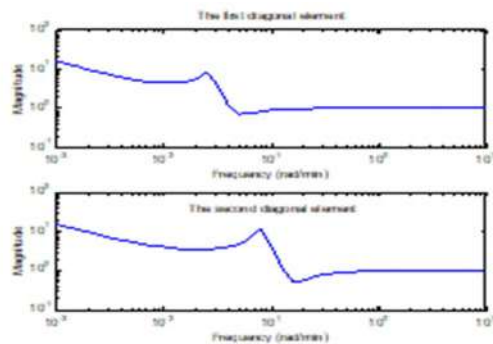


Figure 5a DRMC Diagonal elements stage No=3

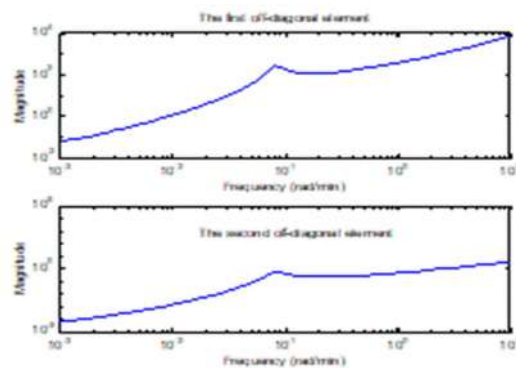


Figure 5a DRMC Off Diagonal elements stage No=3

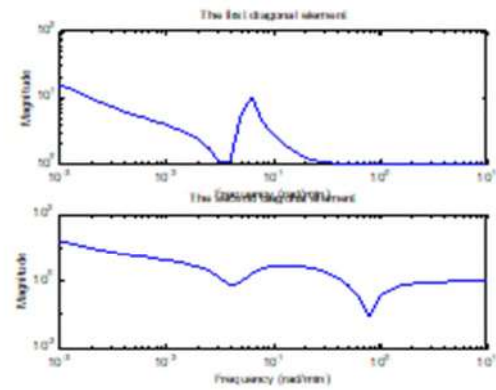


Figure 6a DRMC Diagonal elements stage No=1

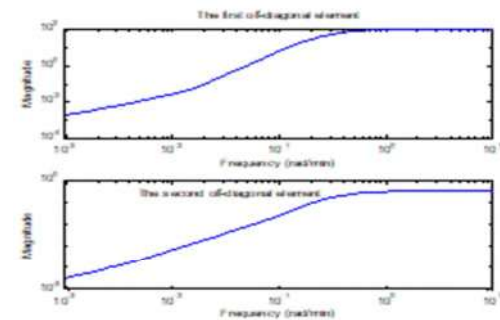


Figure 6a DRMC Off Diagonal elements stage No=1

The DRMC values are summarized in the following Table 2

Table 2. DRMC diagonal and off diagonal elements

Xf	The diagonal elements	
	δ_{11}	δ_{22}
4	1.1	1.2
5	1.3	3
7	1.2	3.5
3	4.7	4
1	4	36

From the above table we deduce that the location that gives a system with small degree of interactions is plate N°4 and whenever we go far from this optimum feed location the degree of interactions gets larger

6. CONCLUSION

In This paper highlights the critical interdependence between the physical design aspects and the dynamic control performance of a binary distillation column, focusing specifically on the impact of the feed plate location. Our results demonstrate that an appropriate choice of feed plate position minimizes the degree of interaction between control loops. This reduction in loop coupling makes the system far more suitable for a conventional, industry-standard multi-loop control

approach. Consequently, evaluating the dynamic behavior and loop interactions of a column is strongly recommended before finalizing any structural design decisions. Moving forward, this research opens up three key avenues for future study. First, this methodology can be extended from binary systems to more complex multi-component distillation columns where loop interactions are inherently more severe. Second, while this study validates the efficacy of a multi-loop framework, benchmarking these design modifications against advanced multivariable strategies, such as Model Predictive Control (MPC), remains an important next step. Finally, integrating this dynamic interaction analysis with steady-state energy optimization tools will help ensure that the chosen feed plate location balances both optimal control performance and minimal operational energy consumption.

Appendix A. NOTATION

- L_i and V_i respectively liquid and vapor flow from stage [Kmole/min]
- x_i and y_i - respectively liquid and vapor composition of light component on stage i [mole fraction].
- M_i liquid holdup on stage i [K mole].
- D and B distillate (top) and bottoms product flow rate [K mole/min].
- $L=LT$ and $V=VB$ reflux flow and boilup flow [K mole/min].
- F, zF feed rate [K mole/min] and feed composition [K mole].
- qF fraction of liquid in feed.
- i. stage n°. i=1=bottomn tray. NF= feed
- α Relative volatility between light and heavy component.
- τ_L time constant [min] for liquid flow dynamics on each stage.

Appendix B . STATE SPACE MODEL

$$A = \begin{bmatrix} -0.0135 & 0.0063 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.0290 & -0.0436 & 0.0168 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.0490 \\ 0 & 0.0290 & -0.0457 & 0.0212 & 0 & 0 & 0 & 0 & 0 & 0 & -0.0908 \\ 0 & 0 & 0.0290 & -0.0502 & 0.0290 & 0 & 0 & 0 & 0 & 0 & -0.1396 \\ 0 & 0 & 0 & 0.0270 & -0.0626 & 0.0346 & 0 & 0 & 0 & 0 & -0.1176 \\ 0 & 0 & 0 & 0 & 0.0356 & -0.0702 & 0.0446 & 0 & 0 & 0 & -0.1369 \\ 0 & 0 & 0 & 0 & 0 & 0.0356 & -0.0802 & 0.0548 & 0 & 0 & -0.1240 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.0356 & -0.0904 & 0.0628 & 0 & -0.0892 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.0081 & -0.0157 & 0 & -0.0123 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -15.240 & -5.0086 & 299.42 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.0004 & 0.0283 & 0.0084 & -0.6868 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0.0533 & -0.0005 & 0 & 0 & 0 & 0 \\ 0.0988 & -0.0009 & 0 & 0 & 0 & 0 \\ 0.1520 & -0.0014 & 0 & 0 & 0 & 0 \\ 0.1653 & -0.0019 & -0.1169 & 0.0086 & 0 & 0 \\ 0.1129 & -0.0011 & 0.1129 & 0 & 0 & 0 \\ 0.1023 & -0.0010 & 0.1023 & 0 & 0 & 0 \\ 0.0736 & -0.0007 & 0.0736 & 0 & 0 & 0 \\ 0.0102 & -0.0001 & 0.0102 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.6229 & 1.4409 \\ 0.0005 & 0 & 0.0005 & 0 & 0 & 0 \end{bmatrix}$$

$$C = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

REFERENCES

- [1] Halvorsen, I. Skogestad, S. (2000) "Distillation Theory" *Encyclopedia of Separation Science*, Academic Press 1117-1134.
- [2] Huang, H. Ohshima, M and Hashimoto, L (1994) "Dynamic interaction and multiloop control system design" *J. Proc. Cont.* 1994 volume 4, number 1 pages 15-27.
- [3] Khelassi A, Bendib, R, A. Benhalla (2012) "control configurations for optimal control for a binary distillation column" UKACC International conference on control Cardiff-UK 793-797. IEEE Xplorer.
- [4] A Khelassi, J Wilson, R Bendib "Assessment of Interaction in Process Control Systems" International Conference: 2004 - Dynamical Systems and Applications 1, 463-471
- [5] Riad BENDIB "Optimization and improvement of the overall performance of an industriale Plant " PHD thesis Université M'hamed Bougara de Boumerdès, Département Electrical engineering IGEE 2017
- [6] Riad Bendib, Hamid Bentarzi (2017) "Contribution to improve the dynamic control of a binary distillation column" International Journal of System Assurance Engineering and Management 663-675, springer
- [7] R. bendib, A. Khelassi (2007) "The Assessment of Interactions in Ratios Control Schemes for a Binary Distillation Column " International Journal Of Applied Science, Engineering And Technology , V 4, 217-222 WASET
- [8] King, C, J. (2013) "Separation Process" 2nd edition dover publication .
- [9] Luyben, W. (1990) "Process Modeling, Simulation and Control for Chemical Engineers " second edition McGraw hill New York.
- [10] Luyben, W. (1992) "Practical Distillation Control " Van Nostrand Reinhold.
- [11] Luo, X., Liu, Y, Xu, F. (2014) "Interaction analysis and decomposition principle for control structure design of large scale systems" *Chinise journal of chemical engineering* 22(1) 59-71. Elsevier
- [12] Perry. H. (2007) "Perry's Chemical Engineer' Handbook" eighth edition Mc Graw hill.
- [13] Shinskey. F. (1996) "Process-Control Systems" fourth edition Mc Graw hill.
- [14] Skogestad, S. Postlethwaite, I. (2007) "Multivariable Feed back Control Analysis and Design" John Wiley and sons
- [15] Skogestad, S "Simple Analytic Rules For Model Reduction And PID Controller Tuning " J.O. Process Control 2003, 291-309.
- [16] Sutanto, H and Widodo .RJ (2000) "Design And Implementation Control Decoupling For Binary Distillation Column" AEESEAP 2000
- [17] Takamatsu, T and Hashimoto, I "Non-Interacting Control System Design Of A Distillation Column By The Partial Model Matching " IFAC 1986