

Volatility Dynamics of the USD/DZD Exchange Rate: Assessment of Shocks and Leverage Effects

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Abstract:

This study investigates the volatility of the USD/DZD exchange rate through ARCH, GARCH, and asymmetric models such as EGARCH. The analysis identifies volatility clustering, deviations from normality with fat-tailed returns, and persistent long-memory effects in exchange rate fluctuations. The EGARCH model captures leverage effects and asymmetries, demonstrating that positive and negative shocks influence volatility differently. Diagnostic tests confirm the robustness of these models, highlighting their usefulness for forecasting exchange rate volatility, managing currency risk, and informing the design of effective monetary and financial policies.

Keywords: Volatility, USD/DZD, ARCH, GARCH, EGARCH, leverage effect, asymmetry, forecasting, exchange rate risk.

JEL Classification Codes : F31, C58, C53, C22

1. Introduction:

Financial time series are characterized by a well-documented phenomenon known as volatility clustering, whereby periods of pronounced fluctuations tend to be followed by similar periods, while phases of low variability persist over time. This empirical regularity was first identified by Mandelbrot (1963) and subsequently formalized through the development of the ARCH framework by Engle (1982) and its extension, the GARCH model, by Bollerslev (1986). In the realm of foreign exchange markets, exchange rate volatility—particularly since the breakdown of the Bretton Woods system—has remained a major concern for policymakers and market participants alike (Obstfeld&Rogoff, 1996). A clear understanding of volatility dynamics is therefore essential, as heightened exchange rate uncertainty can raise transaction costs and diminish the benefits of international trade(De Grauwe, 1994).

The expansion of global economic interactions, coupled with rising exchange rate volatility in recent decades, has intensified the need for effective strategies to manage exchange rate risk. This trend is largely driven by the growth of international trade and the increasing globalization of firms through foreign direct investment (Clark, 1973). According to the World Trade Organization, trade volumes conducted

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within the WTO framework increased by a factor of forty-four between 1950 and 2023(World Trade Organization, 2024).

In the case of Algeria, which has adopted a floating exchange rate regime since the 1990s, exchange rate fluctuations may have significant implications for monetary policy formulation and overall economic performance. These challenges are compounded by the rapid expansion of financial transactions and the growing complexity of financial markets, which can amplify vulnerabilities and systemic risks(Calvo & Reinhart, 2002). Consequently, accurate measurement of exchange rate volatility is crucial for assessing market expectations and policy-related uncertainty. Furthermore, conditional volatility is a key element in asset valuation, portfolio optimization, and risk management practices(Brooks, 2014).

Over the past several decades, volatility modeling has attracted substantial scholarly interest, leading to the development of numerous econometric approaches designed to capture the distinctive features of financial time series. A defining characteristic of such data is that conditional variance depends on past information. Among the most widely applied methods for modeling exchange rate volatility are the ARCH model introduced by Engle (1982) and its generalized version, the GARCH model proposed by Bollerslev (1986), which effectively capture the dynamics of conditional heteroskedasticity(Taylor, 1986).

Nonetheless, earlier empirical research—most notably by Meese and Rogoff (1983)—demonstrated that traditional structural exchange rate models, such as those developed by Dornbusch (1976) and Branson (1977), failed to consistently outperform a random walk model in out-of-sample forecasting. These findings provide further empirical support for the application of ARCH–GARCH-type models in exchange rate volatility and risk analysis.

Since many risk assessment frameworks rely on accurate estimates or forecasts of volatility—commonly measured by variance or standard deviation—the present study aims to examine the volatility behavior of the Algerian dinar against the US dollar using ARCH–GARCH models. By analyzing the key characteristics of exchange rate volatility, this article seeks to contribute to the formulation of sound economic policies that require robust and effective exchange rate management.

Exchange rate volatility generally describes short- to medium-term fluctuations in currency values. It is commonly quantified using the standard deviation of exchange rate returns(Fama, The behavior of stock-market prices, 1965). In this study, exchange rate movements are measured using the logarithmic returns of the USD/DZD exchange rate over the period spanning from January 3, 2000, to February 28, 2025. If y_t and y_{t-1} represent the DZD/USD exchange rates at times t and $t - 1$, respectively, r_t is

defined as:

$$r_t = \log\left(\frac{y_t}{y_{t-1}}\right) = \log(y_t) - \log(y_{t-1}) \dots \dots (1)$$

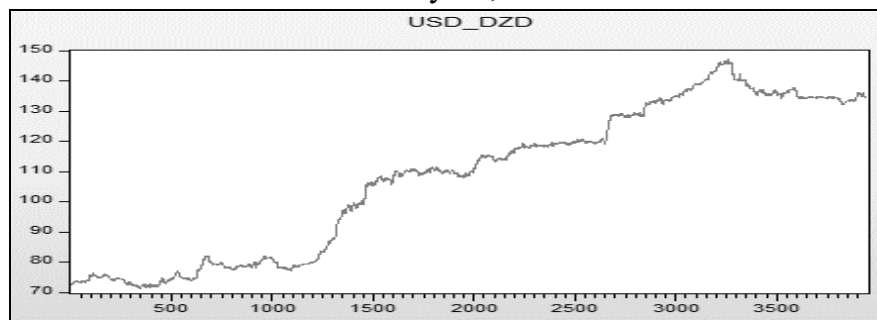
Here, r_t represents the **relative exchange rate return** at time t .

2. Volatility Clustering and Asymmetric (Leverage) Effects

A well-known characteristic of financial asset return series is volatility clustering, a phenomenon first highlighted by Mandelbrot (1963) and later rigorously modeled within the ARCH framework developed by Engle (1982). Volatility clustering describes the tendency for large absolute returns—regardless of sign—to be followed by similarly large movements, while periods of small fluctuations tend to persist. This dependence of conditional variance on past information constitutes a fundamental statistical feature of financial time series(Cont, 2001).

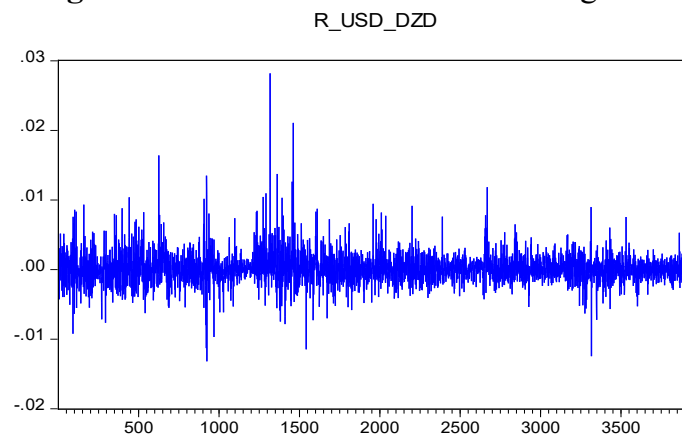
For the exchange rate series analyzed in this study, this behavior is clearly evident in the daily returns of the USD/DZD exchange rate over the sample period(Bank of Algeria, 2025). The series displays pronounced fluctuations, with episodes of elevated volatility occurring in clusters, which is indicative of the presence of ARCH and GARCH effects as described by Bollerslev (1986).

Fig.1.Historical evolution of the USD/DZD exchange rate from January 3, 2000, to February 28, 2025



Source:(Bank of Algeria, 2025)

Fig.2.Returns of the USD/DZD exchange rate



Source: Author's own elaboration

The ARCH and GARCH frameworks introduced by Engle (1982) and extended by Bollerslev (1986) are central tools for modeling such volatility dynamics. These models allow the conditional variance to be influenced by past innovations, thereby accounting for volatility persistence—that is, periods of heightened market uncertainty tend to be followed by continued high volatility.

In addition to volatility persistence, financial return series may exhibit a leverage effect, a phenomenon first discussed by Black (1976). This effect refers to the asymmetric response of volatility to shocks, whereby unfavorable news leads to a larger increase in future volatility than favorable news of equal magnitude. Such asymmetries are explicitly modeled in the GJR-GARCH specification proposed by Glosten, Jagannathan, and Runkle (1993), as well as in Nelson's (1991) EGARCH model, both of which allow positive and negative shocks to have distinct effects on conditional variance.

3. Fat Tails and Deviations from Normality:

Empirical analyses of financial return series frequently reveal pronounced departures from normality, particularly in the form of skewness and excess kurtosis, commonly referred to as fat tails (Fama, 1965; Mandelbrot, 1963). These distributional

characteristics constitute well-established stylized facts of financial data and have been extensively documented in the contemporary finance literature(Cont, 2001).

To formally examine the assumption of normality, this study employs the Jarque–Bera (1980) test, which evaluates whether the empirical distribution of returns is consistent with the symmetry and kurtosis implied by a normal distribution. Under the null hypothesis of normality, the Jarque–Bera statistic follows a chi-squared distribution with two degrees of freedom. The Jarque–Bera (JB) test statistic is computed as follows:

$$JB = \frac{T}{6} \left(S^2 \frac{(K - 3)^2}{4} \right) \dots \dots \dots (2)$$

where

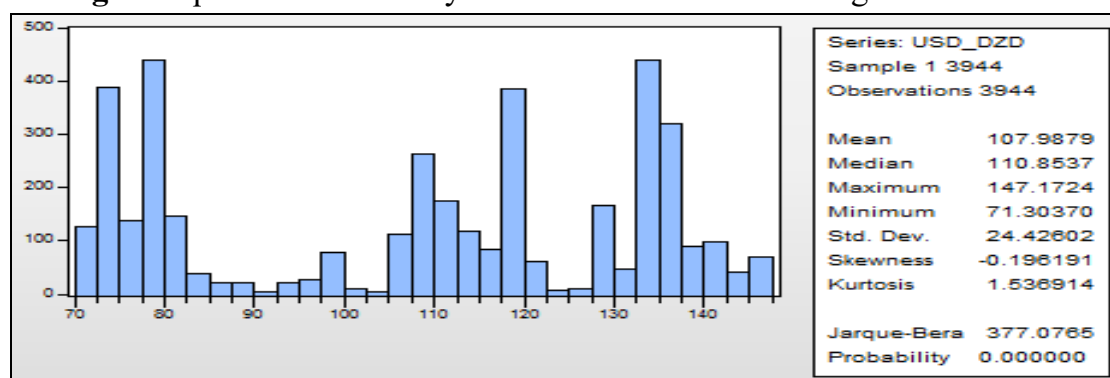
$$S = \frac{1}{T} \sum_{t=1}^T \left(\frac{x_t - \bar{x}}{\hat{\sigma}} \right)^3 \dots \dots \dots (3)$$

where S denotes the sample skewness, representing the third central moment and providing a measure of the asymmetry of the return distribution. The fourth central moment, captured by kurtosis, reflects the degree of peakedness of the distribution. T refers to the sample size, $\bar{x}$ is the sample mean, and $\hat{\sigma}$ denotes the estimated standard deviation. For sufficiently large samples, the Jarque–Bera statistic asymptotically follows a chi-squared (χ^2) distribution with two degrees of freedom. The null hypothesis of the test states that the observed data are normally distributed.

For reference, the probability density function of a normal distribution is given by:

$$f(x_t) = \frac{1}{\sigma\sqrt{2\pi}} \exp \left(-\frac{1}{2} \left(\frac{x_t - \bar{x}}{\sigma} \right)^2 \right) \dots \dots \dots (4)$$

In this study, the evidence presented in Figure 3 leads to the rejection of the null hypothesis at the 5% significance level, indicating that the return series does not conform to a normal distribution. The empirical distribution is characterized by a sharper peak and heavier tails compared to the normal density, features commonly linked to the occurrence of extreme observations and nonlinear volatility behavior (Brooks, 2014). In light of these departures from normality, an appropriate volatility modeling approach must accommodate fat-tailed distributions. Accordingly, this study adopts the Generalized Error Distribution (GED), which is frequently employed in the estimation of GARCH-type models when return series display leptokurtosis (Nelson, 1991).

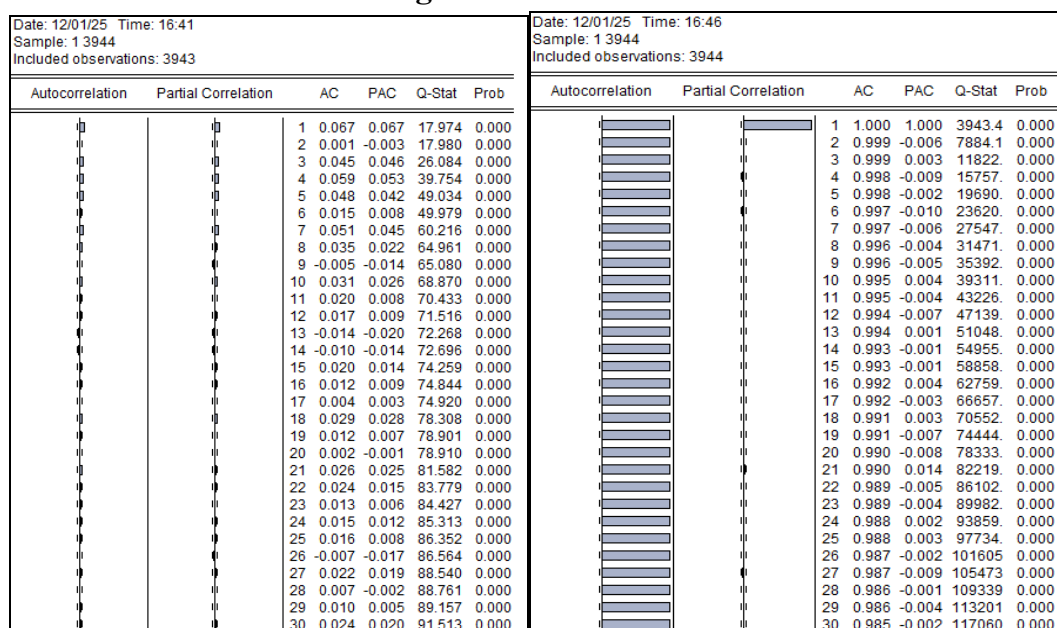
Fig.3. Jarque-Bera Normality Test for USD/DZD Exchange Rate Returns

Source : Author's own calculations

4. Serial Correlation and Unit Root Properties:

Serial correlation in financial return series represents a key stylized fact of market behavior (Cont, 2001). The presence of significant autocorrelation indicates temporal dependence, which, if ignored, may result in an underestimation of risk in return forecasts (Brooks, 2014).

To formally assess this dependence, the Ljung-Box Q statistic (1978) is employed. This test evaluates the null hypothesis that there is no autocorrelation up to a specified number of lags. A p-value below 5% leads to rejection of the null hypothesis, indicating significant temporal dependence. For the USD/DZD return series analyzed here, the results illustrated in Figure 4 reveal persistent autocorrelation, with statistically significant coefficients even at higher-order lags, confirming the presence of strong temporal dependence.

Fig.4. ACF and PACF Functions:

Source : Author's own calculations

The correlogram of the series displays the characteristic pattern of a non-stationary process, with autocorrelations that diminish gradually toward zero, suggesting behavior consistent with a random walk (Fama, Efficient capital markets: A review of theory and empirical work, 1970). This pattern is especially common for nominal exchange rates, whose logarithmic values typically contain a unit root, while their first differences are stationary (Dickey & Fuller, 1979), (Phillips & Perron, 1988).

To formally assess stationarity, the Augmented Dickey-Fuller (ADF) test is applied to both the levels and first differences of the USD/DZD series. As reported in Table 1, the test fails to reject the unit root hypothesis at the level of the exchange rate, confirming its non-stationary nature. Conversely, Table 2 shows that the null hypothesis is rejected for the first differences, indicating stationarity. These findings are in line with the empirical literature on exchange rates (Engle & Granger, 1987; Baillie et al., 1996), which emphasizes the coexistence of long memory in volatility with non-stationarity in price levels.

Tab.1. Augmented Dickey-Fuller (ADF) Test for the USD/DZD Exchange Rate at Level

Null Hypothesis: USD_DZD has a unit root Exogenous: Constant, Linear Trend Lag Length: 1 (Automatic - based on SIC, maxlag=30)		
	t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic	-0.381525	0.9882
Test critical values: 1% level	-3.960367	
5% level	-3.410945	
10% level	-3.127281	
*MacKinnon (1996) one-sided p-values.		

Source : Author's own calculations

Tab.2. Augmented Dickey-Fuller (ADF) Test for the USD/DZD Exchange Rate in First Difference

Null Hypothesis: D(USD_DZD) has a unit root Exogenous: Constant, Linear Trend Lag Length: 0 (Automatic - based on SIC, maxlag=30)		
	t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic	-58.57034	0.0000
Test critical values: 1% level	-3.960367	
5% level	-3.410945	
10% level	-3.127281	
*MacKinnon (1996) one-sided p-values.		

Source : Author's own calculations

These findings imply that a suitable volatility model for exchange rates should be capable of capturing several key features simultaneously: serial correlation, heteroskedasticity, fat-tailed distributions, and potential long memory. The ARCH/GARCH class of models is particularly well-suited for this purpose, as it effectively accommodates these statistical characteristics observed in financial return series.

5. The ARCH (Autoregressive Conditional Heteroskedasticity) Model:

The ARCH (Autoregressive Conditional Heteroskedasticity) model, introduced by Engle (1982), was designed to capture a fundamental feature of financial time series: **conditional heteroskedasticity**, or the tendency for the variance of errors to change over time. This framework provided a formal approach to modeling **volatility clustering**, a phenomenon first highlighted by Mandelbrot (1963) and consistently observed across a wide range of financial asset returns (Cont, 2001).

Let σ_t^2 represent the conditional variance of the stochastic term u_t :

$$\sigma_t^2 = \text{var}(u_t/u_{t-1}, u_{t-2}, \dots) = E \left[(u_t - E(u_t))^2 / u_{t-1}, u_{t-2}, \dots \right] \dots \dots \dots (5)$$

Assuming that $E(u_t) = 0$, equation (5) can be rewritten as:

$$\sigma_t^2 = \text{var}(u_t/u_{t-1}, u_{t-2}, \dots) = E[u_t^2 / u_{t-1}, u_{t-2}, \dots] \dots \dots \dots (6)$$

Equation (6) indicates that the conditional variance of u_t is equal to the conditional expectation of u_t . In the ARCH framework, autocorrelation in volatility is captured by allowing the conditional variance to depend on the **most recent squared innovation**, as expressed by:

$$\sigma_t^2 = \gamma_0 + \gamma_1 u_{t-1}^2 \dots \dots \dots (7)$$

In this context, u_t is assumed to follow a normal distribution with zero mean and a conditional variance given by $\text{var}(u_t) = (\gamma_0 + \gamma_1 u_{t-1}^2)$, that is, $u_t \sim N(0; \gamma_0 + \gamma_1 u_{t-1}^2)$, where γ_0 and γ_1 are parameters to be estimated. Equation (7) illustrates that the variance of u_t follows an **ARCH(1) process**, since the conditional variance depends on only one lag of the squared residual. In the ARCH framework, the mean equation describing the evolution of r_t over time can adopt various forms (Brooks, 2014; Enders, 2014). For the purposes of this study, the full model is specified as:

$$\begin{aligned} r_t &= \mu + u_t \text{ with } u_t \sim N(0; \sigma_t^2) \\ \sigma_t^2 &= \gamma_0 + \gamma_1 u_{t-1}^2 \dots \dots \dots (8) \\ \gamma_0 &\geq 0 \text{ and } \gamma_1 \geq 0 \end{aligned}$$

Because σ_t^2 represents a conditional variance, it must always take strictly positive values. Moreover, the error variance can be influenced not just by a single lagged squared innovation but by multiple lagged squared errors. Accordingly, model

(8) can be generalized to an **ARCH(p) process**, in which the conditional variance depends on **p** past squared residuals:

$$r_t = \mu + u_t$$

$$\begin{aligned} u_t &\sim N(0; \sigma_t^2) \\ \sigma_t^2 &= \gamma_0 + \gamma_1 u_{t-1}^2 + \gamma_2 u_{t-2}^2 + \dots + \gamma_p u_{t-p}^2 \dots \dots \dots (9) \\ \gamma_i &\geq 0, \forall i = \overline{0, p} \end{aligned}$$

In this test, the null hypothesis posits the absence of ARCH effects, whereas the alternative hypothesis asserts that ARCH effects are present. Formally:

$$H_0: \gamma_1 = \gamma_2 = \dots = \gamma_p = 0 \dots \dots \dots (10)$$

Because σ_t^2 is not directly observable, Gujarati & Porter (year) and Engle (1982) suggest estimating it by performing the following regression:

$$\hat{\sigma}_t^2 = \hat{u}_t^2 = \hat{\gamma}_0 + \hat{\gamma}_1 \hat{u}_{t-1}^2 + \hat{\gamma}_2 \hat{u}_{t-2}^2 + \dots + \hat{\gamma}_p \hat{u}_{t-p}^2 \dots \dots \dots (11)$$

This approach allows for a straightforward test of the null hypothesis, which states that no ARCH effects are present. Since the ARCH(1) model is a particular case of the more general ARCH(p) framework, the same testing procedure and interpretation apply to ARCH(1) as to ARCH(p).

6. The GARCH (Generalized Autoregressive Conditional Heteroskedasticity)

Model:

The GARCH (Generalized Autoregressive Conditional Heteroskedasticity) model, introduced by Bollerslev (1986), represents a more parsimonious extension of Engle's (1982) ARCH framework. It was developed to address a key limitation of ARCH models: the requirement of a large number of lags to adequately capture volatility dynamics, which increases the number of parameters and complicates estimation (Brooks, 2014).

Among its variants, the GARCH(1,1) model is the most widely applied in empirical research and defines the conditional variance as:

$$\begin{aligned} r_t &= \mu + u_t \text{ with } u_t \sim N(0; \sigma_t^2) \text{ and} \\ \sigma_t^2 &= \gamma_0 + \gamma_1 u_{t-1}^2 + \lambda \sigma_{t-1}^2 \dots \dots \dots (12) \end{aligned}$$

Model (12) shows that the conditional variance of u_t depends not only on the squared innovation from the previous period but also on the conditional variance from the preceding period. As noted by Brooks (2014), the GARCH framework allows the current conditional variance to be interpreted as a weighted combination of a long-term average variance, the previous period's shock information, and the past period's conditional variance. The GARCH(1,1) specification is both the simplest and the most widely applied model within the GARCH family. Nevertheless, this framework can be generalized to a GARCH(p, q) model, in which the current conditional variance is determined by **p** lagged squared errors and **q** lagged conditional variance terms.

$$\sigma_t^2 = \gamma_0 + \gamma_1 u_{t-1}^2 + \dots + \gamma_p u_{t-p}^2 + \lambda_1 \sigma_{t-1}^2 + \dots + \lambda_q \sigma_{t-q}^2 \dots \dots \dots (13)$$

Furthermore, the constraints $\gamma_0, \gamma_1, \dots, \gamma_p \geq 0$ and $\lambda_1, \dots, \lambda_q \geq 0$ are imposed to guarantee that the conditional variance σ_t^2 remains strictly positive. In practice, the **GARCH(1,1) specification** is typically sufficient to capture the volatility clustering observed in financial return series.

7. Estimation of ARCH–GARCH Models:

The parameters of ARCH and GARCH models are typically estimated using the **maximum likelihood (ML) method**, a widely employed technique in time series analysis that identifies the parameter values that maximize the likelihood of observing the sample data (Hamilton, 1994). This approach is particularly appropriate for conditional heteroskedasticity models, as it explicitly accounts for the time-varying structure of the variance. Under conditional heteroskedasticity, the mean and variance equations of an **AR(1)–GARCH(1,1)** model can be expressed as follows:

$$r_t = \mu + \varphi r_{t-1} + u_t \text{ with } u_t \sim N(0; \sigma_t^2) \text{ and} \\ \sigma_t^2 = \gamma_0 + \gamma_1 u_{t-1}^2 + \lambda \sigma_{t-1}^2$$

where the conditional variance σ_t^2 evolves over time. Assuming that the errors are normally distributed, the log-likelihood function to be maximized is given by:

$$L = -\frac{1}{2} \log(2\pi) - \frac{1}{2} \sum_{t=1}^T \log(\sigma_t^2) - \frac{1}{2} \sum_{t=1}^T (r_t - \mu - \varphi r_{t-1})^2 / \sigma_t^2 \dots \dots \dots (14)$$

Here, $\left(-\frac{1}{2} \log(2\pi)\right)$ is a constant with respect to the parameters, T denotes the

total number of observations, and r_{trt} represents the return of the exchange rate. As noted by Brooks (2014), maximizing the log-likelihood function involves minimizing the terms $\sum_{t=1}^T \log(\sigma_t^2)$, $\sum_{t=1}^T (r_t - \mu - \varphi r_{t-1} - u_t)^2 / \sigma_t^2$, and the conditional

variance. However, the assumption of normally distributed errors fails to capture the heavy tails typically observed in exchange rate returns. To address this issue and properly model the leptokurtic nature of the data, the **Generalized Error Distribution (GED)** is often employed, as discussed earlier.

8. Leverage Effect and Asymmetric GARCH Model:

Standard GARCH models that assume normally distributed errors often fail to adequately capture the heavy tails observed in financial return series (Cont, 2001). To address this limitation, this study employs the **Generalized Error Distribution (GED)**, originally proposed by Nelson (1991), which is widely used in GARCH-type estimations when returns exhibit pronounced leptokurtosis.

The likelihood function can be maximized using various numerical optimization techniques, such as the **Newton–Raphson or quasi-Newton** methods. In cases where the return distribution deviates from normality, robust **quasi-maximum likelihood**

(QML) estimators, as developed by Bollerslev and Wooldridge (1992), provide reliable parameter estimates.

The **GJR-GARCH model** extends the standard GARCH framework by including an additional term to account for potential asymmetries in volatility, formally expressed as:

$$\begin{aligned}\sigma_t^2 &= \gamma_0 + \gamma_1 u_{t-1}^2 + \lambda \sigma_{t-1}^2 + \theta u_{t-1}^2 I_{t-1} \dots \dots \dots (15) \\ I_{t-1} &= \begin{cases} 1 & \text{if } u_{t-1} < 0 \\ 0 & \text{otherwise} \end{cases} \\ \gamma_0 &> 0, \gamma_1 > 0, \lambda \geq 0 \text{ and } \gamma_1 + \theta > 0, \text{ for } \theta > 0\end{aligned}$$

In addition, **asymmetric power ARCH models**, such as the **Exponential GARCH (EGARCH)**, are commonly used in empirical finance. For instance, Nelson (1991) applied the exponential formulation to model U.S. stock market returns, while Hsieh (1989), Theodossiou (1998), and Koutmos&Theodossiou (1997) employed EGARCH specifications to capture the dynamics of exchange rate series. The **EGARCH model** is formally specified as follows:

$$\ln(\sigma_t^2) = \omega + \lambda \ln(\sigma_{t-1}^2) + \theta \frac{u_{t-1}}{\sqrt{\sigma_{t-1}^2}} + \gamma \left[\frac{|u_{t-1}|}{\sigma_{t-1}^2} - \sqrt{\frac{2}{\pi}} \right] \dots \dots \dots (16)$$

In this specification, σ_t^2 represents the **conditional variance**, providing a one-step-ahead forecast of volatility based on all relevant past information. The parameters ω, λ, θ and γ are to be estimated. No constraints are imposed on ω, θ and γ , whereas λ must lie between 0 and 1 to ensure stationarity. Unlike the standard GARCH model, the **logarithmic formulation of the variance** guarantees its positivity.

The parameter γ reflects the magnitude or symmetric component of volatility—the traditional “GARCH effect”—while λ measures the **persistence of conditional volatility**. The **leverage effect**, or asymmetry, is captured by θ . A significant negative value of θ indicates the presence of leverage: negative shocks (bad news) increase volatility more than positive shocks of the same magnitude. When $\theta = 0$, the model is symmetric. Conversely, $\theta > 0$ would imply that positive innovations have a larger impact on volatility than negative ones. In practice, both positive and negative shocks tend to elevate stock market volatility.

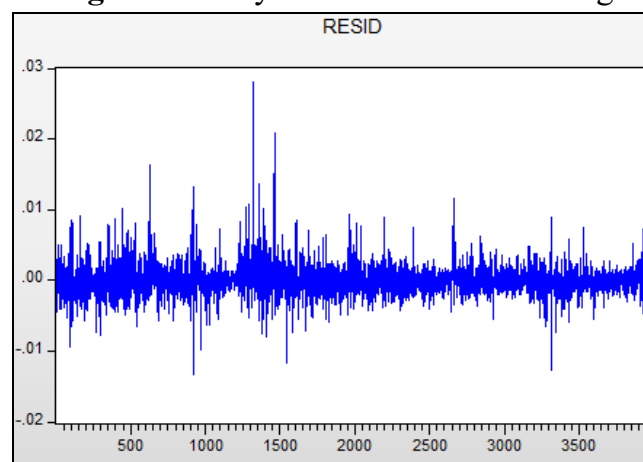
Overall, this estimation approach is well-established in the econometric literature and is particularly suitable for modeling financial time series volatility. In the

context of this study, it allows for a detailed assessment of the **USD/DZD exchange rate volatility**, including potential asymmetries and long-memory effects.

9. Volatility Clustering in the Residuals:

Prior to estimating the ARCH and GARCH models, the exchange rate series is analyzed to identify its statistical characteristics and to verify whether it satisfies the necessary conditions for ARCH–GARCH modeling, specifically **volatility clustering** and the presence of ARCH effects in the residuals. Figure 5 presents a visual examination of the residuals, highlighting volatility clustering. The plot reveals that large and small errors tend to occur in consecutive clusters, indicating that substantial returns are typically followed by other large returns, while minor returns follow other minor returns. Put differently, periods of high exchange rates are often succeeded by additional periods of elevated rates, whereas low exchange rates are likely to be followed by similarly low rates. This observed clustering of volatility confirms that the residuals are **conditionally heteroskedastic**, making them suitable for modeling with ARCH and GARCH frameworks.

Fig.5.Volatility and Residual Clustering



Source: Author's own calculations

10. Heteroskedasticity Testing: The ARCH Effect

The ARCH effect reflects a specific form of heteroskedasticity, often interpreted as serial dependence in the variance. This phenomenon becomes apparent when the variance of a variable exhibits clustering, forming patterns driven by underlying factors. Because exchange rate volatility serves as a proxy for financial risk, the presence of an ARCH effect can be interpreted as an indicator of the risk associated with the asset. With this in mind, we reconsider the model as follows:

$$r_t = \mu + u_t, \quad u_t \sim N(0; \gamma_0 + \gamma_1 u_{t-1}^2) \dots \dots \dots (17)$$

The error term is assumed to follow a **normal distribution with zero mean**, and its **conditional variance** depends on lagged squared innovations. Specifically, the

conditional variance is defined as the variance of the error term **given its past values** at one lag, two lags, and so forth:

$$\sigma_t^2 = \text{var}(u_t/u_{t-1}, u_{t-2} \dots) = E(u_t^2/u_{t-1}, u_{t-2} \dots) \dots \dots \dots (18)$$

Here, σ_t^2 denotes the **conditional variance** of the error term. The presence of the

ARCH effect is then captured by modeling it as follows:

$$\sigma_t^2 = \gamma_0 + \gamma_1 u_{t-1}^2 \dots \dots \dots (19)$$

This specification corresponds to an **ARCH(1) model**.

To model the volatility of the exchange rate, as illustrated in Figure 6, we consider the simple model:

$$r_t = \mu + u_t$$

where r_t represents the percentage change in the exchange rate and u_t is the error term. Applying this framework to the daily USD/DZD exchange rate series spanning from January 3, 2000, to February 28, 2025, the following OLS regression results are obtained:

$$\hat{r}_t = 0,000156$$

$$t = (4,41) \quad \dots \dots \dots (20)$$

$$D.W = 1,8638$$

It should be noted that, aside from the intercept, the model does not include any additional explanatory variables. The intercept represents the **average percentage return** of the exchange rate series, or equivalently, the mean of r_t . Over the period

under study, this implies that the **average daily return** of the USD/DZD exchange rate is approximately 0.000156 percent.

The results of the **ARCH effect test** applied to the residuals of model (20) are summarized in Table 3.

Tab.3.Arch Test and LM Test

Heteroskedasticity Test: ARCH					Breusch-Godfrey Serial Correlation LM Test				
F-statistic	112.0944	Prob. F(1,3940)	0.0000		F-statistic	18.04240	Prob. F(1,3941)	0.0000	
Obs*R-squared	109.0489	Prob. Chi-Square(1)	0.0000		Obs*R-squared	17.96929	Prob. Chi-Square(1)	0.0000	
Test Equation: Dependent Variable: RESID*2 Method: Least Squares Date: 12/07/25 Time: 16:36 Sample (adjusted): 3 3944 Included observations: 3942 after adjustments					Test Equation: Dependent Variable: RESID Method: Least Squares Date: 12/10/25 Time: 17:12 Sample: 2 3944 Included observations: 3943 Presample missing value lagged residuals set to zero.				
Variable	Coefficient	Std. Error	t-Statistic	Prob.	Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	4.12E-06	3.09E-07	13.35379	0.0000	C	5.29E-08	3.54E-05	0.001495	0.9988
RESID*(-1)	0.166319	0.015709	10.58747	0.0000	RESID*(-1)	0.067524	0.015897	4.247635	0.0000
R-squared	0.027663	Mean dependent var	4.95E-06		R-squared	0.004557	Mean dependent var	-7.58E-20	
Adjusted R-squared	0.027417	S.D. dependent var	1.90E-05		Adjusted R-squared	0.004305	S.D. dependent var	0.002225	
S.E. of regression	1.88E-05	Akaike info criterion	-18.92940		S.E. of regression	0.002220	Akaike info criterion	-9.382300	
Sum squared resid	1.39E-06	Schwarz criterion	-18.92621		Sum squared resid	0.019419	Schwarz criterion	-9.379115	
Log likelihood	37311.84	Hannan-Quinn criter.	-18.92827		Log likelihood	18499.20	Hannan-Quinn criter.	-9.381170	
F-statistic	112.0944	Durbin-Watson stat	2.011523		F-statistic	18.04240	Durbin-Watson stat	1.998315	
Prob(F-statistic)	0.000000				Prob(F-statistic)	0.000022			

Source : Author's own calculations

The results of the **ARCH-LM test** offer compelling evidence to reject the null hypothesis for the exchange rate series. The test is significant at the 1% level, indicating that the null hypothesis of no ARCH effects can be confidently rejected. This outcome confirms the presence of **ARCH effects** in the residuals of the mean equation, justifying the use of ARCH/GARCH models to capture the conditional heteroskedasticity in the series.

11. Estimation of the GARCH(1,1) Model:

The **GARCH(1,1) model** is estimated for the daily percentage changes of the exchange rate using the sample data, and the estimation results are presented below:

$$\hat{\sigma}_t^2 = (4,86E - 8) + 0,065 u_{t-1}^2 + 0,929 \hat{\sigma}_{t-1}^2$$

$$t = \quad (0,000)(0,003)(0,003) \quad \dots \dots \dots (21)$$

$$DW = 1,86$$

The estimated coefficients for the **lagged squared residual** (u_{t-1}^2) and the **lagged conditional variance** (σ_{t-1}^2) in the conditional variance equation are both statistically significant at the 1% level. This indicates that past volatility has a strong explanatory power for current volatility.

A commonly used measure of **volatility persistence** in the GARCH model is the sum of these two coefficients. In this study, the sum equals 0.994, which is very close to one, suggesting that shocks to volatility are highly persistent. In practical terms, this means that large positive or negative returns tend to lead to elevated volatility for an extended period, consistent with the prolonged volatility clustering illustrated in Figure 6. Additionally, the positive values of the GARCH(1,1) coefficients satisfy the model's **non-negativity condition**, ensuring that the conditional variance remains positive.

Tab.4. Estimation Results of the GARCH(1,1) Model

Dependent Variable: R_USD_DZD				
Method: ML ARCH - Normal distribution (BFGS / Marquardt steps)				
Date: 12/08/25 Time: 09:40				
Sample (adjusted): 2 3944				
Included observations: 3943 after adjustments				
Convergence achieved after 31 iterations				
Coefficient covariance computed using outer product of gradients				
Presample variance: backcast (parameter = 0.7)				
GARCH = C(2) + C(3)*RESID(-1)^2 + C(4)*GARCH(-1)				
Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	9.24E-05	2.89E-05	3.201521	0.0014
Variance Equation				
C	4.86E-08	6.28E-09	7.737796	0.0000
RESID(-1)^2	0.065314	0.003011	21.69215	0.0000
GARCH(-1)	0.928767	0.003345	277.6395	0.0000
R-squared	-0.000824	Mean dependent var	0.000156	
Adjusted R-squared	-0.000824	S.D. dependent var	0.002225	
S.E. of regression	0.002226	Akaike info criterion	-9.555405	
Sum squared resid	0.019524	Schwarz criterion	-9.549034	
Log likelihood	18842.48	Hannan-Quinn criter.	-9.553145	
Durbin-Watson stat	1.862237			

Source : Author's own calculations

12. Leverage Effects and Asymmetric Results:

To account for potential **asymmetric effects** and the **leverage effect** in the USD/DZD exchange rate, the **EGARCH model** is estimated. According to the model's specification, the parameter θ is expected to be positive and statistically

significant if asymmetry is present. The results of this estimation are summarized in the following regression equation (23):

$$\hat{\sigma}_t^2 = -0,297 + 0,137 \hat{u}_{t-1}^2 + 0,015 \sigma_{t-1}^2 + 0,984 u_{t-1}^2 I_{t-1} \\ t = (0,031)(0,007)(0,004)(0,002) \quad \dots \dots \dots (22)$$

$D.W = 1,86$

All estimated parameters are statistically significant at the 1% level. Concerning **asymmetric volatility**, the estimated coefficient θ is positive, indicating that **positive shocks** to the USD/DZD exchange rate generate a greater increase in conditional variance in the subsequent period compared with negative shocks of the same size. This result confirms the presence of **leverage effects** in the exchange rate series.

Tab.5. Estimation Results of the EGARCH(1,1) Model (Leverage Effect)

Dependent Variable: R_USD_DZD				
Method: ML ARCH - Generalized error distribution (GED) (BFGS / Marquardt steps)				
Date: 12/11/25 Time: 15:35				
Sample (adjusted): 2 3944				
Included observations: 3943 after adjustments				
Convergence achieved after 67 iterations				
Coefficient covariance computed using outer product of gradients				
Presample variance: backcast (parameter = 0.7)				
LOG(GARCH) = C(2) + C(3)*ABS(RESID(-1))/@SQRT(GARCH(-1))) + C(4)*RESID(-1)/@SQRT(GARCH(-1)) + C(5)*LOG(GARCH(-1))				
Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	6.50E-05	2.36E-05	2.751374	0.0059
Variance Equation				
C(2)	-0.297324	0.054932	-5.412556	0.0000
C(3)	0.137584	0.014720	9.346868	0.0000
C(4)	0.015242	0.008337	1.828156	0.0675
C(5)	0.984199	0.004068	241.9235	0.0000
GED PARAMETER	1.135517	0.026735	42.47342	0.0000
R-squared	-0.001683	Mean dependent var	0.000156	
Adjusted R-squared	-0.001683	S.D. dependent var	0.002225	
S.E. of regression	0.002226	Akaike info criterion	-9.680102	
Sum squared resid	0.019541	Schwarz criterion	-9.670546	
Log likelihood	19090.32	Hannan-Quinn criter.	-9.676712	
Durbin-Watson stat	1.860639			

Source : Author's own calculations

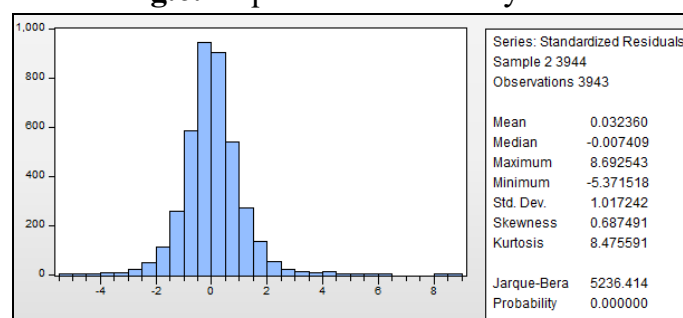
13. Verification and Diagnostics of the EGARCH(1,1) Model:

The adequacy of the ARCH-GARCH model is evaluated through an analysis of the residuals. These residuals are assumed to be **independently and identically distributed**, typically following a normal or standardized distribution. A good model fit is indicated if the **histogram of the residuals** appears approximately symmetric. Additionally, the **ACF and PACF of the standardized residuals** are examined to assess the suitability of the conditional variance specification. Once the model demonstrates an adequate fit, it can be confidently used for **forecasting future volatility**.

13.1. Residual Normality Test:

Because the standard GARCH model assumes **normally distributed innovations**, the **Jarque-Bera (JB) test** is employed to assess the normality of the residuals. The results, shown in Figure 6, indicate that the residuals **deviate significantly from normality at the 1% level**, supporting the use of the **Generalized Error Distribution (GED)** for estimating the model.

Fig.6. Jarque-Bera Normality Test

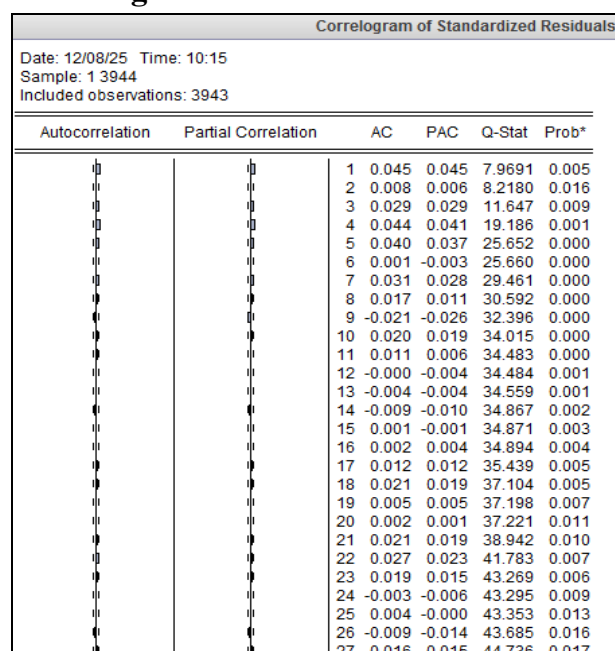


Source : Author's own calculations

13.2. Serial Correlation:

A straightforward diagnostic for assessing the adequacy of the model involves analyzing the residuals along with their **autocorrelations (AC)** and **partial autocorrelations (PAC)** across various lags. The estimated AC and PAC values are displayed in Figure 7, providing a visual check of the model's fit.

Fig.7. AC and PAC Functions



Source : Author's own calculations

As illustrated in the figures, none of the autocorrelations or partial autocorrelations are statistically significant at the 5% level, which confirms the **adequacy of the GARCH(1,1) model** in capturing the volatility dynamics of the series. Moreover, the test results indicate the **absence of remaining ARCH effects**

(i.e., no residual heteroskedasticity), validating the model's specification. Consequently, the fitted model is suitable for **forecasting future values** of the exchange rate series.

13.3. Forecast Evaluation and Accuracy:

Forecasts derived from the model serve as a foundation for **economic and business planning, inventory and production management**, and the **optimization of industrial processes**. To evaluate the accuracy of the EGARCH(1,1) model's predictions, standard **forecast error metrics**—namely the **Mean Absolute Error (MAE)** and the **Root Mean Square Error (RMSE)**—are employed. These metrics are defined as follows:

$$MAE = \frac{1}{T} \sum_{t=1}^T |r_t^2 - \hat{\sigma}_t^2| \dots \dots \dots (23)$$

$$RMSE = \sqrt{\frac{1}{T} \sum_{t=1}^T (r_t^2 - \hat{\sigma}_t^2)^2 \dots \dots \dots} (24)$$

In these formulas, $\hat{\sigma}_t^2$ for $t = 1, \dots, T$ represents the **estimated conditional variance** obtained from the ARCH-GARCH model, while r_t^2 serves as a proxy for the **realized variance**. Both MAE and RMSE are symmetric loss functions and are widely used to assess a model's **forecasting performance** due to their simplicity. The **RMSE** places greater emphasis on larger forecast errors, whereas the **MAE** treats overestimation and underestimation of volatility equally. Lower values of these metrics indicate better predictive accuracy.

Table 6 reports the **MAE and RMSE** for the forecasted volatility. The results suggest that the **EGARCH(1,1) model** provides relatively accurate forecasts, as evidenced by its low MAE and RMSE values. In particular, the smaller errors produced by this model highlight its **superior forecasting capability** relative to alternative specifications.

Tab.6. RMSE and MAE Indicator Results

MAE	0,0023
RMSE	0,0027

Source : Author's own calculations

14. Conclusion:

The examination of USD/DZD exchange rate volatility using ARCH and GARCH models has revealed several key features of financial time series. First, the presence of **volatility clustering** is evident, confirming that periods of large fluctuations tend to be followed by other high-volatility periods, while tranquil periods are also observed to persist. Statistical analyses further indicate that returns deviate from normality, exhibiting **fat tails and asymmetry**, which supports the use of

appropriate distributions, such as the **Generalized Error Distribution (GED)**, in GARCH model estimation.

The findings also highlight significant **temporal dependence and long-memory effects** in volatility, demonstrating the suitability of ARCH–GARCH models for capturing the complex dynamics of foreign exchange markets. The application of the **EGARCH(1,1) model** proved particularly effective, as it simultaneously accounts for volatility persistence, the asymmetric impact of past shocks, and inherited variance, all while maintaining a parsimonious parameter structure.

In summary, **ARCH and GARCH models** provide a robust framework for modeling exchange rate volatility, supporting risk management and forecasting in financial markets. For Algeria, understanding these volatility characteristics is essential for anticipating USD/DZD exchange rate movements and informing more effective monetary and financial policies. Finally, this study lays the groundwork for *future research*, including the use of multivariate GARCH models and further exploration of asymmetric leverage effects, to better capture the interplay between different economic variables and exchange rate volatility.

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