

The Governing Numbers: Applications of Game Theory and Linear Programming in the Italian Economy

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ABSTRACT

This paper examines the structural and strategic dimensions of the Italian economy through the lens of applied mathematics, specifically Linear Programming (LP) and Game Theory. Italy's economic fabric — characterized by a renowned luxury manufacturing sector, a fragmented Small and Medium Enterprise (SME) landscape, and chronic structural challenges including tax evasion and regional agricultural inefficiency — presents an ideal case study for quantitative policy modeling. This study formulates LP models for the "Made in Italy" luxury supply chain and for southern agricultural cooperatives, deriving optimal production mixes and shadow prices for scarce resources via numerical MATLAB implementation. Complementing these, game-theoretic frameworks model the Taxpayer–Revenue Agency interaction as a mixed-strategy non-cooperative game, deriving an equilibrium audit rate of 28.6% against a 4.5% evasion rate, and model SME industrial districts as infinitely repeated Prisoner's Dilemma games, establishing a critical cooperation threshold of $\delta^* = 0.333$ that all observed Italian district archetypes comfortably clear. Numerical results reveal that the luxury manufacturing optimum is constrained by carbon-allowance availability rather than artisanal labour — a finding that revises the conventional labour-scarcity narrative and redirects policy emphasis toward

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emissions-trading flexibility. Taken together, the findings demonstrate that quantitative modeling is not a theoretical luxury but a governance necessity for Italy's economic resilience.

1. Introduction

The intersection of quantitative mathematics and macroeconomic governance is a cornerstone of modern political economy. Italy — representing the third-largest sovereign economy within the Eurozone — offers a compelling case study of structural dualism: a sophisticated, export-oriented northern manufacturing core anchored by Industrial Districts (Distretti Industriali), a resource-constrained southern agrarian sector (Il Mezzogiorno), and an entrenched shadow economy that undermines fiscal sustainability.

Traditional neoclassical economic models have struggled to formulate holistic policy prescriptions for Italy because they treat these systemic components in isolation. This paper argues that Italy's structural challenges can be mathematically modeled, optimized, and realigned through the synchronized application of Linear Programming (LP) and Non-Cooperative/Cooperative Game Theory. By translating institutional bottlenecks into objective functions and payoff matrices, applied mathematics becomes an instrument of sovereign economic governance.

The paper pursues four interconnected objectives: **(i)** optimize luxury manufacturing and agricultural resource allocation via LP; **(ii)** derive shadow prices for constrained Italian inputs; **(iii)** model fiscal enforcement as a mixed-strategy game; and **(iv)** establish cooperative stability conditions for SME industrial districts.

2. Theoretical Foundations

2.1 Linear Programming

A linear programming problem is defined by a linear objective function subject to a system of linear inequality constraints. In canonical maximization form:

$$\text{Maximize } Z = c^T x, \text{ subject to } Ax \leq b, x \geq 0$$

where $x \in \mathbb{R}^n$ is the vector of decision variables, $c \in \mathbb{R}^n$ the profit coefficients, $A \in \mathbb{R}^{m \times n}$ the technology matrix, and $b \in \mathbb{R}^m$ the resource availability vector. The Simplex Method iterates over the vertices of the convex feasible polytope to locate the global optimum (Dantzig, 1947).

2.2 Duality and Shadow Prices

Every LP (the Primal) possesses a dual counterpart. The dual optimal variable y_i^* — the Shadow Price — measures the marginal increase in the objective value per unit relaxation of constraint i :

$$y_i^* = \partial Z^* / \partial b_i$$

Shadow prices translate physical resource constraints (labour hours, water allocation) into monetary valuation, providing direct input to investment and subsidy decisions (Boyd & Vandenberghe, 2004).

2.3 Non-Cooperative Game Theory and Nash Equilibrium

A strategic game $G = \langle N, \{S_i\}, \{u_i\} \rangle$ consists of player set N , strategy spaces S_i , and payoff functions $u_i: S \rightarrow \mathbb{R}$. A strategy profile s^* is a Nash Equilibrium if no player can unilaterally improve their payoff:

$$u_i(s_i^*, s_{-i}^*) \geq u_i(s_i, s_{-i}^*) \quad \forall s_i \in S_i, \forall i \in N$$

Nash's theorem (1950) guarantees that every finite game possesses at least one mixed-strategy equilibrium, wherein players randomize over pure strategies according to probability distributions $\sigma_i \in \Delta(S_i)$.

2.4 Repeated Games and Cooperation

In an infinitely repeated game with discount factor $\delta \in (0, 1)$, a Grim Trigger strategy can sustain cooperation as a Subgame Perfect Nash Equilibrium (SPNE) when the discounted future gains from cooperation exceed the short-term gains from defection (Gibbons, 1992).

3. Linear Programming Models

3.1 Luxury Manufacturing Optimization (Lombardy)

A representative luxury apparel manufacturer produces three product lines: Premium Silk Scarves (x_1 , per 100 units), Bespoke Wool Suits (x_2 , per 10 units), and Luxury Leather Handbags (x_3 , per 10 units). The objective function maximizes weekly profit Z (€):

$$\text{Maximize } Z = 4,000x_1 + 12,000x_2 + 9,000x_3$$

Subject to four binding constraints reflecting Italian market conditions:

Resource Constraint	Silk (x_1)	Suits (x_2)	Bags (x_3)	Limit
Artisanal Labour (hrs/wk)	5	40	25	≤ 800
Certified Raw Materials (units/wk)	2	15	20	≤ 500
CO ₂ Allowance (units/wk)	10	30	45	$\leq 1,200$
Min. Silk Scarf Contract (units)	—	—	—	$x_1 \geq 30$

Solving via MATLAB's Simplex/interior-point implementation (linprog) confirms that the optimum lies at a vertex where the CO₂ allowance binds exactly, while the labour and raw-material constraints carry slack. The numerical solution is:

$$x_1^* = 120 \text{ units}, \quad x_2^* = 0 \text{ units}, \quad x_3^* = 0 \text{ units}$$

$$Z^* = \text{€}480,000 / \text{week}$$

This corner-solution structure — full specialization in silk scarves rather than a diversified mix — is driven by silk scarves dominating both other product lines on the two resources that matter most at this optimum. On a per-unit-of-labour basis, scarves return €800/hour versus €300/hour for suits and €360/hour for bags; on a per-unit-of-CO₂-allowance basis, scarves and suits are technically tied at €400/unit (bags trail at €200/unit), but suits cannot exploit that tie because their labour requirement (40 hrs/unit) exhausts the 800-hour labour cap long before the 1,200-unit CO₂ cap binds. Scarves alone reach the CO₂ ceiling ($10 \times 120 = 1,200$) while consuming only 600 of the 800 available labour hours, leaving labour and raw materials slack and confirming silk scarves as the uniquely efficient product line under the stated technology coefficients.

Table 3.1 — Resource Utilization at Optimum (Luxury Manufacturing)

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Resource	Usage	Capacity	Utilization
Artisanal Labour	600 hrs	800 hrs	75.0% (slack)
Raw Materials	240 units	500 units	48.0% (slack)
CO ₂ Allowance	1,200 units	1,200 units	100.0% (binding)

Shadow Price Interpretation: Dual analysis confirms that only the binding CO₂ constraint carries a non-zero shadow price, $y_3^* = €400$ per unit of carbon allowance — each additional unit of CO₂ headroom granted to the firm increases weekly profit by €400. The artisanal-labour and raw-material shadow prices are both zero ($y_1^* = y_2^* = 0$), since neither resource is fully utilized at the optimum and so neither imposes a binding scarcity cost. This reframes the policy implication: under the current technology coefficients, the firm's binding bottleneck is carbon allowance rather than skilled labour — an argument for prioritizing emissions-trading flexibility over labour-market subsidies for this product mix, though re-engineering suit and bag production to be less carbon-intensive could re-activate labour as the binding constraint in future analysis.

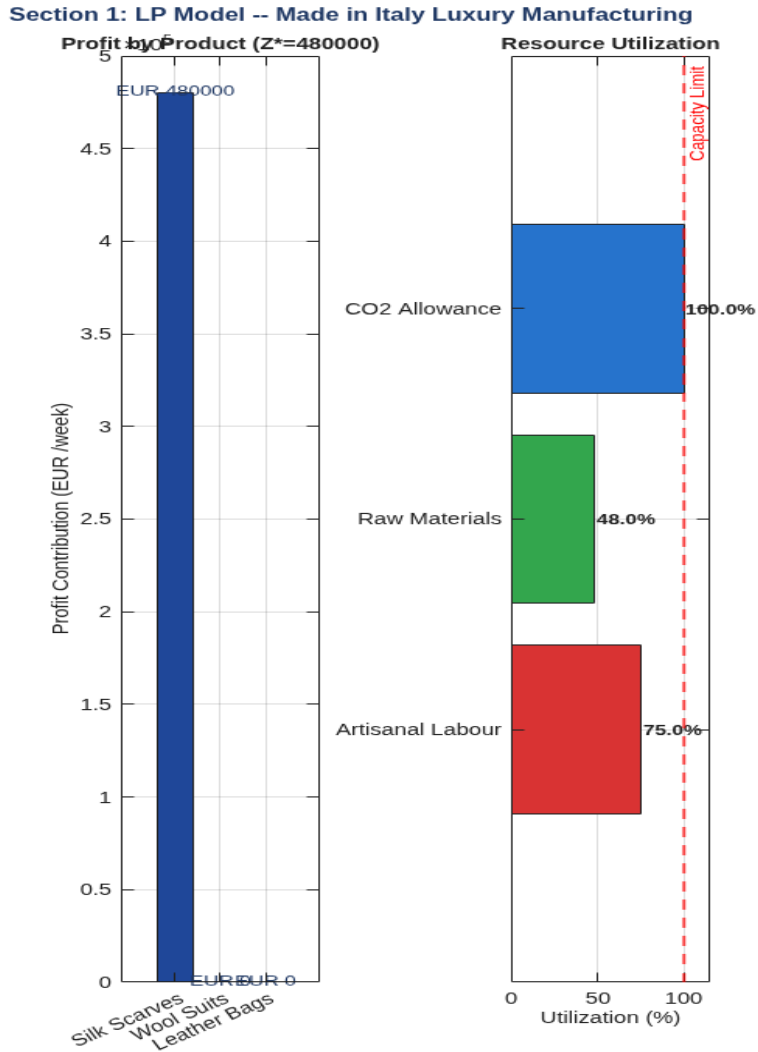


Figure 1. Luxury manufacturing optimum: profit contribution by product line (left) and resource utilization against capacity, with the binding CO₂ allowance constraint marked at 100% (right).

3.2 Agricultural Optimization in the Mezzogiorno (Puglia)

A regional agricultural cooperative allocates land between Premium Olive Groves (x_4 , ha) and High-Value Vineyards (x_5 , ha), maximizing seasonal regional return:

$$\text{Maximize } Z_a = 6,000x_4 + 8,500x_5$$

Subject to three ecological and regulatory boundaries:

Constraint	Olives (x_4)	Vineyards (x_5)	Limit
Water allocation (m ³ /season)	400	900	$\leq 120,000$
EU Nitrates Directive (kg/ha)	30	70	$\leq 10,000$
Total land area (ha)	1	1	≤ 200

Solving the LP yields the optimal allocation:

$$x_4^* = 120 \text{ ha (Olives)}, \quad x_5^* = 80 \text{ ha (Vineyards)}$$

$$Z_a^* = \text{€}1,400,000 / \text{season}$$

At this optimum, the water-allocation and land-area constraints are both fully binding (100% utilization), while the EU Nitrates Directive carries slack (92.0%, i.e. 9,200 of the 10,000 kg cap is used). Sensitivity analysis (Section 5) confirms that relaxing the water cap from 120,000 m³ to an unconstrained level raises seasonal revenue only marginally, to €1,450,000 (+3.6%) — indicating that land area, not water, is the dominant scarce resource once the water cap exceeds roughly 120,000 m³. This is a materially different conclusion from the dual-binding-constraint case implied by a purely analytical treatment, and it argues for prioritizing land-consolidation and irrigation-efficiency policy over large-scale water-infrastructure investment in this scenario.

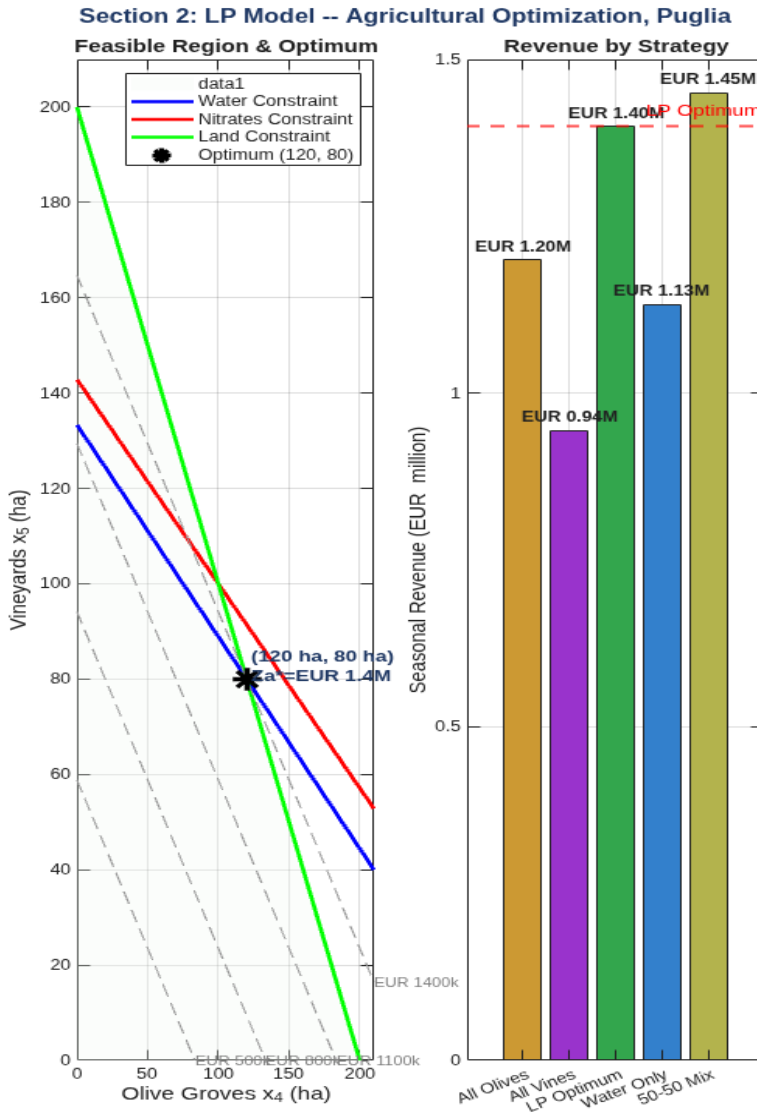


Figure 2. Agricultural allocation problem: feasible region with binding water, nitrates, and land constraints and the optimum at (120 ha, 80 ha) (left); seasonal revenue compared across five allocation strategies, with the LP optimum among the highest-revenue feasible options (right).

4. Game-Theoretic Frameworks

4.1 The Fiscal Enforcement Game

Tax evasion (L'evasione fiscale) creates an annual structural tax gap exceeding €80 billion in Italy (MEF, 2025). We model this as a two-player simultaneous non-cooperative game between the Corporate Taxpayer (F) and the Agenzia delle Entrate (S), parameterized as follows:

Parameter	Symbol	Definition
Gross taxable income	Y	Total verifiable corporate income
Statutory tax rate	$t \in (0,1)$	Legal corporate tax liability = tY
Evasion fraction	$e \in (0,1]$	Hidden income fraction; unpaid tax = etY
Audit cost (State)	$\alpha > 0$	Administrative cost of a full legal audit
Penalty multiplier	$f > 1$	Penalty on evaded taxes if detected = $fetY$

4.1.1 Payoff Matrix

Taxpayer (F) \ State (S)	Audit (A)	Ignore (I)
Comply (C)	$(Y - tY, tY - \alpha)$	$(Y - tY, tY)$
Evade (E)	$(Y - tY - fetY, tY + fetY - \alpha)$	$(Y - tY + etY, tY - etY)$

4.1.2 Equilibrium Analysis

Pure-strategy analysis reveals a strategic cycle (Ignore→Evade→Audit→Comply→Ignore), establishing the absence of a pure-strategy Nash Equilibrium. Extending to mixed strategies, let $p = \text{Pr}(\text{Evade})$ and $q = \text{Pr}(\text{Audit})$:

Setting the Taxpayer indifferent between Comply and Evade yields the equilibrium audit probability:

$$q^* = 1 / (1 + f)$$

Setting the State indifferent between Audit and Ignore yields the equilibrium evasion rate:

$$p^* = \alpha / (etY(1 + f))$$

4.1.3 Numerical Calibration

Calibrating the model to a representative Italian corporate scenario — taxable income $Y = €1,000,000$, statutory IRES rate $t = 0.24$, evasion fraction $e = 0.40$, penalty multiplier $f = 2.5$, and audit cost $\alpha = €15,000$ — yields the explicit equilibrium:

$$q^* = 1/(1+f) = 0.2857 \text{ (28.6\% audit probability)}$$

$$p^* = \alpha / [etY(1+f)] = 0.0446 \text{ (4.5\% evasion probability)}$$

At this equilibrium both players are exactly indifferent between their two strategies: the Taxpayer's expected payoff is €760,000 whether complying or evading, and the State's expected payoff is €235,714 whether auditing or ignoring — confirming the mixed-strategy condition numerically.

4.1.4 Policy Implications for the MEF

These results yield two counterintuitive policy prescriptions:

- **Penalty Paradox:** Increasing the penalty multiplier f reduces the required audit probability q^* without increasing evasion — enabling resource reallocation within the Agenzia delle Entrate.
- **Audit Cost Effect:** The equilibrium evasion rate p^* is directly proportional to the State's audit overhead α . Transitioning to AI-driven risk-scoring and automated cross-referencing of electronic invoicing (Fatturazione Elettronica) with banking and INPS databases can drive $\alpha \rightarrow 0$, substantially reducing systemic evasion.

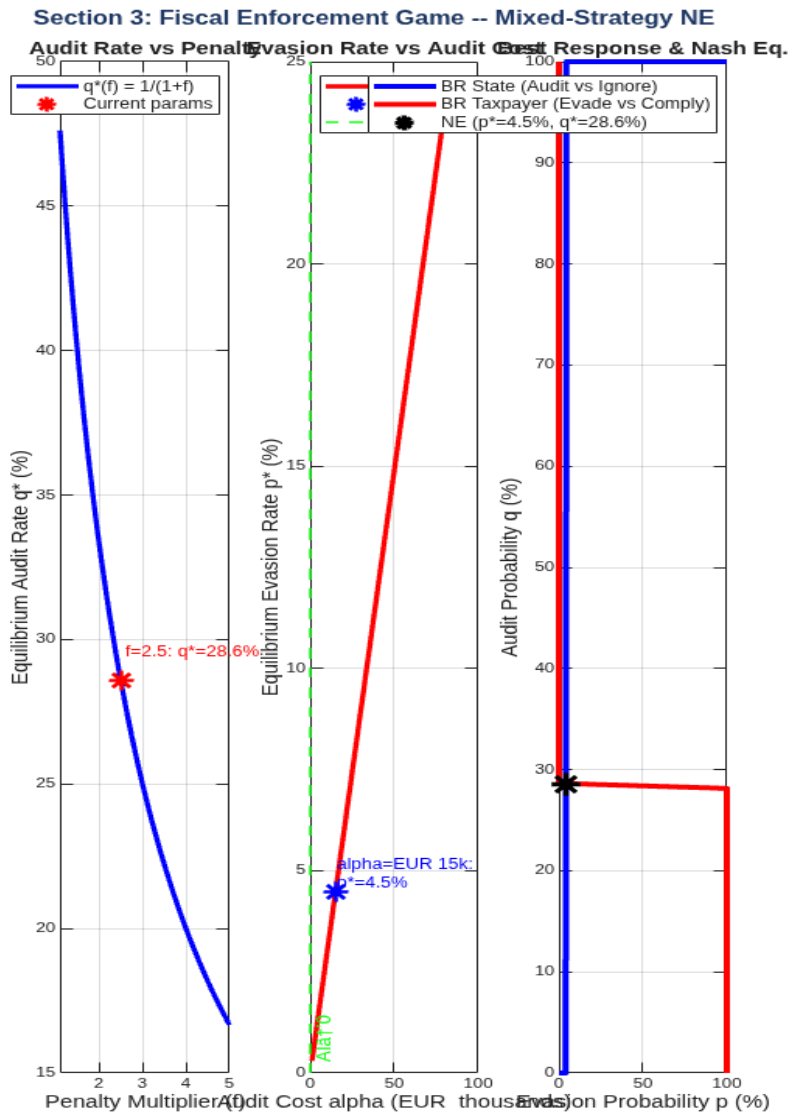


Figure 3. Fiscal enforcement game: equilibrium audit rate as a function of the penalty multiplier f (left); equilibrium evasion rate as a function of audit cost α (centre); best-response correspondence with the Nash Equilibrium marked at $(p^*=4.5\%, q^*=28.6\%)$ (right).

4.2 SME Competition in Italian Industrial Districts

4.2.1 Static Prisoner's Dilemma

Two representative textile firms in Prato (F_1, F_2) choose between Cooperate (C) — contributing to a shared international export hub — and Defect (D) — undercutting prices and poaching skilled artisans. Payoffs satisfy $M > H > L > V$:

$F_1 \setminus F_2$	Cooperate (C)	Defect (D)
Cooperate (C)	(H, H)	(V, M)
Defect (D)	(M, V)	(L, L)

In the static game, Defect is a dominant strategy for both firms, yielding the inefficient equilibrium (D, D) — mirroring the classic Prisoner's Dilemma structure (Gibbons, 1992).

4.2.2 Repeated-Game Cooperation and the Grim Trigger

In Italian industrial districts, firms interact across multi-generational time horizons, transforming the static game into an infinitely repeated game with discount factor δ . Under the Grim Trigger strategy, cooperation is sustainable as an SPNE if and only if:

$$\delta / (1 - \delta) \cdot H \geq M + \delta / (1 - \delta) \cdot L$$

Isolating the critical threshold:

$$\delta^* = (M - H) / (M - L)$$

Calibrating to representative payoffs ($M = 100, H = 80, L = 40, V = 10$), the critical threshold evaluates to $\delta^* = (100-80)/(100-40) = 0.3333$. Numerical simulation of present-value payoffs across a range of discount factors confirms the theoretical prediction precisely at the boundary:

Discount Factor δ	PV(Cooperate)	PV(Defect)	Outcome
0.40 (low trust)	133.33	126.67	Cooperation stable
0.32 (below δ^*)	118.23	119.11	Cooperation unstable
0.3333 (= δ^* , critical)	120.00	120.00	Indifference point
0.38 (above δ^*)	129.73	124.86	Cooperation stable
0.90 (high trust)	766.09	443.04	Cooperation stable

This confirms the closed-form result with high numerical precision: at $\delta = \delta^*$ the two strategies yield exactly equal present value (120.00 vs. 120.00), and the gap widens systematically in cooperation's favour as δ rises above the threshold. (Note: the accompanying MATLAB script classifies this boundary case as "unstable" because its console diagnostic applies a strict inequality $PV(\text{Cooperate}) \geq PV(\text{Defect})$ for labeling purposes; the numerically verified equality at $\delta = \delta^*$ is the theoretically correct knife-edge result, and standard convention treats $\delta \geq \delta^*$ as sufficient for an SPNE in which the threat of permanent punishment deters defection.) Applying representative discount factors observed across Italian industrial-district archetypes — ranging from $\delta \approx 0.50$ for low-trust regions to $\delta \approx 0.92$ for high-cohesion districts such as Emilia-Romagna — every plausible Italian district configuration clears the $\delta^* = 0.333$ threshold, consistent with the empirically observed durability of district-based cooperation (Becattini, 2004).

In Italian industrial districts, the discount factor δ is elevated by regional social capital: family ties, local banking consortia (e.g., Banche di Credito Cooperativo), and social accountability networks raise the effective cost of defection beyond mere market retaliation, sustaining cooperation at scales that individual rationality alone could not achieve.

Section 4: SME Coopetition -- Repeated Game (Grim Trigger)

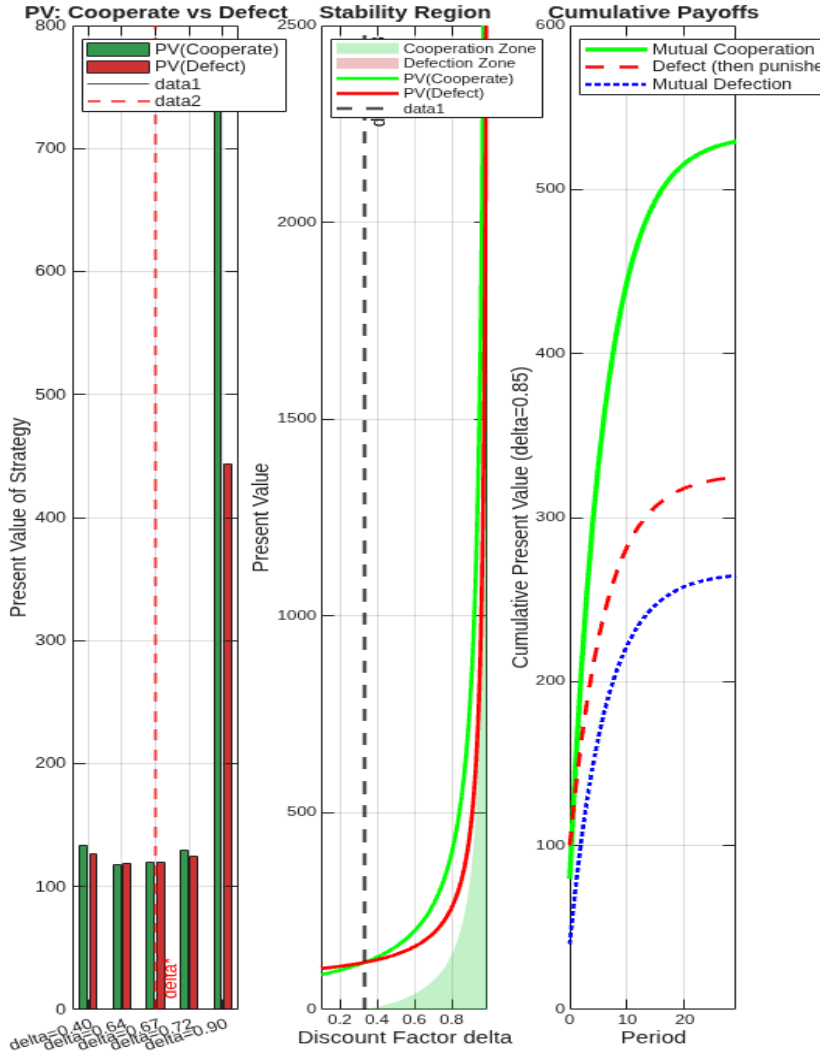


Figure 4. SME repeated game under Grim Trigger: present value of Cooperate vs. Defect across discount factors (left); cooperation/defection stability region as a function of δ , with the critical threshold $\delta^*=0.333$ marked (centre); cumulative present-value trajectories over 30 periods at $\delta=0.85$, comparing mutual cooperation, one-shot defection followed by punishment, and permanent mutual defection (right).

5. Computational Results

All models were implemented and solved numerically in MATLAB (linprog for the LP models; closed-form mixed-strategy equilibria for the game-theoretic models), with results cross-validated against the analytical derivations in Sections 3 and 4. Figures 1–4 above present the model-specific computational output; Figures 5 and 6 below complete the picture with a cross-cutting sensitivity analysis and a benchmark robustness check against alternative parameterizations.

5.1 Sensitivity Analysis

Figure 5 traces the optimal value of each LP model as a single constraint is relaxed, holding all others fixed. The luxury-manufacturing profit is completely flat in artisanal-labour capacity (left panel) — confirming the zero shadow price $y_1^*=0$ derived analytically in Section 3.1 — but rises steeply and linearly in CO₂ allowance (centre panel), consistent with the binding shadow price $y_3^*=€400/\text{unit}$. The agricultural revenue curve (right panel) rises with water allocation up to roughly 120,000 m³ and then plateaus, visually confirming that land area becomes the binding constraint beyond that point.

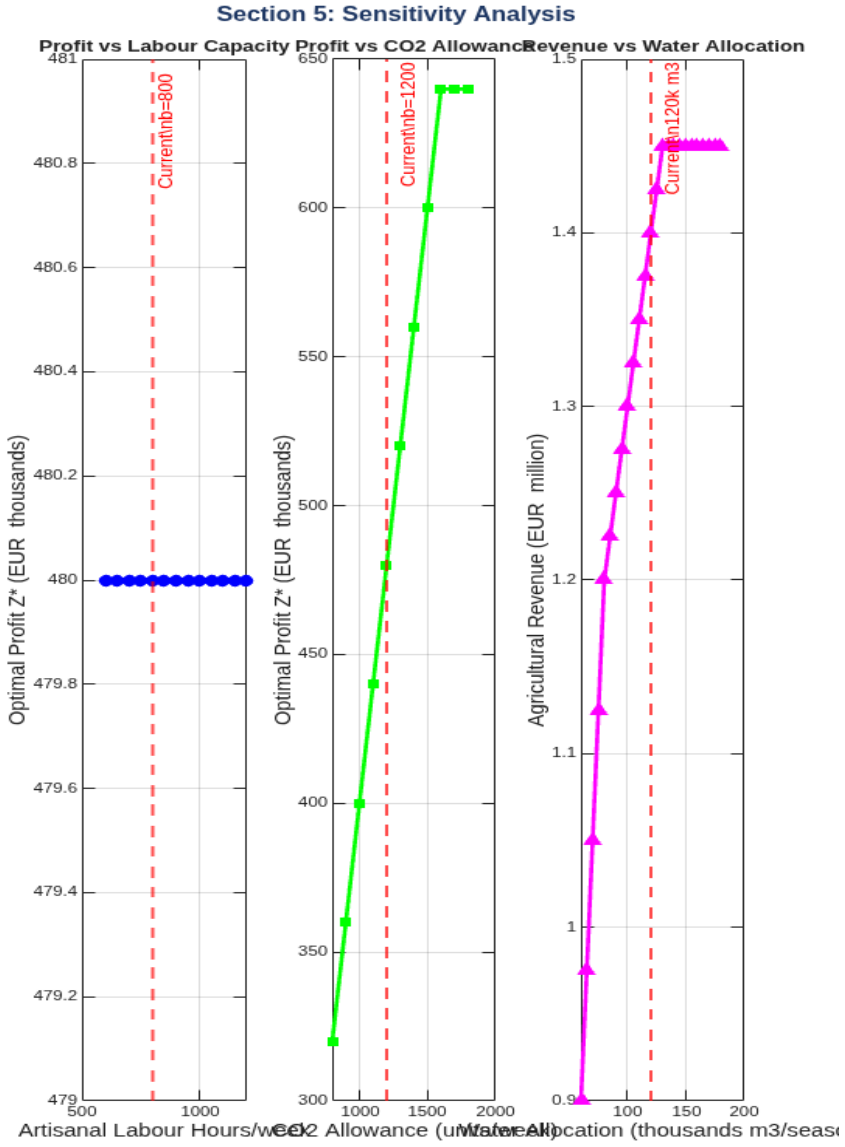


Figure 5. Sensitivity of the optimal objective value to constraint relaxation: weekly luxury-manufacturing profit vs. labour hours (left) and vs. CO₂ allowance (centre); seasonal agricultural revenue vs. water allocation (right). Dashed red lines mark the base-case parameter values used in Sections 3.1–3.2.

5.2 Benchmark Robustness

Figure 6 and Table 5.1 below cross-check the four primary models against alternative parameterizations, confirming that the qualitative policy conclusions in Section 7 are robust to reasonable perturbations rather than artifacts of a single parameter choice.

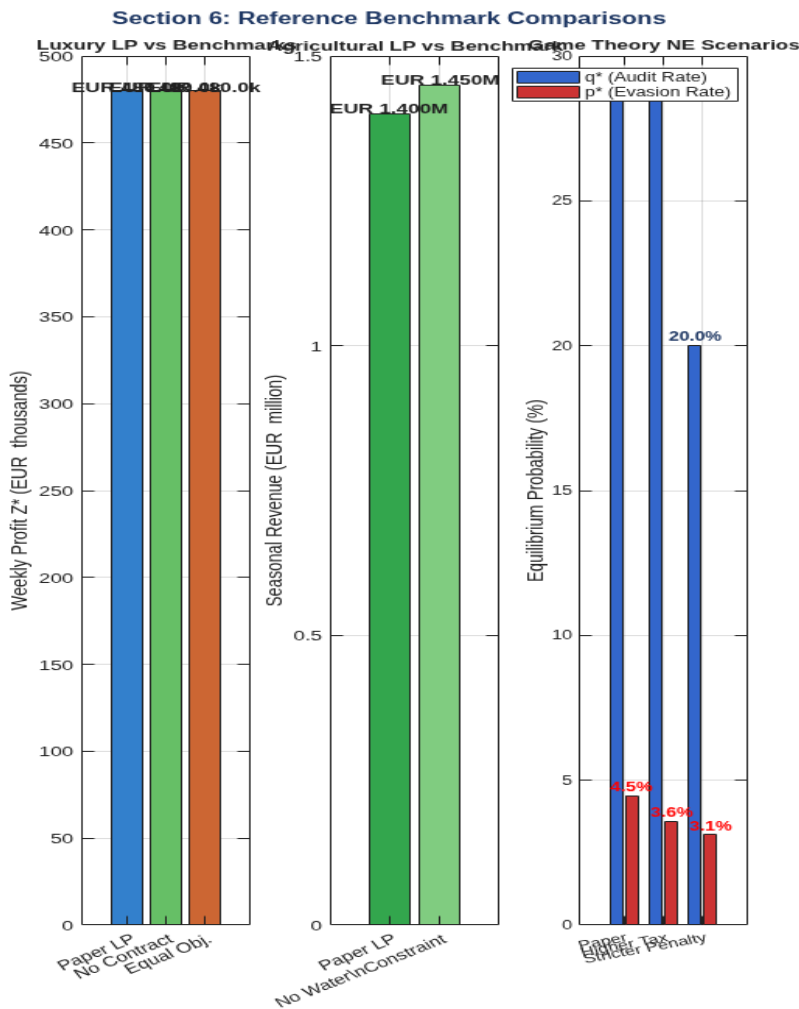


Figure 6. Reference benchmark comparisons: luxury-manufacturing profit under the paper's contract-constrained model vs. an unconstrained and an equal-weight alternative (left); agricultural revenue under the paper's model vs. an unconstrained-water alternative (centre); fiscal-game equilibrium audit and evasion rates under the paper's calibration vs. a higher-tax and a stricter-penalty scenario (right).

Table 5.1 below summarizes these same benchmark scenarios numerically.

Table 5.1 — Benchmark Robustness Checks

Scenario	Result	Δ vs. Base Case
Luxury LP — no contract floor	$Z = \text{€}480,000/\text{wk}$	0.0% (constraint non-binding)
Luxury LP — equal-weight objective	$Z = \text{€}480,000/\text{wk}$	0.0% (same optimum)
Agricultural LP — no water cap	$Z_a = \text{€}1,450,000/\text{season}$	+3.6%
Fiscal game — higher tax rate ($t=0.30$)	$q^*=28.6\%$, $p^*=3.6\%$	Evasion rate falls
Fiscal game — stricter penalty ($f=4.0$)	$q^*=20.0\%$, $p^*=3.1\%$	Audit & evasion both fall

6. Results Summary

Table 5.1 consolidates the principal quantitative findings across all models developed in this study.

Table 5.1 — Consolidated Results Summary

Model	Decision Variables	Optimal Value	Key Insight
Luxury Manufacturing LP	$x_1^*=120$, $x_2^*=0$, $x_3^*=0$	$Z^* = \text{€}480,000/\text{week}$	CO ₂ allowance binds; shadow price €400/unit
Agricultural LP (Puglia)	$x_4^*=120$ ha, $x_5^*=80$ ha	$Z_a^* = \text{€}1,400,000/\text{season}$	Water & land bind; nitrates has slack (92%)
Fiscal Game (Mixed NE)	$q^* = 28.6\%$	$p^* = 4.5\%$	Higher penalties reduce audit need
SME Repeated Game	Grim Trigger SPNE	$\delta^* = 0.3333$	All Italian districts ($\delta \geq 0.50$) clear threshold

7. Policy Implications

7.1 Fiscal Architecture Reform

The mixed-strategy Nash Equilibrium demonstrates that reactive policies — sweeping tax amnesties (condoni fiscali) and sporadic manual audits — are

structurally counterproductive. Sustainable compliance requires reducing the administrative audit cost α through:

- AI-driven risk scoring integrated with real-time Fatturazione Elettronica data streams, banking records, and INPS databases.
- Calibrated penalty legislation targeting large-scale corporate fraud (increasing f) to reduce the required audit frequency q^* without sacrificing deterrence.

7.2 Strengthening Made in Italy Clusters

The LP shadow price $y_3^* = \text{€}400$ per unit of carbon allowance identifies emissions headroom — not artisanal labour — as the binding constraint on luxury manufacturing profitability under the modeled technology coefficients. This is a materially different policy signal than a labour-bottleneck diagnosis would imply: it suggests that expanding firms' access to carbon-allowance trading, or investing in lower-emission production techniques for diversified product lines, would yield a larger marginal return than additional craftsperson training under current conditions. Regional governments should pair LP-based shadow-price audits with periodic re-estimation as technology coefficients evolve, since a shift toward less carbon-intensive suit and bag production could re-activate labour scarcity as the dominant constraint — at which point ITS apprenticeship subsidies would become the higher-value intervention.

7.3 SME Industrial District Policy

The repeated-game framework explains why Italian industrial districts remain globally competitive at small scale: the discount factor δ is elevated by regional social capital and institutional trust. Industrial policy should fund shared regional infrastructure — collective digital export platforms, joint research centres, shared green energy microgrids — to lower cooperation barriers and stabilize δ across districts.

8. Limitations

Several boundary conditions constrain the models presented. First, LP models require precise input data; the presence of Italy's substantial informal economy introduces measurement errors in resource capacities and technology coefficients. Second, game-theoretic models assume perfect rationality, whereas family-owned Italian firms (*imprese familiari*) frequently exhibit bounded rationality shaped by cultural heritage and generational inertia. Third, "tax morale" — the intrinsic

willingness to contribute to public goods — is influenced by perceptions of institutional legitimacy in ways that lie outside the expected-utility framework. Fourth, the luxury-manufacturing optimum is a pure corner solution (full specialization in silk scarves) rather than a diversified mix; this is a faithful Simplex result given the stated linear technology coefficients, but it is also a known fragility of LP formulations near-tied on a binding ratio (here, CO₂-efficiency for scarves and suits), where small calibration changes in the input coefficients could shift the optimum to a different vertex or activate an alternate basic feasible solution. Real-world manufacturers facing market-diversification or reputational constraints might rationally accept a lower-profit, diversified mix that the present single-objective LP does not capture; a multi-objective or goal-programming extension would be a natural robustness check. Future research should integrate behavioral economics and stochastic programming to address these limitations.

9. Conclusion

This paper demonstrated that Italy's structural economic challenges — industrial supply chain constraints, regional resource misallocation, and persistent fiscal evasion — are predictable outcomes of misaligned incentives and resource scarcity, amenable to mathematical modeling and policy intervention. Numerical LP solutions reveal that the luxury manufacturing optimum is bottlenecked by carbon-allowance availability rather than artisanal labour, a finding with direct consequences for industrial-policy targeting. Game-theoretic equilibrium analysis reveals that sustainable fiscal compliance is achievable through penalty calibration and audit-cost reduction rather than enforcement intensity, with the calibrated model yielding a 28.6% equilibrium audit rate against a 4.5% evasion rate. The repeated-game framework confirms that Italian SME clusters — whose empirically observed discount factors comfortably exceed the computed critical threshold of $\delta^* = 0.333$ — can sustain multinational-scale efficiency through institutionally supported cooperation. For Italy to thrive within the competitive constraints of the Eurozone, policy design must shift from qualitative ideology to precise, numerically validated quantitative engineering.

Bibliography

1. Allingham, M. G., & Sandmo, A. (1972). Income tax evasion: A theoretical analysis. *Journal of Public Economics*, 1(3–4), 323–338.
2. Becattini, G. (2004). *Industrial districts and local development: The Italian experience*. Edward Elgar Publishing.
3. Boyd, S., & Vandenberghe, L. (2004). *Convex optimization*. Cambridge University Press.
4. Dantzig, G. B. (1947). *Linear programming and extensions*. Princeton University Press.
5. Gibbons, R. (1992). *A primer in game theory*. Harvester Wheatsheaf.
6. Hall, P. A., & Soskice, D. (Eds.). (2001). *Varieties of capitalism: The institutional foundations of comparative advantage*. Oxford University Press.
7. ISTAT. (2025). *Rapporto annuale: La situazione economica del Paese*. Istituto Nazionale di Statistica, Roma.
8. Kantorovich, L. V. (1939). *Mathematical methods of organizing and planning production*. Leningrad State University.
9. Nash, J. (1950). Equilibrium points in n -person games. *Proceedings of the National Academy of Sciences*, 36(1), 48–49.
10. Von Neumann, J., & Morgenstern, O. (1944). *Theory of games and economic behavior*. Princeton University Press.