Analysis of competitive bidding strategy in Government Construction Contracts under Risk and Uncertainty

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Abstract:

In public procurement, some firms adopt aggressive strategy by submitting a very low bid price to increase their probability of winning, However, such behavior may lead to execution failure due to financial inability, Therefore, this study aims to develop a probabilistic framework for assessing pricing-related risks and determining safe *mark-up* bounds. Assuming that bids follow log-normal distribution, the suggested model explains the risk levels in last project of the training data, which aide contracting authorities to evaluate winner bid prior then allocate project, the model can also predict the winner and all provided strategy for the competitors in a new project**.**

Keywords: Log-normal distribution; probabilistic modeling; loss probability; competition analysis; anomalous bid.

Jel Classification Codes: C13, C44, D44

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1. INTRODUCTION

Public procurement is a large using method for allocating the execution of public projects, when a public contracting authority intends to carry out a Public infrastructure such as roads and hospitals, it launches a call for bids, and begins to receiving bids from interested companies, bid evaluation phase focuses on a several criteria such as the lowest-bid criterion[1] which is the central focus on this study, in this context, some bidders adopt aggressive strategy by submitting low bids,

Given uncertainty in project cost, it become difficult to filter serious bids that properly balance the profitability with financial risk, This complexity ultimately lead back to the fundamental basis of any bid which represented by underlying cost, and arises the question of whether a firm's bid is actually below or close to its real cost. Bids sometimes appear close to one another, but it is impossible to assess each company's capabilities through a preliminary glance at the offers.

Public projects planning of execution vary from one bidder to another, because its complexity and degree of difficulty, and bidders usually don't adopting the same plan. Therefore, considering cost as a Radom variable is the close approach to the reality. This lead that the bid is also can consider as a random variable that exceeds the cost by a percentage known as the *mark-up* (profit multiplier), Therefore, the cost and the bid follow the same probability distribution.

Problematic:

The core problematic of this study is that increasing the winning probability requires lowering the bid price at minimum levels, which can lead to very high levels of losing, this raise a key question: what is the lower bound of mark-up when the company adopt aggressive strategy, and what are the safe interval bounds? To address this

Research hypothesis:

In this study we assume that bids are independent and follow log-normal distribution

Research objectives:

Develop a quantitative framework for identifying each bidder's pricing strategy patterns and Enable contracting authority to perform statistical evaluation of winning bid prior to project allocation, with precise verification of bid seriousness.

Methodology:

This study relies on An iterative estimation algorithm (ALS), Probabilistic modeling and analysis

Previous studies:

Research on bidding pricing strategy has significantly evolved over recent some decades; it has developed from descriptive statistical analysis toward economic behavioral approach aimed to explaining bidder's strategy.

Early studies focused on the optimal excepted profit, one of the most important models examining the relationship between profit margin (*mark-up*) and winning probability (assuming a fixed cost) was proposed by Freidman[2] (1956).

In the same field, Skitmore (1989-1991) and Skitmore & Pemberton (1991)demonstrated that there is significant variation in pricing strategy of bidders and showed non-homogeneous.

Within the modern statistical models, Liu, Wang & Lai (2005) proposed a multivariate approach[3] based on a probability, aimed at analyzing profit margins and winning probabilities, under the assumption that cost is a random variable.

Despite the growing use of machine learning methods, and their strong predictive performance, they suffer from limited interpretability and an inability to represent the probabilistic structure of competitive bidding under uncertainty and heterogeneous risk. Moreover, they typically require large volumes of high-quality data, which are often unavailable in public procurement contexts.

1. 2. Theoretical framework of the model:

2.1 Parameters estimation

Bid of each bidder follows a log normal distribution around a baseline cost:

$$log(bid_{ij}) = y_{ij} \sim N(\mu_i + \beta_j, \sigma_i^2)$$

$$log\,(bid_{ij}) = y_{ij} = \mu_i + \beta_j + \epsilon_{ij}$$

With:

$$\epsilon_{ij} \sim N(0, \sigma_i^2)$$

Where:

- μ_i : average log bid of firm *i*
- β_j : the level of complexity of project *j*
- σ_i^2 : variance of bids for firm i
- ϵ_{ij} : a random error

This mean that the density of each observation is:

$$f(y_{ij} | \mu_i, \beta_j, \sigma_i^2) = \frac{1}{\sqrt{2\pi\sigma_i^2}} e^{\left(-\frac{(y_{ij}-\mu_i-\beta_j)^2}{(2\sigma_i^2)}\right)}$$

The likelihood Function is:

$$L(\mu, \beta, \sigma^2 \mid y) = \prod_i^c \prod_j^r \left[\frac{1}{\sqrt{2\pi\sigma_i^2}} e^{\left(-\frac{(y_{ij}-\mu_i-\beta_j)^2}{(2\sigma_i^2)}\right)}\right]^{d_{ij}}$$

The log likelihood Function is:

$$log\ L = -\frac{1}{2}\sum_{i=1}^{c}\sum_{j=1}^{r} d_{ij}\left(\frac{1}{\sqrt{2\pi\sigma_i^2}} e^{\left(-\frac{(y_{ij}-\mu_i-\beta_j)^2}{(2\sigma_i^2)}\right)}\right)$$

$$log\ L\ = \sum_{i=1}^{c}\sum_{j=1}^{r} d_{ij}\left(-\frac{1}{2}\log\left(2\pi\sigma_i^2\right) - \left(-\frac{\left(y_{ij}\ -\ \mu_i\ -\ \beta_j\right)^2}{\left(2{\sigma_i}^2\right)}\right)\right)$$

For estimating parameters of each company and project, we use weighted least squares, however, since the data of bids usually are incomplete (firms do not participate in all projects), the objective function is defined as:

$$\min_{a_i,b_j} \sum_{i=1}^{r}\sum_{j=1}^{c} d_{ij}(y_{ij} - \mu_i - \beta_j)^2$$

where:

$$d_{ij} = \begin{cases} 1,\ if\ firm\ (i)\ participated\ in\ project\ (j) \\ 0,\ if\ firm\ (i)\ not\ participated\ in\ project\ (j) \end{cases}$$

Updating ($\boldsymbol{\mu_i}$):

Fixing (β_j) and minimizing the objective function :

$$\min_{\mu_i} \sum_{j=1}^{c} d_{ji}(y_{ij} - \mu_i - \beta_j)^2$$

Taking the derivative with respect to $\boldsymbol{a_i}$ and setting it to zero:

$$\frac{\partial}{\partial \mu_i} = -\ 2\sum_{j=1}^{c} d_{ij}\left(y_{ij} - \mu_i - \beta_j\right) = 0$$

Which gives:

$$\mu_i = \frac{1}{n_i}\sum\nolimits_{i:d_{ij}=1}\left(y_{ij} - \beta_j\right)$$

where:

$$n_i = \sum_{j=1}^{c} d_{ij}$$

):$\boldsymbol{\beta_j}$Updating (

Fixing (μ_i) and minimizing the objective function

$$\beta_j = \frac{1}{m_j}\sum\nolimits_{i:d_{ij}=1}\left(y_{ij} - \mu_i\right)$$

where:

$$m_j = \sum_{i=1}^{r} d_{ij}$$

And m_j is the number of firms participating in project j.

Identification constraint:

$$\sum_{j=1}^{c} \beta_j = 0$$

Variance update:

$$\sigma_i^2 = \frac{1}{n_i} \sum\nolimits_{i:d_{ij}=1} \left(y_{ij} - \mu_i - \beta_j\right)^2$$

Iterative procedure:

This procedure is a practical application of the Expectation–Maximization (EM) algorithm[4]. The method relies on an alternating update scheme to solve the maximum likelihood estimation (MLE) problem via fixed-point iterations. Firm effects and project effects are updated as conditional means, project effects are normalized to ensure identifiability, and firm-specific variances are updated as mean squared residuals. The process is repeated until convergence, i.e., when changes in the parameter estimates between iterations fall below a predefined threshold.

2.2 Strategic Performance Functions: Win Probability, Expected Profit, and Conditional Loss Probability:

Win Probability Function *P*(win):

In government procurement auctions awarded on a lowest bid basis, the *markup*(v) plays a crucial role in determining win probability[5]:

$$v = markup = \frac{Bid}{Cost}$$

- Higher v → higher bid → **lower win probability**
- Lower v → lower bid → **higher win probability**

Additionally, each competitor's cost probability density function $f(x)$ fundamentally shapes the estimation of win probabilities.

Let $y_1, y_2, y_2 \ldots . y_n$ be random variables representing the logarithms of bids submitted by competing bidders in a set of tenders, such that :

$$y_{ij} \sim N\left(\mu_i + \beta_j, \vartheta_i^2\right)$$

Then, the probability that the bid y_1 submitted by bidder A_1 is the lowest bid among all submitted bids in a given tender is given by:

$$P(win|y_1) = P(y_1 < y_2 \cap y_1 < y3 \cap \ldots \cap y_1 < y_n)$$

The probability of winning equal to the product of the cumulative probability distributions of all competing bidders in the same project over the interval $]y_1, \infty[$

$$p(win|y_1) = \prod_{i \neq 1} \int_{y_1}^{\infty} f_i(y_i) dy_i$$

Where $f_i(y_i)$ is the probability density function of bidderA_i, is given by a following expression:

$$f_i(y_i) = \frac{1}{\sqrt{2\pi}}\frac{1}{\vartheta_i x}e^{\frac{-(y_i-\mu_i-\beta_j)^2}{\vartheta 2_i^2}}$$

By integrating over the interval of all possible costs of company A_1 , the win probability weighted cost density is given by a following expression:

$$Ce(x,v) = \frac{1}{\sqrt{2\pi}}\frac{1}{\vartheta_1 x}e^{\frac{-(\ln(x)-\mu_1-bj)^2}{2\vartheta_1^2}}\left[\prod_{k\neq 1}\int_{y_1=vx}^{+\infty}\left(\frac{1}{\sqrt{2\pi}}e^{\frac{-(\ln(xv)-\mu_1-bj)^2}{2\vartheta_1^2}}\right)dz\right]$$

Unconditional probability of winning is obtained by integrating the function $Ce(x)$ over the entire range of possible cost of company A_1:

$$p(win) = \int_0^{+\infty} Ce(x)dx$$

Expected profit:

Expected profit is the profit realizing by the company multiplied by the probability of win, while the cost is random variable, and with considering that this cost is unknown[6], Expected profit formula is given by a following expression:

$$E[P] = E\left(\frac{vx-m_e}{m_e}|win\right)p(win)$$

$$E[P] = \int \frac{vx-m_e}{m_e}\,Ce(x)dx$$

Where m_e represents the expected cost under the log normal distribution principal rules:

$$m_e = exp\ \left(\mu_1 + \beta_j + \frac{\vartheta_1^2}{2}\right)$$

The optimal *mark-up* can be expressed mathematically as:

$$v^* = arg\ \max_v E[P(v)]$$

Conditional probability of loss given winning:

This represents the probability of incurring financial losses in the case of that this firm win the contract. or the possibility that actual project costs exceed the submitted bid value.

It is calculated by integrating the winning probability weighted cost density function over the interval upper then the expected cost (m_e) to the maximum possible cost value, divided by the unconditional probability of winning. Its formula is given by a following expression:

$$p(Loss|win) = \frac{\int_{m_e}^{+\infty} Ce(x,v)dx}{\int_{-\infty}^{+\infty} Ce(x,v)dx}$$

Based on the previous relationship, different bidding strategies can be targeted by specifying a desired probability of loss[7] and solving the equation with a single unknown variable (the markup multiplier). In this study, two probability-of-loss thresholds are considered (5% and 25%):

$$p(Loss|win) \leq 0.05 \leftrightarrow No_loss\ strategy$$
$$0.25 > p(Loss|win) > 0.05 \leftrightarrow Balanced\ strategy$$
$$p(Loss|win) > 0.25 \leftrightarrow Loss_oriented\ (risky)\ strategy$$

Markup multiplier estimation:

The conditional markup multiplier applied by each firm can be estimated from its observed bid using the conditional cost excepted value:

$$Bid = v.E[X|v]$$

Applying Bayes' conditional formulation:

$$E[Bid|v] = \frac{\int f(vx).Ce(x,v)dx}{\int Ce(x,v)dx}$$

This leads to:

$$E[Bid|v] = v.\frac{\int f(x).Ce(x,v)dx}{\int Ce(x,v)dx} = v.E[X|win.v]$$

Under the assumed equilibrium, the conditional winning bid is equal to observed bid. Therefore, the markup multiplier can be estimated by solving the following equation:

$$Bid_{actual} = E[Bid|v]$$

That is,

$$Bid_{actual} = v.E[x|v]$$

Or equivalently:

$$\hat{v} = \frac{Bid_{actual}}{E[x|win,v]}$$

3. Case Study

3.1 Data

This study is based on simulated data generated to represent bidding behavior in public procurement. The simulation aims to reproduce a semi-realistic competitive environment involving 15 construction firms competing over 38 projects with different characteristics and cost levels. The generated data take the following form:

Table 1. bids data

Project_ID	Company_ID	bid
4	1	25786634
5	1	23616344
7	1	32053292
1	2	24388396
5	2	16811726
7	2	19735417
1	3	29146188
4	3	19256484
5	3	21321708
2	4	30298616
1	5	20133643

Source: Simulation results generated by the author using R

Results of firm and project parameter estimation:

The iterative estimation algorithm showed rapid convergence after only 12 iterations, confirming the efficiency of the proposed estimation procedure.

Using the "BidAnalysis" package available on GitHub[8] with use The function “Estimate_Bid_Parameters” in the R environment the result parameters estimation show as follows:

Table 2. Results Estimation Companies Parameters

Company_ID	(μ_i)	(ϑ_i)	Number of bids
1	17.14103	0.087783	15
2	16.79719	0.102933	16
3	17.05124	0.052102	15
4	17.06026	0.083296	15
5	16.82305	0.116135	16
6	16.9972	0.095618	15
7	16.9972	0.10072	14
8	17.06877	0.118484	12
9	17.3151	0.077116	15
10	17.07217	0.126091	15
11	16.91833	1.083263	16
12	17.17759	0.127541	15
13	17.13917	0.136941	15
14	16.95506	0.113447	14
15	17.23772	0.123094	14

Source: author’s name, year, page

Table 3. Estimated parameters of the last eight projects

Project_ID	(β_i)	Number of

		bidders
30	-0.126073	7
31	0.130052	8
32	-0.048690	4
33	-0.045383	5
34	-0.308090	5
35	-0.187749	8
36	-0.210245	3
37	0.344348	2

From the estimated parameters of firms and projects, it is observed that firms differ in their average cost levels. The results indicate that firms 2 and 5 exhibit a clear competitive advantage. The projects parameters β_i reflect the level of project complexity.

3.2 Model application

Application of the model on training data:

The last project in training data is the project No.37 which include tow participants (firm No.5 and firm No.11), The model was implemented using the "*BidAnalysis"* package on the last project in the training dataset (Project No. 37). The aim is to infer and analyze the bidding strategies adopted by participating firms, as well as their efficiency in this specific project. This analysis focuses in particular on comparing two levels: actual pricing and optimal pricing.

In addition, applying the model to the last project in the test dataset aims to detect actual and preferred strategy and assist contracting authorities in evaluating submitted bids.By applying the "Analyzing_Strategy" function from the same package to Project No. 37, the following results are obtained:

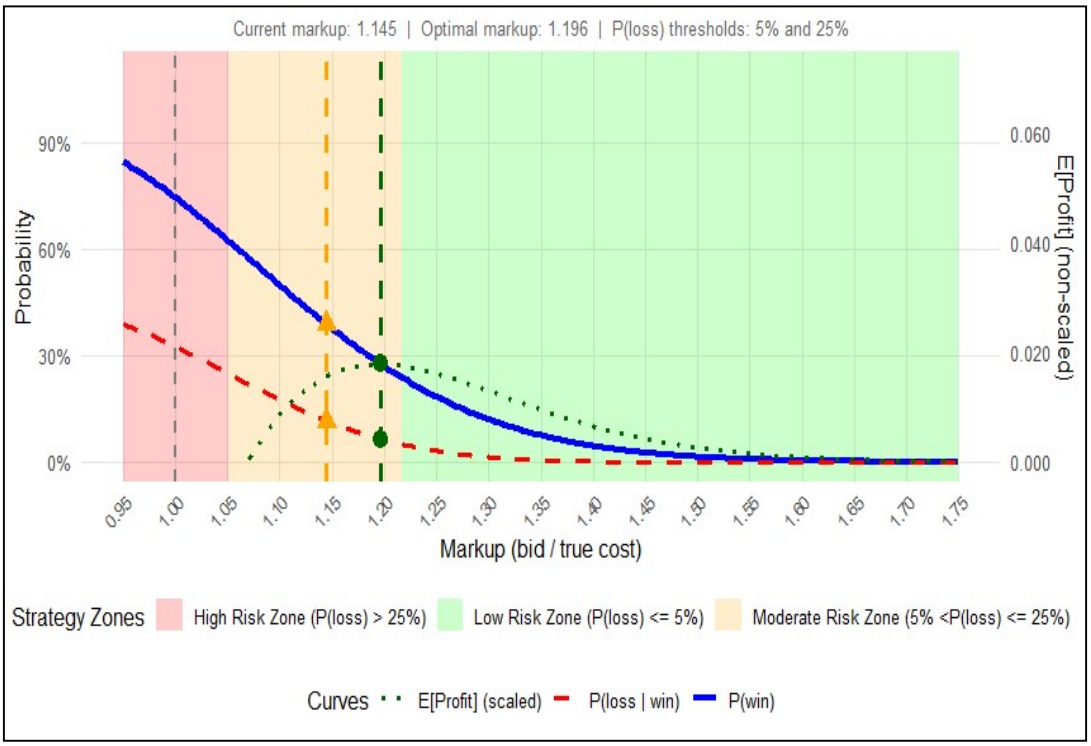
Table 4. Analyzing the winning bid in Project No. 37

Indicators	Wining firm	Second-lowest Bidder
Actual Bid	29 900 000.00	30 000 000.00
Win Probability	0.7476	0.2524
Conditional Win Cost	27 622 749.00	29 452 064.00
Estimate Mark-Up	1.1448	1.0175
Optimal Mark-Up	1.1965	1.1163
Mark-Up at 5% Loss	1.2174	1.1925
Mark-Up at 25% Loss	1.051	0.9392
Max Expected Profit	0.01784	0.002024
Current Expected Profit	0.015605	-0.010382
Profit Improvement (%)	-14.32	-119.49
Optimal P(win)	0.2781	0.0406
Current Loss Probability	11.64%	16.57%

Source: Author's own elaboration based on the results of the proposed probabilistic model using R.

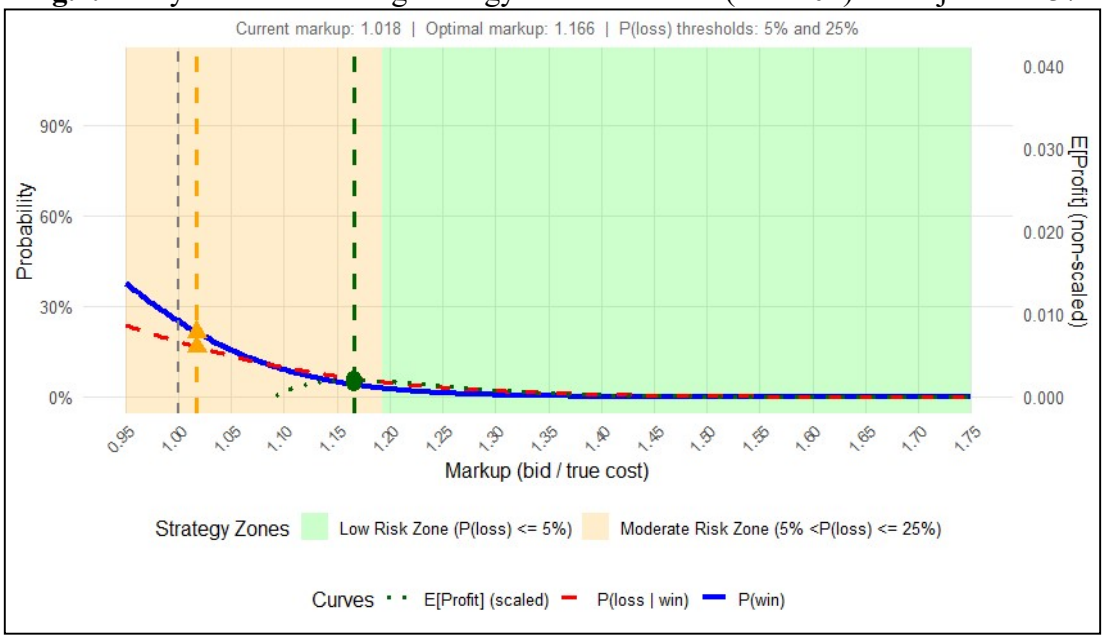
The output also includes Figure 01 below:

Fig.1. Analysis of the bidding strategy of the lowest bidder in Project No. 37



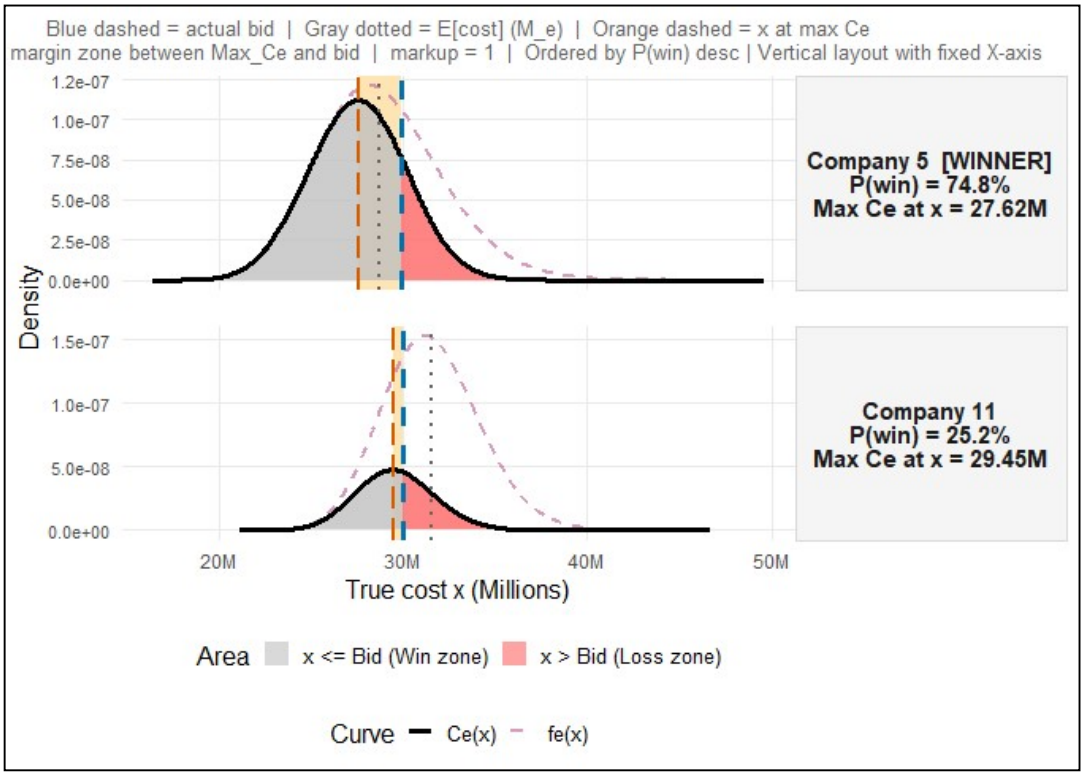
Source: Author's own elaboration using proposed model in R.

Fig.2. Analysis of the bidding strategy of Firm No. 11 (Rank 02) in Project No. 37



Source: Author's own elaboration using proposed model in R.

Fig.3. Bidders Ce(x) function in Project No. 37



Source: Author's own elaboration using proposed model in R.

Figure 01 and the above Table 1 illustrate the effect of mark-up on probability of winning, probability of loss, and expected profit, in addition to the three strategic regions (risky, balanced, and no-loss zones). The results indicate that Firm 05 adopted a moderate strategy with an estimated markup of 14.48% (orange line). This level is relatively low compared to the optimal mark-up (green line). In order to increase its probability of winning from 27.81% to 39%, also increased its financial risk, with the probability of financial loss reaching 11.77%. and the loss probability curve (red) stays clearly below the winning probability curve.

Figure 01 also shows that in case of adopting the optimal strategy by firm 05 it would have increased profitability by approximately 12.42%, without significantly affecting competitiveness (winning probability close to 30%). The risk curve indicates that the firm's submitted bid lies within a cautious region, reflecting that the firm's attempt to secure the contract without exploiting its competitive advantage.

Figure 02 indicate that Firm 11 adopted a cautious strategy, reducing its markup multiplier to approximately 1.0175 in order to increase its probability of winning from 3.9% to 21.4% (a six fold increase). This was achieved by submitting a bid very close to its expected conditional cost; however, The results also show that the loss probability curve is very close to the winning probability curve, which is another indicator of the fragility of the adopted strategy.

If Firm 11 had won this project, it would have faced a probability of financial loss exceeding 16.5%. Moreover, the negative expected profit (-1.04%) indicates that the submitted bid is not economically viable and fails to ensure a proper risk–return balance, exposing the firm to significant financial risk.

Based on its bid value which very close to its conditional cost, and a high probability of loss comparing to win probability, a negative expected profit, and the strong proximity between the

winning and loss probability curves, the bid of the firm 11 can be classified as an abnormally low bid (Abnormally Low Bid).

Ultimately, No matter how close the bids may be, they are related to two different strategies patterns and risks, so The model is a capable tool to revealing these hidden differences.

Application of the model on test data:

Test data include a project No.38, and participated in it: firm 5, firm 11, firm 7.

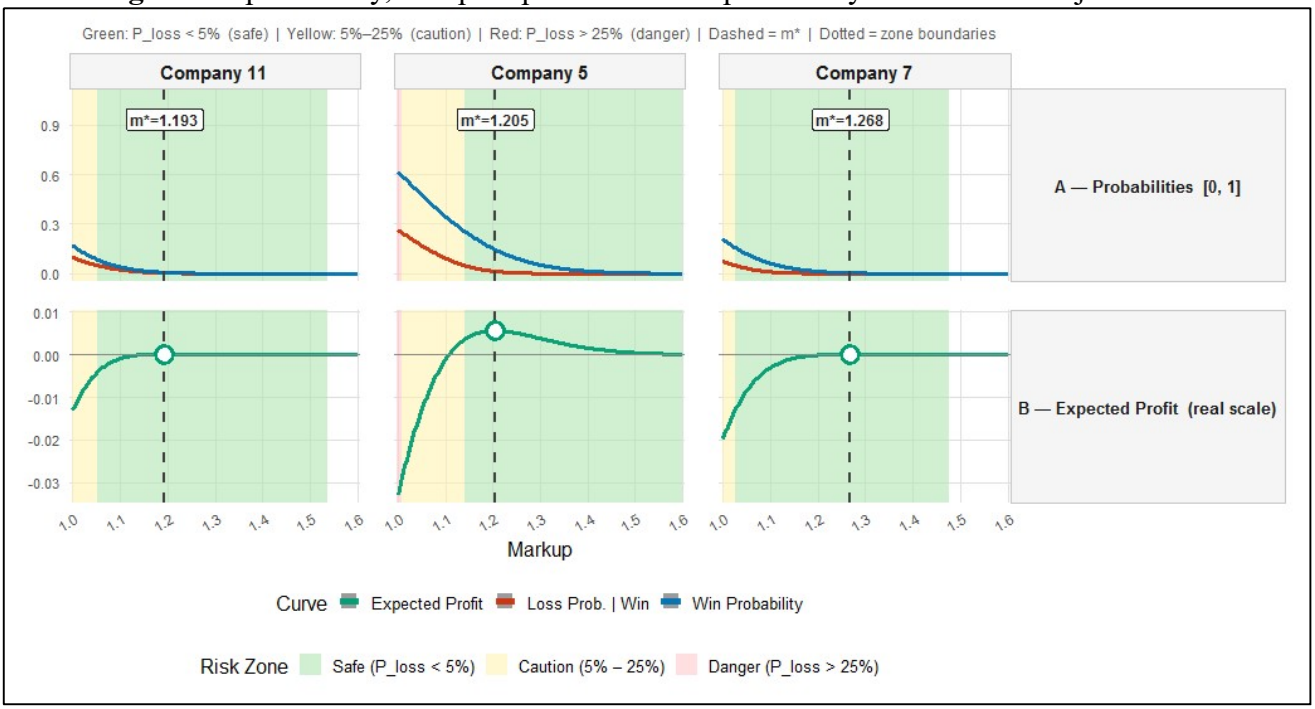
The model gives a set of output indicators that determine the competitive performance of each firm in test data. By applying the "Analyzing_Strategy_test" function from the *BidAnalysis* package in R, the following results are obtained:

Table 5. bidders main indicators in Project No. 38

Company	Win Probability at Mark-up=1	Optimal Mark-up	Max Excepted Profit	Win Probability at Opt Mark-Up	Loss Probability at Opt Mark-Up	Conditional Winning Cost
5	0.6197	1.205025	0.005661048	0.146494797	0.0137135303	19164861
7	0.2099	1.268342	0.000087195	0.003836024	0.0000848702	19971921
11	0.1704	1.192965	0.000203312	0.007546292	0.0041896745	20389833

Source: Author's own elaboration based on the results of the proposed probabilistic model using R.

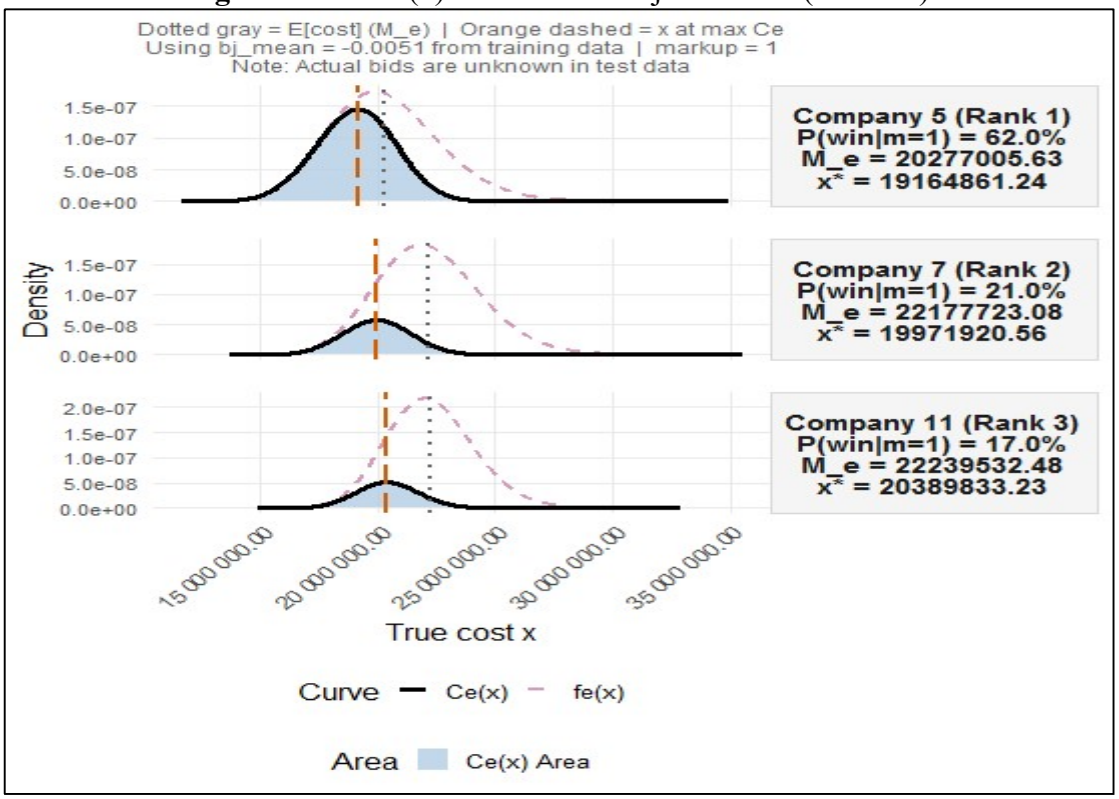
Fig.4. Win probability, excepted profit and Loss probability functions in Project No.



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Source: Author's own elaboration based on the results of the proposed probabilistic model using R.

Fig.5. bidders Ce(x) function in Project No. 38 (test data)



Source: Author's own elaboration based on the results of the proposed probabilistic model using R.

Reconciling Table 5 with the Figures (4, 5) ,confirm that the firm No.5 has a large structural competitive advantage to win project over firm No.7 and firm No.11, which appear a lowest conditional cost, and greatest excepted profit among the three competitors, also it possess a larger strategic space which provides a wider solutions for adjusting its pricing decisions and manipulating mark-up without risks, this is reflected in the winning probability curve (blue), which remains far from the conditional loss probability curve(red).

In contrast, other companies have a low competitive efficiency and insufficient strategic scope, this appear in their conditional cost density, which is concentrated in a higher value compared to Company No. 5.

4. CONCLUSION

In this research, we addressed an important public procurement issue, primarily developing a quantitative model that allows contractors to assess the price they should bid while considering the financial risks associated with them (in relation to their bidding strategy). It is assumed in the model that bids are log-normal distributed; the parameters of the log-normal are calculated using the least squares method and provide a rapid convergence (less than 12 iterations), thus proving that the model is efficient and feasible for practical implementation. Based on these estimates, the model was developed using the cost density function, Ce (x), which is based on the available competitive ability of contractors and their corresponding internal cost uncertainty, weighted by the probability of winning.

The major findings of this research include:

With respect to bidding strategies; The use of the model with respect to the previous training project (Project No. 37), a winning contractor with a historical competitive advantage of winning

(conditional win ratio = 61%) used an aggressive pricing strategy with a marginal markup of approximately 1.104 which increased the contractor's financial risk of 12% and reduced his expected profit by approximately 15% from his optimal profit level.

Thirdly, in terms of decision-making locations; The model identifies three pricing strategy regions:

A risk region, characterized by P(loss l win) > 25%, should be avoided.

A caution zone where 5%< P(loss ｜ win)<25%, requiring monitoring.

A safe zone where P(loss ｜ win)<5%, considered the optimal pricing region.

Recommendations:

For construction firms: Avoid the aggressive strategy which is accompanied with financial risk, and stay the pricing close to the optimal position that ensure an acceptable input.

For contracting authorities: Mathematical tools and risk indicators offered in the model should be utilized by contracting authorities to detect potential abnormally low bids prior to awarding a contract so as to avoid potential delays or terminating a contract prior to completion.

For researchers: This research provides new avenues for future research--e.g. learning and experience(i.e., learning curves), asymmetric auction model extensions, and validation of the model through various construction industry sectors to measure its generalizable nature.

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