

Numerical investigation on the behavior of saturated clayey sloping soils submitted to seismic action

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Abstract

The present paper treats the stability of the saturated clayey sloping soil under a seismic action (under the principle of total stress (PTS)). The influence of rupture criterion of Drucker-Prager is treated in comparison to the Mohr-Coulomb criterion. Moreover, the reformulation of the safety factor (Fs) is made to include all types (rigid or flexible) and nature (normally or overconsolidated) of saturated clay soils. This approach is in fact an improvement of study made by Cascone and al. Also, this numerical study confirms the results obtained in certain experimental studies. A new model is advanced based on the notion of crossing to estimate if a given slope is failed or not. To make our task easier, a visual computing program (with Interactive Database Access software using the concept of object-oriented programming) is created

Keywords : Clayey slope; seismic action; undrained shear strength; Mohr-Coulomb; Drucker-Prager

1. Introduction

The General Limit Equilibrium Method (GLEM) (pseudostatic) is often used to evaluate slope stability. In fact, since Fellenius (1936) [5], who has quantified sloping soil stability with linear safety factor; the GLEM has known much refinement in the aim to approach the in-situ real soil behavior (Bishop, 1955 [1]; Janbu, 1954 [8] [9]; Morgenstern and Price, 1965; Spencer, 1967 [15]; Newmark, 1965 [10] and Zhu, 2005 [20] etc.) [6]. Beginning in the 1920s, the seismic stability of earth structures has been analyzed by pseudostatic approach in which the effects of an earthquake are represented by constant vertical and/or horizontal accelerations. The first explicit application of the pseudostatic approach to the analysis of seismic slope stability has been attributed to Terzaghi (1950) [10]. The vertical component of seismic action has less influence on the factor of safety since it reduces (or increases, depending on its direction) both the driving and resisting forces; as a result, the effects of vertical accelerations are frequently neglected [10].

The basic concept of this method (GLEM) consists to evaluate the stability of sloping soils in terms of ratio (safety factor) of the stabilizing and destabilizing forces, acting on the inclined ground. The sloping soil is

subdivided into vertical slices, delimited by a probable circle of rupture (Figure 1). After that, the equilibrium of all slices is obtained in terms of ratio (safety factor, Eq.1) of the stabilizing and destabilizing forces, acting on this inclined ground [6]. However, beyond the simplicity displayed by the classical pseudostatic (GLEM) method, this last one suffers, however, of some limitations. Indeed, experience has clearly shown, for example, that the classical pseudostatic method can be unreliable for soils that build up large pore pressures or show more than about 15% degradation of strength due to earthquake shaking [10]. Moreover, it is generally recognized that the stress-strain response of clayey soils is dependent upon the rate of strain [3].

In fact, the principle of the effective stress (PES) is applied [20], [6] if the ground does not lose significant resistance under the dynamic forces. Otherwise, the principle of total stresses (PTS) (Under earthquake) is used [3].

The influence of application rate of dynamic load on the rate of dissipation of pore pressure of the water, on the one hand, and saturation and permeability of the soil on the other hand, will be the key factors to estimate which one of the two principles of stresses (effective or total) is appropriate. Indeed if the velocity of application of the charge is more important compared to the velocity of dissipation of the water, in saturated sloping soils with a

feeble permeability, the PTS is more suitable. In laboratory testing, this corresponds to an undrained test.

To approach the in-situ real conditions, and because of the confining intermediate effective normal stress (σ'_2) (which is an inherent property of soil), one proposes in this work to replace of the well known Mohr-Coulomb model (which ignore this intermediate effective normal principal stress in its formulation) by the Drucker-Prager criterion (which includes all principal normal stresses in its formulation). In this study, one reformulates the sloping soil stability to estimate safety through a seismic time analysis (beneath the PTS). This paper inscribes as a continuous work made by Haderbache & al [6] in the frame of PES. This last study has shown that the effect of the effective intermediate normal stress is great when it is used in the frame of the (PES). In the present work, one wants to know the effect of this principal intermediate stress in the frame of the PTS.

The originality of this study is that the key element in studies of sloping soil is, of course, the rupture. Therefore, the modelling of the rupture close to the actual conditions in the soil is more than necessary. Indeed, the flexibility of the formulation of the finite element method (FEM) allows it to use any one of the models of rupture criteria. For the case of classical limit equilibrium method (method of slices), except the model of Mohr Coulomb, this is not possible. Also, this study focuses on the use of rupture criteria of Drucker-Prager and Mohr-Coulomb. Furthermore, in studies of slope stability, one often prefers the method of slices (GLEM) to the FEM; especially for soil under seismic loading. Because, the FEM is very cumbersome to develop, despite its flexibility in the use of any one of rupture criteria. Therefore, this study satisfies the condition of flexibility in the formulation that is lacking in the method of slices. Thus, one obtains a new method that shows how to include any other rupture criterion than the classical criterion of Mohr- Coulomb in its formulation. This improvement is well came because it give a solution for soil saturated submitted to earthquake. Moreover, the degradation of soil's stiffness during cycles of seismic loading is often made in experimental studies [14]. In the numerical studies of sloping soils subjected to seismic action (by using the classical GLEM), except the study of Cascone and al [3], it is almost rarely evidenced. This study is among one of the first that indicates the way to follow for showing numerically the degradation of stiffness of sloping soil by using the GLEM. The evident relation between the reactions of water to the seismic loading is clearly seen in the curves plotted.

It is, also important to note that this study improves the method of Cascone and al [3] to encompass all types (rigid or soft) and natures (normally or overconsolidated) of saturated clayey soils.

But unlike to the static model, it is hardly possible to quantify the rupture of a slope by saying (for the dynamic case) that the $FS > 1$ then the slope is stable. Why? Simply

because in the case of the seismic loading, the accelerations may be alternated between destabilizing (> 0) and stabilizing (< 0) (see Figure 3). Each punctual acceleration will correspond to one punctual FS. Therefore, the probability distribution of FS beneath different seismic accelerations will be particularly important for quantifying the stability of a slope under seismic action.

The nonlinear aspect of FS (because one has a rupture of the slope) imposes us a choice of a non-Gaussian distribution of the peaks. In the literature ([11], [19]), we often refer to the Rayleigh distribution and Weibull distribution to model the distribution of the peaks. That of Rayleigh (Gaussian-type) is, however, a special case of the Weibull (which may be non-Gaussian). In addition to these two proposed distributions, a more direct distribution that takes into accounts the effect of all accelerations upon the slope is proposed in this study. The quantification of the rupture will be modelled by a probability of failure (P_f) for each aforementioned distribution. This approach is more suitable than the punctual FS in the static case. Moreover, the proposed distribution in this study, avoids us the problem shown in Figure 2; i.e. it avoids us the inconvenient of Rayleigh distribution which do not consider two peaks for one up-crossing. One reminds that each FS corresponds to one only acceleration. Here also, one estimates that it is another improvement compared to the work of Cascone and al. This latter has not suggested the way to estimate that the slope will be considered as failed or not. This study considers the effect of all the seismic accelerations. So, one has, in this case, another improvement compared to study like that of Newmark [10], who considers the effect of one only seismic coefficient when he has modelled the effect of the earthquake action on the slope.

To make our task easy, a visual computer program (interactive with access software database and using a concept of programming oriented object -P.O.O-) is developed.

2. Stability of slopes subjected to the principle of total stress (PTS)

The pseudo-static procedures (in the frame of the PES) are appropriate only for cases of soils that do not lose significant strength during of application of a dynamic loading [3], [4] [14]. Otherwise, the formulation will be more rigorous if it is treated in terms of total stresses. In this case, the lost of resistance during an earthquake, is due to the pore pressure buildup along the anticipated slip surface. This phenomenon induces a decrease of the surface contact between particles of soil, then by consequent a decrease of the effective stress [16]. In the laboratory tests, this behavior corresponds to an undrained test. When we deal with saturated clay soils of low permeability, the

seismic loading velocity (compared to the dissipation of water velocity) has a great importance in the choice of the calculation method used. So if the seismic velocity is greater than the rate of dissipation of the water, the use of the formulation of the total stress is required. Otherwise, the calculation must be done with PSE.

2.1- Problem formulation, for the case of saturated clay sloping soils subject to the principle of total stresses (PTS).

The safety factor (FS) expression, as established by Cascone and al [3] for the slope failure mechanism shows in Figure 1, is given by:

$$Fs(N) = \frac{\sum_{i=1}^n C_{uNi} \eta_i}{\sum_{i=1}^n \left[W_i (1 - K_c \sin \omega) \sin(\alpha_i) + K_c W_i \cos \omega \frac{b_i}{R} \right]} \quad (1)$$

With:

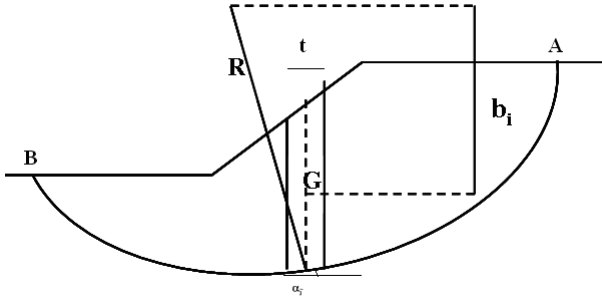


Figure 1: Probable rupture circle
(For details see section 4) [6]

$$\eta_i = t_i \sec(\alpha_i) \quad (2)$$

C_{uNi} : undrained shear strength at successive cycles of seismic loading; K_c : seismic acceleration coefficient (the module of seismic acceleration normalized with respect of gravity acceleration g), n is the number of slices (280 slices are used in this study); W_i is the weight of slice i; α_i is the slope of slice i; t_i represents the thickness of the slice i; $\omega (= 0$; because, one considers in this study only the horizontal acceleration) is the angle of seismic acceleration to the horizontal. Fs is the safety factor. For b_i and R see Figure 1; i is the number of the slice.

Hence, according to [7], [12] and [17], one writes:

$$C_{uN} = C_0 N^{-d} \quad (3)$$

Where, N is the loading cycles to the initial undrained shear strength C_0 . In this study, one has supposed that the counting of cycles is done with reference to the number of zero-crossing of the accelerogram; (d) is an experimental parameter [3]. In the present analysis, (d) is assumed to be constant, and its values are ranging between: [3]

$$d = 0.01; 0.05; 0.1; 0.15; 0.2.$$

As suggested by Yasuhara [14], the undrained shear strength at successive cycles of seismic loading given by Eq. (3) is better considered by using $(1-d \cdot \log_{10}(N))$ rather than (N^{-d}) . Thus, in this study, one replaces (N^{-d}) by $(1-d \cdot \log_{10}(N))$ in Eq. (3). Therefore, each once that one will speak in the rest of this paper of the Eq 3, this change of (N^{-d}) by $(1-d \cdot \log_{10}(N))$ is implicitly supposed as being made.

However, the undrained cohesion is generally obtained by experimental means. Therefore, one proposes, in this work, of replace the experimental values of undrained cohesion by mathematical formulations. In these last ones, the undrained cohesion is formulated by using the model of Mohr-Coulomb criterion the first once and the Drucker-Prager criterion the second once. One thinks that this approach is advantageous, because it will permit us to show the effects of the two models of rupture on the slope stability. This will also permit us to avoid the unnecessary fees generated by undrained experimental cohesion. One reminds that in the PES [6], one had had a great influence on the stability of the slope by comparing the two criteria. At the end, this mathematical reformulation of undrained cohesion, included in the Eq.1, will permit us, to have safety factor versus Skempton parameter (A). This action is beneficial, because the soils overconsolidated and normally consolidated, may be studied; which is not possible if one uses the old formulation of the Eq. 1; i.e. if one does not use the proposed reformulation of undrained cohesion.

2.2. Undrained cohesion established by using the Mohr-Coulomb criterion (without (σ'_2) stress)

The formulation of undrained cohesion is established here below as shown in [13]:

$$q = 2C_0 = \sigma_1 - \sigma_3 = \sigma'_1 - \sigma'_3 \quad (4)$$

Where, C_0 is the initial undrained shear of soil; i.e. at $d = 0$; σ_1, σ_3 are respectively the major and minor totals principals normal stresses, σ'_1, σ'_3 are respectively the major and minor effectives principals normal stresses and q is deviator stress.

With the total stress, one can writes, [13]:

$$q = \sigma_1 - \sigma_3 = \sigma_0 + \Delta\sigma_1 - K_s\sigma_0 - \Delta\sigma_3$$

Rearranging the above equation leads to:

$$q = (1 - K_s)\sigma_0 + (\Delta\sigma_1 - \Delta\sigma_3) \quad (5)$$

Where, K_s is the classical earth coefficient at rest; σ_0 is the in-situ principal major stress before seismic loading action; $\Delta\sigma_k$ is the variation of principal major or minor stresses.

Moreover, one of the forms of Mohr-Coulomb criterion (based on the PES) is:

$$\sigma_1' - \sigma_3' = 2 \left[\frac{\sigma_3' \sin(\varphi') + C' \cos(\varphi')}{1 - \sin(\varphi')} \right] \quad (6)$$

After some manipulations, the equation (6) becomes:

$$C_0(1 - \sin(\varphi')) = \sigma_3' \sin(\varphi') + C' \cos(\varphi') \quad (7)$$

One supposes for the undrained case [2], [16] and [13], the following assumptions:

$$\begin{aligned} \Delta\sigma_1' &= \Delta\sigma_1 - \Delta u \\ \Delta\sigma_3' &= \Delta\sigma_3 - \Delta u \end{aligned} \quad (7-1)$$

Where, Δu is the excess pore pressure of the water.

At the undrained state (constant volume), this excess pore pressures of water in the soil is equal to the difference between the stresses of the soil after and before of application of seismic load. To keep a volume constant, the stresses (loss by the soil) are substituted by the incremental excess pore pressures of water. Then, according [13], one can write:

$$\sigma_3' = \sigma_3 + \Delta\sigma_3 - \Delta u$$

$$\text{With: } \Delta u = B[\Delta\sigma_3 + A(\Delta\sigma_1 - \Delta\sigma_3)]$$

For saturated soil, $B = 1$, then the minor effective normal stress becomes:

$$\sigma_3' = K_s\sigma_0 - A(\Delta\sigma_1 - \Delta\sigma_3) \quad (8)$$

Where, (A) and (B) are the famous parameters of Skempton (1954).

Substituting (8) into (7) and replacing (q) by $2C_0$ in (5) lead respectively to equations (9) and (10):

$$\begin{aligned} C_0(1 - \sin(\varphi')) &= K_s\sigma_0 \sin(\varphi') - \\ &A \sin(\varphi')(\Delta\sigma_1 - \Delta\sigma_3) + C' \cos(\varphi') \end{aligned} \quad (9)$$

$$\Delta\sigma_1 - \Delta\sigma_3 = 2C_0 - (1 - K_s)\sigma_0 \quad (10)$$

Finally, the combination of equations (9) and (10) leads to: [13]

$$\begin{aligned} C_0 &= \frac{\sigma_0 \sin(\varphi')(K_s + A(1 - K_s))}{1 + \sin(\varphi')(2A - 1)} \\ &+ \frac{C' \cos(\varphi')}{1 + \sin(\varphi')(2A - 1)} \end{aligned} \quad (11)$$

Eq. (11.), thus established, may be used in Eq. (3)

In the present analysis, (A) is assumed constant, and its values are ranging between:

$$A = -0.5 ; -0.4 ; -0.3 ; -0.2 ; -0.1 ; 0.2 ; 0.4 ; 0.6 ; 0.8.$$

It is important to note that the negative values of A ($A < 0$) wants to say that the soil is overconsolidated. Otherwise, if ($A > 0$), the soil is much more normally consolidated. Eq (11), thus established, may be used in Eq. (3).

2.3. Undrained cohesion established by using the Drucker-Prager criterion (with (σ_2') stress)

Drawing on a methodology similar to that in paragraph 2.2 above, we shall establish in this study the shape of the undrained cohesion based on the criterion of Drucker-Prager.

$$\sigma_1' = \sigma_0 + (1 - A)(\Delta\sigma_1 - \Delta\sigma_3) \quad (12)$$

For q' , p' by using $\sigma_2' = \sigma_3'$, one found (see Appendix) [6], [18]:

$$p' = \frac{\sigma_1' + 2\sigma_3'}{3} \quad (13)$$

$$q = \sigma_1 - \sigma_3 = \sigma_1' - \sigma_3' \quad (14)$$

Where: σ_1' , σ_3' are the effective normal stresses major and minor. σ_1 , σ_3 : are the total normal stresses major and minor

By substituting equations (8) and (12) into (13), one writes (see Appendix) [6]:

$$P' = \left(\frac{1+2K_s}{3}\right)\sigma_o + \left(\frac{1-3A}{3}\right)(\Delta\sigma_1 - \Delta\sigma_3) \quad (15)$$

The Drucker-Prager criterion is (see Appendix) [6]:

$$q' = \left(C' M / \tan(\phi')\right) + MP' \quad (16)$$

Compared to Eq (6), Eq. (16) has the advantage that the parameters, P' (Eq. 13) and q' (Eq. 14) are invariants terms, and another advantage of Eq (16) is that it includes the effect of σ'_2 in its formulation, but Eq (6) does not encompass this effect of σ'_2 .

Hence, relation (15) into equation (16) leads to:

$$q' = \left(\frac{1+2K_s}{3}\right)M\sigma_o + \frac{C' M}{\tan(\phi')} + \left(\frac{1-3A}{3}\right)M(\Delta\sigma_1 - \Delta\sigma_3) \quad (17)$$

Finally, by combining equations (10), (17) and (4), one derives the expression for C_o (in the frame of Drucker-Prager criterion) as below:

$$C_o = \frac{(\mu_1 - \mu_2(1 - K_s))\sigma_o}{2(1 - \mu_2)} + \frac{C' M}{2(1 - \mu_2)\tan(\phi')} \quad (18)$$

$$\mu_1 = \frac{(1+2K_s)M}{3}$$

With,

$$\mu_2 = \frac{(1-3A)M}{3}$$

Also, Eq (18), thus established, may be used in Eq. (3).

2.4. Methodology

The methodology to follow to calculate the F_s of the Eq.1 begins by choosing the rupture circle the most probable. Then, by subdividing the circle in question by vertical slices (280 in our case), it is possible to calculate the Eq.1. When one arrives to the step of calculation of C_o (Eq.3),

this last one is replaced for the first once by the Eq.11 and for the second once by the Eq.18.

3. The crossing

3.1. Introduction

In the process of deformation of sloping soil submitted to seismic loading, the obstacle which will interest us firstly, is to know what criteria to use to estimate that a considered slope has reached the rupture or no? In the frame of the static load, the safety factor (F_s) will play the role assigned to the rupture criterion ($F_s \leq 1$ one has rupture). But, if the soil is submitted to seismic loading, the rupture of the sloping soil can not be estimated for one only safety factor (corresponding to only one accelerating of seismic load), but, it is an estimate upon of all accelerations which compose the seismic loading. Indeed, if a punctual given seismic acceleration is destabilizing (> 0) temporarily, the following acceleration, can stabilize (< 0) temporarily the sloping soil. Such stabilizing and destabilizing nature of the seismic accelerations demolishes the bearing capacity of the slope and generates the degradation of its shearing stiffness. The degradation of shearing stiffness is represented by the undrained shear strength during the process of seismic loading. So, the rupture or not of sloping soil, must be done upon all the accelerations which compose the seismic loading.

Thus, one has to study the response to seismic excitation; symbolized in this study by the F_s . The distribution of the peaks of this output (F_s), in this case, will have special significance.

3.2. The distribution of the peaks in the literature

Among the distributions modeling the distribution of peaks, one will present those established by Rayleigh and Weibull. The Rayleigh distribution is based mainly on the relationship of Rice (1944), giving the rate of crossing following: [11]

$$\nu_a = \int_{m_{FS}}^{\infty} p(a, \dot{y}) \dot{y} d\dot{y} \quad (19)$$

$p(a, \dot{y})$ Is the joint distribution; ν_a represent the crossing rate; m_{FS} is the average of the safeties factors (F_s); $\dot{y}$ represent the derivative of F_s .

If $p(a, \dot{y})$ follows a normal distribution, the crossing rate ν_a will have the following form: [11]

$$\nu_a = \frac{\sigma_{FS}}{2\pi\sigma_{FS}} e^b ; \quad (20)$$

$$b = -\frac{(a - m_{FS})^2}{2\sigma_{FS}^2}$$

Where, (a) being the value of crossing and σ_{FS} is the standard deviation of FS.

$$P(\text{Peak} > a) = \frac{\nu_a}{\nu_{m_{FS}}} \quad (21)$$

$$P(\text{Peak} > a) = e^c ; \quad (22)$$

$$c = -\left[\frac{a - m_{FS}}{2\sigma_{FS}^2} \right]$$

If one decides to establish the probability of failure for a value of crossing ($a=1$), one writes:

$$P(\text{Peak} \leq a) = 1 - e^c ; \quad (23)$$

$$c = -\left[\frac{a - m_{FS}}{2\sigma_{FS}^2} \right]$$

Here, one takes the $FS = a = 1$ (see Figure 2), because the $FS \leq 1$ implies that the stabilizing forces are $\leq$ than the destabilizing ones. In this case, the slope will fail. Thus, Eq (23) gives us the probability that the FS will be $\leq a = 1$; i.e. the probability of failure.

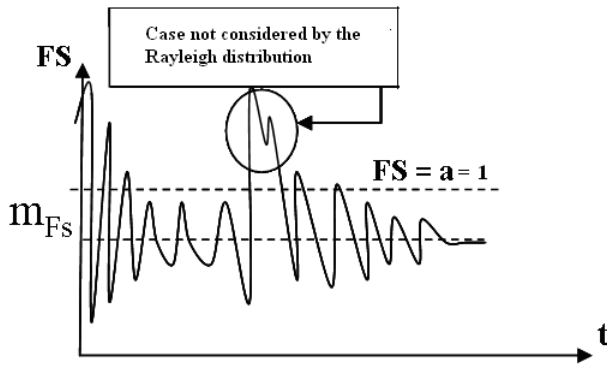


Figure 2: More than one peak for the same up crossing is not considered by Rayleigh distribution

The Weibull distribution peak, which is more general, is not based on the Gaussian assumption ([11], [19]). Indeed, in a number of applications, the Gaussian assumption is not valid and, in this case, the distribution of peaks will be far from the Rayleigh distribution. Mention will be made for this purpose the calculation of wave inducing bending moments in ships, or the calculation of buildings subjected to wind load [11]. The distribution of Weibull [11] is:

$$P\left(\frac{\text{Peak}}{a_0} > \frac{a}{a_0}\right) = e^d ; \quad (24)$$

$$d = -\text{Ln}(2) \left[\frac{a - m_{FS}}{a_0 - m_{FS}} \right]^k$$

$$P\left(\frac{\text{Peak}}{a_0} < \frac{a}{a_0}\right) = 1 - e^d ; \quad (25)$$

Where:

$$d = -\text{Ln}(2) \left[\frac{a - m_{FS}}{a_0 - m_{FS}} \right]^k$$

$$a_0 = \sigma_{FS} \sqrt{2\text{Ln}(2)} + m_{FS} \quad (26)$$

Note that for $k = 2$ (Eq 25), one will have the Rayleigh distribution (Eq 23). For $k \neq 2$ corresponds to an approximation for other distributions more general or non-Gaussian.

3.3. The distribution of the peaks proposed in this study

By retaking Eq 22 with the following form:

$$\nu_a = N_{Kc(FS < a)} \quad (27)$$

$$\nu_{m_{FS}} = N_{Kc} \quad (28)$$

$N_{Kc(FS < a)}$: Number of all accelerations Kc , where $FS \leq a = 1$ (Figure 3); N_{Kc} : Number of all accelerations in the Fig. 3.

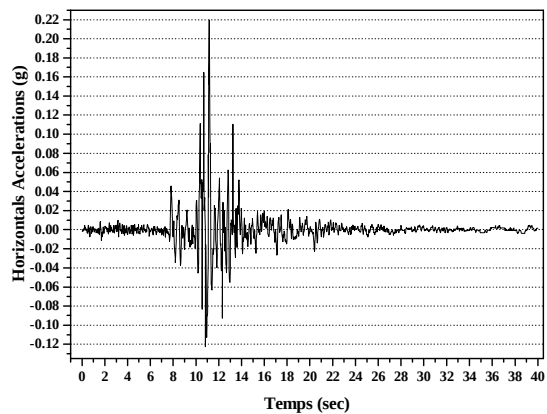


Figure 3: Record of the seismic horizontal accelerations. ($\Delta t = 0.02$ s)

Following the formulas (27) and (28), one will establish the probability of failure of our slope as:

$$P(\text{Peak} < a) = \frac{N_{KC(FS < a)}}{N_{KC}} \quad (29)$$

The expressions of N (Eq 27, 28), are easily obtained by using a query in an Access database, because this program is interactive with this kind of database. This query is filtered by the following criterion: $FS < a = 1$. This process will avoid us the problems of Figure 2 (here, each value of acceleration will correspond to one value of FS); i.e. it avoids us the inconvenient of Rayleigh distribution which do not consider two peaks for one up-crossing. One reminds that each FS corresponds to one only acceleration.

3.4. Methodology

Hence, Eq 23 and 25 (for different values of k parameter) and 29 are used to estimate the probability of rupture of our sloping soil. These equations will be established by making the calculation of m_{FS} and σ_{FS} for the FS ; with (a) is equal to the unity ($a=1$).

4. Example

In order to study the influence of the effective principal intermediate normal stress, the 2 layers of the sloping soil (Figure 4) are considered with characteristics given in Table 1. The two layers of sloping soil are considered as wholly saturated with feeble permeability and the seismic vertical load is neglected. The component of the seismic horizontal load is shown in Figure 3. The calculation under the PTS is performed to analyze the stability of slope.

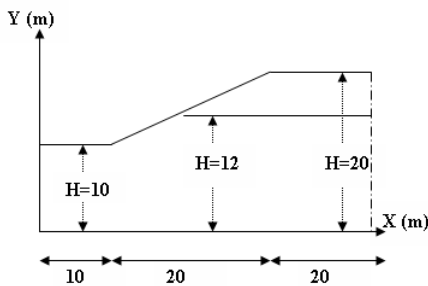


Figure 4: Example of sloping soil with two saturated layers. (For details see Table 1) [6]

Table 1

Characteristics of two layers (Figure 4) sloping soil [6]

N° Layer	1	2
Height (m)	8	12

C' (KN/m ²)	28.74	23
γ_{Sat} (KN/m ³)	18.85	19
ϕ' (Deg)	20	18

280 slices had been used in this example

γ_{Sat} : Saturate density of soil (KN/m³)

ϕ' : Effective friction angle (Degree)

C' : Effective Cohesion (KN/m²)

H : Height of the layer (m)

The coordinates of the circle of failure most probable (Seek the circle of rupture most probable is not the aim of this study) are: Radius $R = 20.6m$, circle centre C ($x_C = 16m$ and $y_C = 22m$) and the point A ($x_A = 34.8m$ and $y_A = 20m$) and B ($x_B = 2m$ and $y_B = 10m$) (See Figure 1). 280 slices of soil are used in this example.

For the crossing calculation, the same example of Figure 4 with characteristics given by Table 1 is used. The seismic excitation is given by the Figure 3.

5. Presentation of results

5.1. Results of the slope stability beneath seismic action

The results obtained by programming of the Eq.1 (By taking first once C_0 , by using Eq. 11 and another once by using Eq 18) are concentrated in Figures 5, 6, 7, 8, 9. The figures 5 and 6 are plotted to see the effect of the presence of σ'_2 or no in the formulation of the safety factor versus the time (Figure 5) and for peak ground acceleration (Pga) equal to 0.22 g (time=11.16 s) (Figure 6).

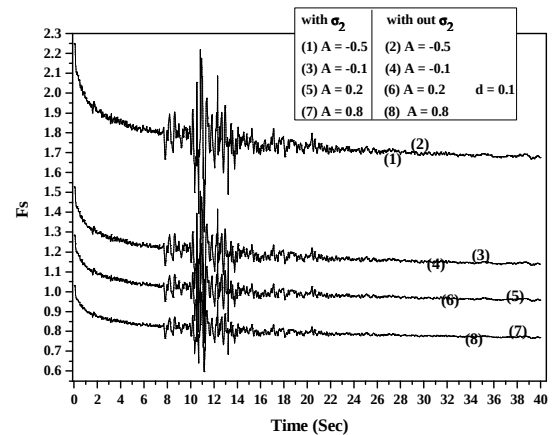


Figure 5: The influence of the parameter (A) computed with and without (σ'_2)

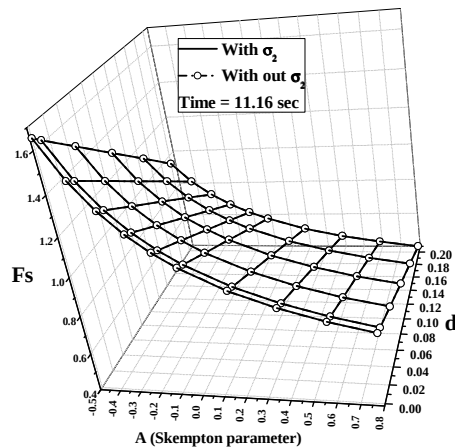


Figure 6: the safety factor versus A and d, computed beneath peak ground acceleration (= 0.22 g)

The Figure 7 shows the influence of parameter (d) versus time; for constant parameter (A = - 0.5).

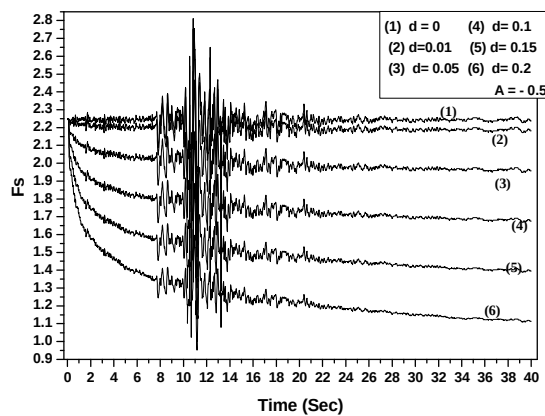


Figure 7: The influence of the (d) parameter

The figure 8, is plotted to show the importance of the loss of the resistance between the first F_s (at $t = 0.02s$) and the last F_s (at $t = 40s$) under the horizontal seismic loading of the Figure 3. Thus the formula used for the Figure 8 is:

$$\frac{(F_s(t = 0.02 \text{ sec}) - F_s(t = 40 \text{ sec})) * 100}{F_s(t = 0.02 \text{ sec})} \quad (30)$$

Where, t is the time in second (sec).

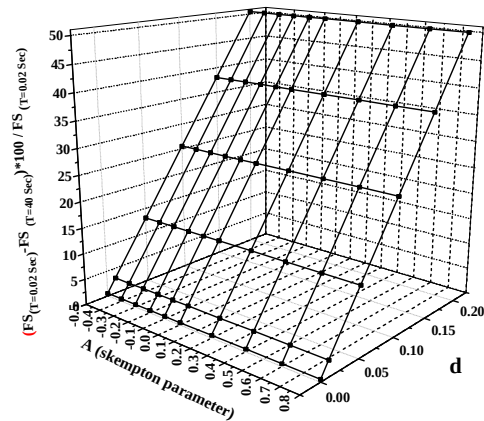


Figure 8: Reduction of soil resistance between the first and the last F_s , Versus the two parameters (A) and (d)

The equation (30) models the fall of the resistance of the soil, from the beginning to the end of the time of the applying of seismic loading; i.e. between the first and the last F_s .

At the last, one plots the Figure 9 to show the lost of resistance for the first cycles of zero crossing.

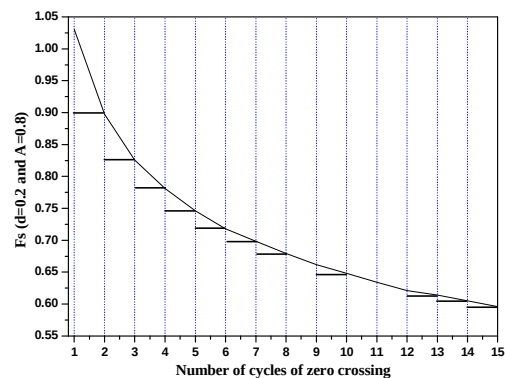


Figure 9: The fall of the resistance of soil, with the firsts cycles of zero crossing.

5.2. Results of the crossing

The results obtained by programming the equations 23, 25 and 29, are concentrated in Figures 10, 11, 12, 13, 14 and 15. Thus, Figure 10 ($d = 0.05$) is plotted to compare the failure probabilities calculated by Eq 23, 25 ($k = 1, 3$ and 4) and 29. To avoid overloading Figure 10, Figure 11 ($d = 0.05$) is plotted to compare Eq 29 and 25 ($k = 4, 5, 6$ and 7).

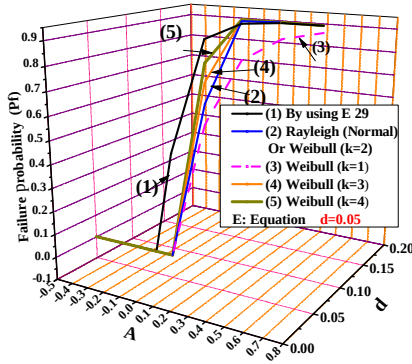


Figure 10: The failure probability of the slope versus Skempton parameter (A) and $d = 0.05$ ($k = 1$ to 4)

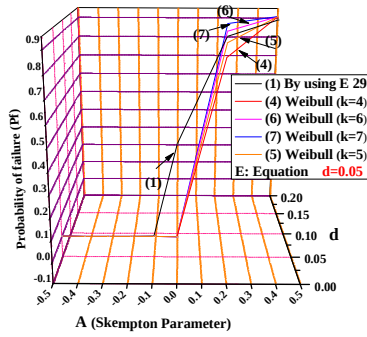


Figure 11: The failure probability of the slope versus Skempton parameter (A) and $d = 0.05$ ($k = 4$ to 7)

One will remind, for this purpose, that Eq 23 (Rayleigh) is none other than Eq 25 (Weibull) with $k = 2$.

However, the figure 12 (with $A = -0.2$) is plotted to compare Eq 23, 25 ($k = 3, 4$) and 29. In the same manner and to avoid overloading Figure 12, Figure 13 ($A = -0.2$) is plotted for Equations 29 and 25 ($k = 4, 5$ and 7).

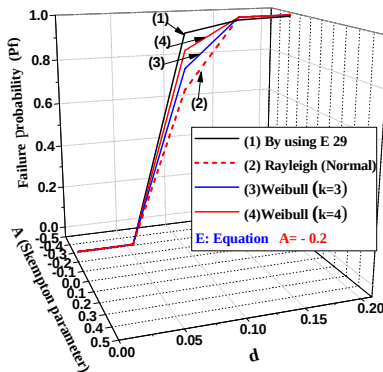


Figure 12: The failure probability of the slope versus Skempton parameter ($A = -0.2$) and d ($k = 3$ and 4)

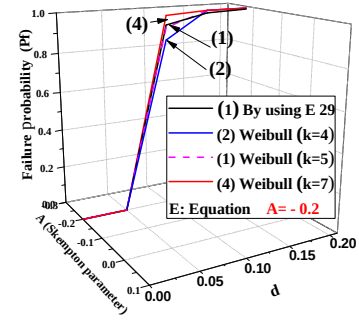


Figure 13: The failure probability of the slope versus Skempton parameter ($A = -0.2$) and d ($k = 4, 5$ and 7)

At the end, the Figures 14 and 15 show the evolution of the probability of failure (Pf) as a function of Skempton parameter (A) and parameter (d).

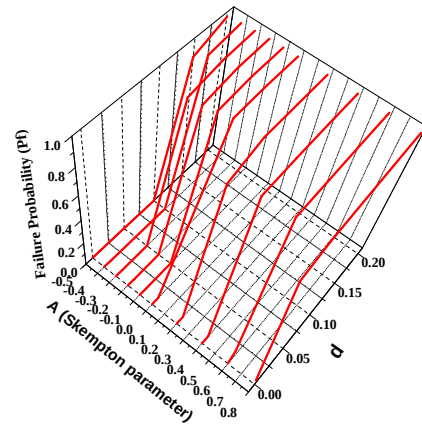


Figure 14: The failure probability of the slope versus Skempton parameter (A) and (d) by using E 29.

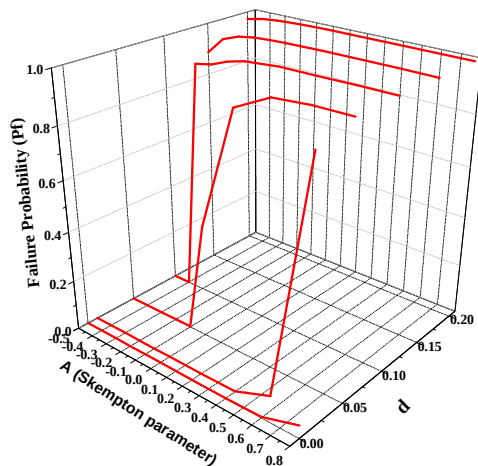


Figure 15: Other view of the failure probability of the slope versus Skempton parameter (A) and (d) by using E 29.

6. Discussions

According to the Figures 5 and 6, the plotted curves do not show any influence of the effective intermediate normal stress (σ'_2). To explain this result, we must know that in the case of the PES [6], the soil (effective stress) and the water (pore pressure) participate together to retaking the external load. However, in the undrained case (by using PTS), the soil does not take part in the resumption of the earthquake loading (Figure 3), because the seismic velocity is greater than the water dissipation velocity. In this case, the contribution of (σ'_2) is null; especially at the first cycles of zero crossing. The excess pore pressures of water in the soil are equal to the difference between the stresses after and before of application of seismic load [2]. One thinks, however, that this conclusion seems pragmatic; being given that the deviator of principal stresses (total and effective; see Eq.14) is constant. Therefore, the undrained cohesion is constant too, for each incremental time treated.

Figures 5 to 7 show the variation of the safety factor (Fs) versus of the Skempton parameter (A) and the experimental parameter (d) (for PGA equal to 0.22 g in Figure 6). In general, (Fs) (with or without (σ'_2)) decreases with the increase of both (A) and (d) which means a strength degradation (implicitly represented by a degradation of undrained shear stress C_u (Eq 3)) of corner sliding (see Figures 5, 6 and 7).

These results are in agreement with the fact that the negative values of (A) implicates that the soil is overconsolidated, otherwise if ($A > 0$), the soil is much normally consolidated. One knows that the stresses (especially, the stresses of confinements (σ'_2 , σ'_3) are more powerful for the soil overconsolidated ($A < 0$) than the soil normally consolidated ($A > 0$). The influence of

incrementing of the water pressures (which have as formula:

$$\Delta u = B[\Delta\sigma_3 + A(\Delta\sigma_1 - \Delta\sigma_3)];$$

With $B=1$ for saturated soil) are relatively lower for ($A < 0$) and relatively higher for ($A > 0$). Then, the link of inter granular contacts is much resistant for soils more overconsolidated ($A < 0$) than it is for normally consolidated soils ($A > 0$). Because, if σ'_1 , σ'_2 and σ'_3 , are important, this will inevitably lead to one link inter granular contacts stronger in proportion to the nature of the overconsolidated soil. The water, in this case, experiences some difficulty relating to unsolder the intergranular connection. But, for soils normally consolidated and soft (large d), this process will be outright reversed. That is why; the soils overconsolidated have better resistance than normally consolidated ones. However, for whatever the nature of the soil (normally or overconsolidated) and the degree of resistance of the intergranular contact, the path constraints most vulnerable (the most sensitive to the action of water) will correspond (for case of sloping soils) to the circle of the rupture (most probable). In Figure 5, one shows that the Fs values of soils overconsolidated ($A < 0$) are better than their corresponding values of soils normally consolidated ($A > 0$).

In the Figure 7, the values of safeties factors decrease with the increasing of the (d) parameter. Actually, this (d) parameter plays the role of damping coefficient, in the formulation of one method like finite element method (FEM). Therefore, more the (d) parameter is great, more the soil is soft, and thus its deformation is great; especially in the first times of application of seismic load. In these first times, one observes very great falling of the values of the FS. This point will be explained more far in this section. Indeed, the degradation of soil's stiffness during cycles of seismic loading is not much proved before in the numerical studies of sloping soil by using the classical GLEM. This study indicates the way to follow for showing numerically the degradation of stiffness of sloping soil by using the GLEM thus reformulated. Furthermore, one has improved the Cascone and al method by including the effect of consolidation of soil.

It is important to know, how will be the loss of strength of the soil after the earthquake. For that, one plots in Figure 8, the ratio of the Eq. 30 versus (A) and (d). The plotted curves show that the loss of the resistance increases with the increase of (d) for reaching 50% for ($d = 0.2$). Cascone and al [3] stated that for real accelerograms, the reduction of soil strength can reach a maximum value of 60% of its initial value. In addition, there is no variation with Skempton parameter (A). This result has been confirmed by experimental study made by Tavakoli and al [21]. This means that the falling of the resistance (between the first and the last Fs) is the same for all Skempton parameter (A). But for the (d) parameter, this loss of the resistance is not

the same from one (d) parameter to other, and it is linear. This shows that the degradation of slope stability is not influenced by the degree of overconsolidation, but rather by the nature of the soil, rigid or flexible. We recall here that the soft soil can be overconsolidated or normally consolidated. The same thing can be advanced for rigid soils. It was found that strength degradation affects dramatically the development of slope displacements [3]. We recall also that the degree of overconsolidation of the soil is indicated by the constraint that the soil had sustained throughout its history, for whatever its nature rigid or soft. For man-made (embankment dams for example) slopes (for example), it is preferable that the soil will be more rigid. In this case, the parameter (d) will be low. Thus, the degradation will be small too. However, be careful here, one speaks about the no influence on the degradation of shear stiffness, when (A) is constant. But, the inter granular connections for the overconsolidated soil is better than the normally consolidated one (See Fig.6). So when we talk about degradation, implicitly, we must think about the falling the shear stiffness between the first and last seismic acceleration for the same soil. Here, the lateral confined constraints of the overconsolidated soil, are more powerful than their analogues normally consolidated. In addition, the rigid ground deformation is small compared to the soft ground. Precisely, the excessive deformation of soft soils leads to the generation of excess pore pressure. The Soils mixed are shown to reduce water reactions to the earthquake. When the quantity of the aggregate increases, the shear strength increases [21]. Indeed, the soils mixed compositions include (in addition to the clay) sand and aggregate. These others elements than the clay, will have two effects. Firstly they reduce the deformations of the ground to the extent that they are less deformable, and then, they provide drainage. The soils composite are often used for the dams which are also another kind of the sloping soil. But this mixture of soils, other than the clay, must be with a certain percentage to not submit the slope to the liquefaction phenomenon. This practical example highlights the importance of a good understanding of the behavior of saturated clay subjected to earthquake.

A case is evidenced in Figure 9, for the soft soil normally consolidated ($d = 0.2$, $A=0.8$). The plotted curve shows (F_s) versus the first cycles of zero crossing and underlines a fast decrease of the safety factors. This is due to the pore pressure build up along the sliding surface. This phenomenon induces a decrease of the surface contact between particles of soil (which conducts to rupture) and by consequent a decrease of the effective stress. The Figure 9 shows the degradation of Soil's resistances. It begins relatively great and then decrease with the cycles of zero crossing. This decrease of resistance indicates that the water retakes the seismic loading. The velocity of the loss of the resistance of soil is conditioned by the velocity of application of the seismic loading. Indeed, quickest is the velocity of application of the seismic loading, quickest will

be the reaction of water contained in the soil to this seismic load. Of course, in this case, the loss of the resistance in the soil (due to the loss of contact of particles of soil) will follow the same velocity of the reaction of water to the earthquake; with the remark, however, that the velocity of earthquake is greater than the water dissipation velocity. This is the undrained case. At the end, one must not forget the fact that, this loss of soil strength is much felt in the early cycles of the earthquake (Figure 9), and that this loss of resistance gradually decreases with sequence of first cycles of the earthquake. At the sliding plan of rupture in the slope, the decrease of the surface contact between particles of saturated sloping soil is optimal and conducts justly to the rupture of the slope. Soltani-Jigheh and al [22] have deduced in their experimental study that most of degradations have taken place during the first 10 cycles. Anderson and al [23] have stated the degradations take place during 10 to 15 cycles. The Figure 9 shows that the degradation is happened at about 11 cycles. For man-made slopes, it is safer for the stability of the slope to use unsaturated soil (drainage of soil must be assured). It is, also, suggested here of compacting the soil. In this case, the reaction of water to the earthquake will be less. Hence, the inter-granular contacts of the soil are then more powerful.

It is important to note that the reformulation (in this study) of the safety factor expression, integrating the (d) parameter and the Skempton parameter (A) together can be of an interest in the investigation of the slope stability under seismic motion. Indeed, as well as, the type of clayey soil (soft or rigid) and nature (normally or overconsolidated) can be explicitly studied.

For the crossing, Figures 10 to 15 show that the rupture of given slopes (submitted to the same seismic action) is much more sensitive when their soils are soft and normally consolidated. Indeed, the failure probability (P_f) is greater for normally consolidated soils ($A > 0$) and soft soils ((d) relatively large) (Figures 10, 11 and 15) than it is for soils overconsolidated ($A < 0$) and rigid ((d) relatively small) (Figures 12, 13 and 14).

However, being given the observed deficiency in the Rayleigh distribution (Figure 2) (which neglects the case of more than one peak for the same up crossing in its formulation), one proposes the (P_f) of Eq 29. Hence, the differences between the curves of Eq.29 and the curves describing the failure probability of Rayleigh, implies the effect of the accelerations neglected by the distribution of Rayleigh. The Weibull distribution is more general, but it depends on the parameter value (k) which includes the Weibull distribution of Gaussian shape ($k = 2$) and non-Gaussian ($k \neq 2$). But, its disadvantage is that this value of (k) corresponding truly to the distribution of our output (F_s in this study) is unknown to us. One believes that the Eq 29 responds correctly to these two problems of Rayleigh and Weibull. Why? Simply because every single acceleration of seismic action, will correspond to one and only one F_s .

Thus, the non-Gaussian nature (non-linear) of the Output is modelled faithfully by the approach of Eq 29 in

the sense that it takes in account all the points of the output. In this sense, the Weibull distribution ($k=5$) in the figures 11 and 13, is the most closer to the Eq 29; especially for a constant value of Skempton parameter ($A=-0.2$) and a variable value for the (d) parameter (Figure 13). Actually, this (k) parameter of Weibull distribution, implicitly, represents the degree of non-Gaussianity of the distribution.

7. Conclusion

In their efforts to approach the real conditions in the soil, scientists have turned to powerful formulations to satisfy this requirement. These formulations have taken a great importance, especially in their stochastic form. However, the idea to include, at first, the maximum parameters that exist really (inherent) in the soil and have an influence on the studied phenomenon in our formulation is, also a non negligible manner to describe the in-situ reality of soil. To satisfy this aim, we focused our efforts to reformulate the slope stability by including the intermediate effective normal stress σ'_2 . The introduction of this stress σ'_2 is, in fact, rightful because, it exists really in the soil. [6]

However, the introduction of (σ'_2) in the formulation of stability of slope under the earthquake doesn't have an influence in the undrained case (with total stresses). The soil does not take part in the resumption of the earthquake (Especially in the first cycles), because the velocity of the seism is greater than the water dissipation velocity. The comportment of soil (by introducing of (σ'_2)), in the frame of the PES (the cases of static loading, an unsaturated soil or a soil under a feeble frequency of dynamics excitations), has a great effect as it was been shown by Haderbache and al [6]. However, in this study by using the PTS, one has seen that this (σ'_2) doesn't have a great influence in the comportment of the soil. Of course, this is due to a buildup of the excess pore pressure of the water.

It is important, however, to note that the reformulation (in this study) of the safety factor expression, integrating the (d) parameter and the Skempton parameter (A) together can be of an interest in the investigation of the slope stability under seismic motion. Indeed, as well as the type of clay soil (soft or rigid) and nature (normally or overconsolidated) can be studied. This improvement in the formulation of the Cascone and al method [3] is welcomed, as it allows us to see the behavior of a large number of natures of sloping soils.

For quantifying the rupture of a given slope beneath a given earthquake, the concept of crossing is introduced in this study. This concept is much more seen in the signal treatment approach. One has bypassed the problem shown in the figure 2 (which is not handled by the Rayleigh distribution) by proposing the Eq. 29. Hence, the non-Gaussianity of the output (FS) is better treated by this last equation (Eq. 29).

References

- [1] Bishop AW. (1955) "The use of slip circle in the stability analysis of slopes." *Géomécanique* 1955; 5:7-17.
- [2] B-W.Song (2002). "The influence of initial static shear stress on post-cyclic degradation of non-plastic silt." *Lowland Technology International* Vol.4.No. 2, 14-24, December 2002. International Association of lowland Technology (IALT), ISSN 1344-9656 (2002)
- [3] Cascone E, Maugeri M and Motta E (1998). "Seismic response of clay slopes." 11th European Conference on Earthquake Engineering © 1998 Balkema, Rotterdam, ISBN 90 54 10 982 3 (1998)
- [4] Duncan J.Michael and Wright Stephen.G. (2005). "Soil strength and slope stability." John Wiley & sons; 2005.
- [5] Fellenius W. (1936) "Calculation of the stability of earth slope." *Transactions of 2nd Congress on Large Dams*, Washing- ton, DC, 1936. p. 445-62.
- [6] Haderbache, L., and Laouami, N. (2010) "The Effect of Failure Criterion on Slope Stability Analysis". *The Electronic Journal of Geotechnical Engineering (EJGE)*, vol. 15, Bund. A, pp. 11. (<http://www.ejge.com/2010/Ppr10.006/Ppr10.006.pdf>)
- [7] Idriss, I.M., Dobry, R. & Singh, A.M. (1978). "Nonlinear behaviour of soft clays during cyclic loading." *Journal of the Geotechnical Engineering Division, ASCE*, vol. 104 (12): 1427-1447.
- [8] Janbu N. (1954) "Application of composite slip surface for stability analysis." In: *Proc., Eur. Conf. On Stability of Earth Slopes*, Stockholm, Sweden, 1954. 43-9.
- [9] Janbu N. (1973) "Slope stability computations." In: *Hirschfeld E, Poulos S, editors. Embankment Dam Engineering Casagrande Memorial Volume*. New York: John Wiley, 1973. p. 47-86.
- [10] Kramer, S. L. (1996). "Geotechnical earthquake engineering", Prentice-Hall, Englewood Cliffs, New Jersey. ISBN :0-13-374943-6
- [11] Newland D.E (1984) "An introduction to random vibrations and spectral analysis" Second edition John Wiley & sons; 1984.
- [12] Procter, D.C. & Khaffaf, J.H. (1984). "Cyclic triaxial tests on remoulded clays." *Journal of the Geotechnical Engineering Division, ASCE*, vol. 110 (10): 1431-1445.
- [13] Sanglerat Guy and Costet Jean (1981). "Cours pratique de mécanique des sols ." Tome 1 troisième édition Dunod 1981.
- [14] Shambhu S. Sharma and Martin Fahey (2003). "Degradation of Stiffness of Cemented Calcareous Soil in Cyclic Triaxial Tests." *Journal of Geotechnical and Geoenvironmental Engineering © ASCE / July 2003*
- [15] Spencer EE. (1967) "A method of the analysis of the stability of embankments assuming parallel inter-slice forces." *Geotechnique* 1967; 17:11-26.
- [16] Terzaghi K and Peck Ralph.B and Mesri Gholamreza (1996). "Soil mechanics in engineering practice." Third edition John Wiley & sons; 1996.
- [17] Vucetic, M. & Dobry R. 1988. "Degradation of marine clays under cyclic loading." *Journal Geotechnical Engineering, ASCE*, vol. 114 (2): 133-149.
- [18] Wood David Muir (1990). "Soil behaviour and critical state soil mechanics." Cambridge University Press (1990).
- [19] Yang C.Y (1986) "Random vibration of structures" John Wiley & sons; 1986.
- [20] Zhu D.Y and Lee C.F and Qian Q.H and Chen G.R (2005). "A concise algorithm for computing the factor of safety using the Morgenstern-Price method." *Can Geotech. J.*42:272-278 (2005).
- [21] TAVAKOLI.H.R and SHAFIEE..A and JAFARI. M.K (2008). "Effect of cyclic loading on undrained behaviour of compacted sand/clay mixtures." *The 14th World Conference on Earthquake Engineering* October 12-17, 2008, Beijing, China
- [22] Soltani-Jigheh. H and Soroush. A (2010) "Cyclic Behavior of Mixed Clayey Soils" *International Journal of Civil Engineering*. Vol. 8, No. 2, June 2010
- [23] Anderson. Donald G, Martin. Geoffrey R, Lam Ignatius (Po) and Wang. J. N. (Joe) (2008) "Seismic Analysis and Design of Retaining

Walls,Buried Structures, Slopes,and Embankments” N A T I O N A L C O O P E R A T I V E H I G H W A Y R E S E A R C H P R O G R A M (NCHRP rapport 611) 2008.

Appendix

The Drucker-Prager criterion versus effective friction angle and effective cohesion is given below: [6]

$$\sqrt{J_2} = A + BI_1;$$

$$A = \frac{6C' \cos(\varphi')}{(3 - \sin(\varphi'))\sqrt{3}} ; \quad B = \frac{2\sin(\varphi')}{(3 - \sin(\varphi'))\sqrt{3}}$$

$$I_1 = \sigma_1 + \sigma_2 + \sigma_3$$

$$J_2 = \frac{1}{6}[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_1 - \sigma_3)^2]$$

Where I_1 is the first invariant of the Cauchy stress and J_2 is the second invariant of the deviatoric part of the Cauchy stress.

With $\sigma'_2 = \sigma'_3$ in the relations of model of Drucker-Prager above by making some elementary manipulations, one obtains the equations below: [6]

$$q' = (C' M / \tan(\varphi')) + MP' \quad q' = \sigma'_1 - \sigma'_3$$

$$p' = \frac{\sigma'_1 + 2\sigma'_3}{3}$$

$$M = \frac{6\sin(\varphi')}{3 - \sin(\varphi')}$$