

Numerical investigation of hydrodynamic nanofluid convective flow in a porous enclosure

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Abstract

In this study, the steady state behavior of natural convection transport on a nanofluid in square enclosure was studied. The model equations were solved using Comsol Multiphysics; a solver for partial differential Navier–Stokes equations based on a two-dimensional Finite Element Method (FEM) over a range of Rayleigh numbers (10^3 - 10^6). Impact of the Rayleigh Number, the Darcy Number, the porosity, the solid fraction volume of porous medium and the nanoparticle concentration on natural convection are depicted. Obtained Results reveal that convection mode increases with rise of Rayleigh number. The simulated results were compared with other numerical data from the literature, which indicate that good agreement is founded.

Keywords: free convection, Hartmann number, volume fraction, Nusselt number.

1. Introduction

Due to its practical engineering applications, the natural convection is one of the most important phenomena in thermal engineering electronic devices, such geothermal energy recovery, crude oil extraction, ground water pollution, thermal energy storage, flow through filtering media, ...etc [1]. Representative studies are focused on natural convection enhancement utilizing nanofluids [2]. The prior research which is most closely related to flow and heat transfer in porous media has been extensively studied via experimental, theoretical or numerical methods [3-6].

The goal of this paper is to study the influence of magnetic field on a nanofluid of free convection in porous medium. Finite Element Method (FEM) is chosen to simulate this problem. The influences of the active parameters on the hydrothermal treatment are examined. The flow field, the temperature field and the average Nusselt number is obtained. Effects of basic parameters on the natural convective heat transfer are analyzed in detail, including Darcy number, Rayleigh number, porosity, thermal conductivities of nanoparticles and solid phase in porous medium, and the nanoparticle concentration. We investigate the change from the conduction regime to the convective regime in the natural

convection, and discuss effects of key parameters on the change in heat transfer regime. We also put forward an average velocity to judge different heat transfer regimes in natural convection and define a medium Rayleigh number to study the combined effect of nanofluid and porous medium.

2. Problem definition

The numerical investigation is carried out for the present problem based on the above finite element method. For the validity of the result, the comparison between the present result and the result of Khanafer *et al.* [8] was conducted. The streamline and the temperature field in the present work are shown in figure 2 and that of Khanafer *et al.* are presented in figure 3. This figure shows the comparison of temperature profile at $y/H=0.5$ of the cavity.

The schematic diagram of the flow configuration is shown in figure 1. The cavity is of width L and height H filled with a viscous, incompressible and electrically conducting fluid, heated from a vertical side wall and cooled from an opposing wall. The horizontal top and bottom walls are thermally insulated. The fluid is permeated by a uniform magnetic field B_0 along the x direction, parallel to gravity.

The working fluid is considered Fe₃O₄-water and its viscosity is a function of nanofluid volume fraction. Results are presented for various values of volume

fraction of Fe₃O₄-Water ($\phi=0.04$ to 0.2), Rayleigh number ($Ra=10^3$ to 10^6).

Table 1

Thermo-physical properties of water and Fe₃O₄ nanoparticles

	$\rho(\text{kg/m}^3)$	$C_p(\text{J/kgK}^{-1})$	$k(\text{W/m} \cdot \text{K}^{-1})$	$\beta \times 10^5(\text{K}^{-1})$	$d_p(\text{nm})$	$\sigma(\text{N} \cdot \text{m})^{-1}$
Water	997.1	4.179	0.613	21	-	0.05
Fe ₃ O ₄	5.200	670	6	1.3	47	25.000

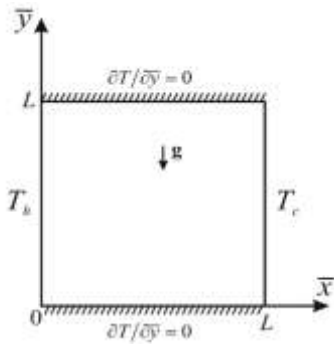


Figure1: Schematic for the physical model.

3. Mathematical model

In this investigation, convection within a two-dimensional vertical cavity filled by an incompressible Newtonian binary fluid (Figure 1) is studied. All boundaries of the cavity considered are impermeable; the top and bottom boundaries are assumed adiabatic whilst the other vertical ones are kept at uniform but different constant temperatures. The gravity acts in the negative direction (y) [7].

3.1. The continuity equation:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

3.2. Momentum equation:

$$\frac{\partial u}{\partial y} v + \frac{\partial u}{\partial x} u = [-\sigma_{nf} B_y^2 u + \sigma_{nf} B_x B_y v \left(\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 v}{\partial x^2} \right) \mu_{nf} - \frac{\partial P}{\partial x} - \frac{\mu_{nf}}{K} u]. (\rho_{nf}) \quad (2)$$

$$\rho_{nf} \left(\frac{\partial v}{\partial x} u + \frac{\partial v}{\partial y} v \right) = \mu_{nf} \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) - \frac{\partial P}{\partial y} + B_y \sigma_{nf} B_x u - B_x \sigma_{nf} B_y v - \frac{\mu_{nf}}{K} v + (T - T_c) \beta_{nf} g \beta_{nf} g \rho_{nf} \quad (3)$$

$$B_x = B_0 \cos \lambda \quad ; \quad B_y = B_0 \sin \lambda$$

$$(\rho C_p)_{nf} \left(\frac{\partial T}{\partial y} v + \frac{\partial T}{\partial x} u \right) = k_{nf} \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) \quad (4)$$

or:

$$(\rho C_p)_{nf} = (\rho C_p)_f (1 - \phi) + (\rho C_p)_s \phi \quad (5)$$

$$(\rho \beta)_{nf} = (\rho \beta)_f (1 - \phi) + (\rho \beta)_s \phi \quad (6)$$

$$\rho_{nf} = \rho_f (1 - \phi) + \rho_s \phi \quad (7)$$

$$k_{nf} = k_f \cdot \left(\frac{k_s + 2k_f + 2\phi(k_s - k_f)}{k_s - \phi(k_s - k_f) + 2k_f} \right) \quad (8)$$

$$\frac{\rho_{nf}}{\rho_f} = 1 + \frac{3 \left(\frac{\sigma_s}{\sigma_f} - 1 \right) \phi}{\left(\frac{\sigma_s}{\sigma_f} + 2 \right) - \left(\frac{\sigma_s}{\sigma_f} - 1 \right) \phi} \quad (9)$$

μ_{nf} is obtained as follows [8]:

$$\mu_{nf} = (0.035B^2 + 3.1B - 27886.4807\phi^2 + 4263.02\phi + 316.0629) \cdot e^{-0.01T} \quad (10)$$

Vorticity and stream function should be used to eliminate pressure source terms:

$$w = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x}, \quad u = \frac{\partial \psi}{\partial y}$$

Introducing dimensionless quantities:

$$P = \frac{p}{\rho_{nf} \left(\frac{\alpha_{nf}}{L} \right)^2}, \quad U = \frac{uL}{\alpha_{nf}}, \quad V = \frac{vL}{\alpha_{nf}},$$

$$\Theta = \frac{T - T_c}{\Delta T}, \quad \Delta T = \frac{q^n L}{k_f}, \quad (X, Y) = \frac{(x, y)}{L}$$

3.3. Energy equation:

$$\frac{\partial T}{\partial x}u + v\frac{\partial T}{\partial y} = \alpha \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) +$$

$$\frac{(\rho C_p)_p}{(\rho C_p)_f} \left[D_B \left\{ \frac{\partial \phi}{\partial x} \cdot \frac{\partial T}{\partial x} + \frac{\partial \phi}{\partial y} \cdot \frac{\partial T}{\partial y} \right\} + \frac{D_T}{T_c} \left\{ \left(\frac{\partial T}{\partial x} \right)^2 + \left(\frac{\partial T}{\partial y} \right)^2 \right\} \right] \quad (11)$$

$$\frac{\partial \phi}{\partial x}u + v\frac{\partial \phi}{\partial y} = D_B \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) + \frac{D_T}{T_c} \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) \quad (12)$$

The stream function and vorticity are defined as follows:

$$w = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}, v = -\frac{\partial \psi}{\partial x}, u = \frac{\partial \psi}{\partial y}$$

The nanofluid's density, ρ is

$$\rho \cong \phi \rho_p + (1 - \phi) [\rho_0 (1 - \beta(T - T_c))] \quad (13)$$

where ρ_0 is the nanofluid's density at the reference temperature, β is thermal expansion coefficient

In addition to the porosity E and the capacity ratio r , the flow governed by equation 12 is characterized by some nondimensional parameters: the Darcy number Da , the viscosity ratio Je , the Prandtl number Pr , the Rayleigh number Ra (for natural convection), and the Reynolds number Re (for forced or mixed convection), which are defined as follows:

$$De = \frac{K}{L^2} \quad ; \quad Je = \frac{\nu_c}{\nu}$$

$$Pr = \frac{\nu}{\alpha} \quad ; \quad Ra = \frac{g\beta\Delta TL^3}{\nu\alpha}$$

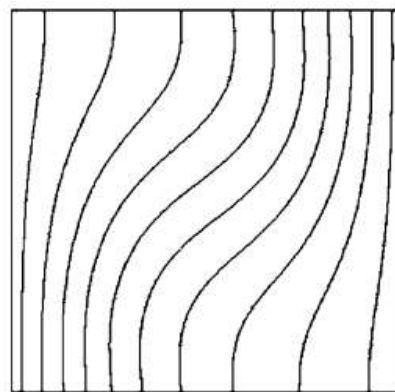
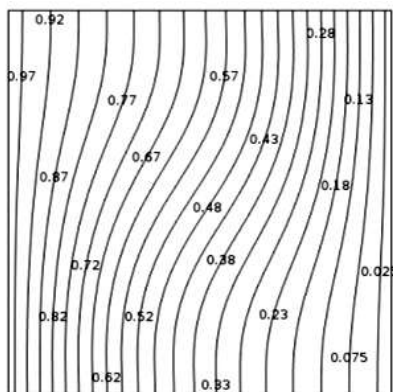


Figure 2: Isotherms inside the cavity for different Rayleigh and Darcy numbers

and $Re = \frac{LU}{\nu}$ where L and U are the characteristic length and the characteristic velocity.

3.4. Boundary conditions

The boundary conditions are in the following forms:

$$u = v = 0 \quad \text{at } x = 0, L \quad 0 \leq y \leq H$$

$$u = v = 0 \quad \text{at } x = 0, H \quad 0 \leq y \leq L$$

$$T = T_c \quad \text{at } x = L \quad 0 \leq y \leq H \quad \text{for case 1,2}$$

$$\frac{\partial T}{\partial x} = 0 \quad \text{at } x = 0 \quad 0 \leq y \leq H$$

$$\frac{\partial T}{\partial x} = 0 \quad \text{at } y = 0, H \quad 0 \leq x \leq L$$

At the heat source's surface: $\begin{cases} u = v = 0 \\ \frac{\partial T}{\partial x} = -\frac{q''}{k} \end{cases}$

4. Results and discussion

Results are obtained from the solution of the governing equations subject to the mentioned boundary conditions, and are presented in terms of Nusselt number, entropy generation and performance coefficient (PEC). First the numerical solution is validated through comparison with recently published works. Afterward, effects of Rayleigh and Darcy numbers and nanoparticle volume fraction on thermal field, entropy generation and PEC for different geometries are presented in the subsequent subsections. In these sections, relevant diagrams and contours are illustrated and discussed.

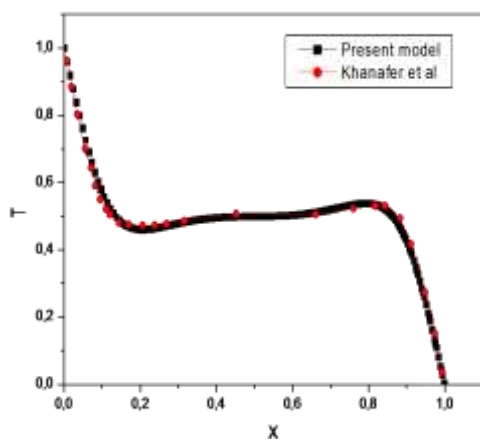


Figure 3: Comparison of the temperature distribution on axial midline between the present results and numerical results by Khanafer *et al.* [8] for $\phi=0.1$ and $Pr=6:8$ Cu-Water

Increasing nanoparticles concentration from one side can improve the thermal conductivity of the fluid and enhance the heat transferrate, and from the other side will increase the fluid viscosity with subsequent increment in frictional losses that can weaken the convection field. In addition to this fact, interaction of the nanofluid with porous fin layers for various Ra and Da numbers can show different impacts on heat transfer characteristics. Accordingly, in this section it is focused on the nanoparticles effects on heat transfer enhancement and results are analyzed in terms of the relative Nu .

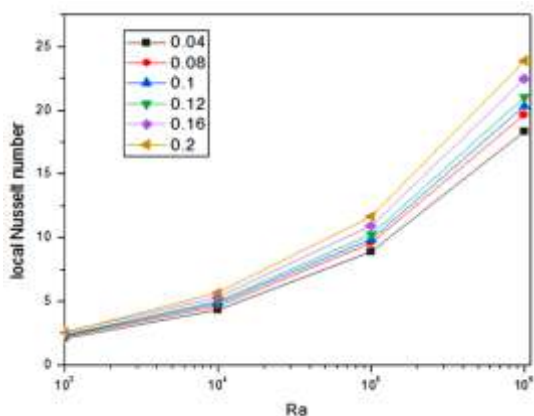


Figure 4: The relationship of the local Nusselt number and the medium Rayleigh number

Figure 3 shows the relationship of the local Nusselt number and the medium Rayleigh number. It can be seen that Nu increases with the increase of Ra , and it means that the increased Ra of whole medium can enhance the natural convective heat transfer.

5. Conclusion:

The flow and thermal performance of the natural convection of a nanofluid flowing in a porous medium cavity is numerically investigated with the proposed FEM method by using Boussinesq approximation. The velocity and temperature fields and the local Nusselt number were obtained and analyzed in detail. The effects of the Rayleigh, the fraction volume, the porosity, and the nanoparticle concentration on the nanofluid natural convective transport in the porous medium were examined. The result shows that the average Nusselt number increases with an increase in the Rayleigh.

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