



Propagation of Classical Waves in One-Dimensional Disordered Media: II- Results

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Résumé :

La propagation des ondes classiques dans les milieux aléatoires est étudiée en présence des corrélations à courtes distances dans le désordre. Un analogue classique du modèle de Kronig-Penney est proposé en considérant une chaîne de sous-systèmes identiques, chacun étant constitué d'une masse reliée à socle rigide par un ressort. Les masses sont reliées entre elles par un fil soumis à une tension uniforme. La nature des modes est examinée par le formalisme des matrices de transfert. Les propriétés de propagation du système sont étudiées d'une manière statistique mettant en valeur un grand nombre de grandeurs physiques telles que le coefficient de transmission, la longueur de localisation et les exposants critiques. En particulier il est mis en évidence que la présence de corrélation dans le désordre permet de restaurer un grand nombre de modes de propagation du type onde de Bloch en contradiction avec les conclusions obtenues par d'autres modèles de la littérature.

Mots-clés: Désordre, système unidimensionnel, Corrélation, localisation

Abstract :

The propagation of classical waves in one-dimensional random media is examined in presence of short-range correlation in disorder. A classical analogous of the Kronig-Penney model is proposed by means a chain of repeated sub-systems, each of them constituted by a mass connected to a rigid foundation by a spring. The masses are related to each other by a string submitted to uniform tension. The nature of the modes is investigated by using different transfer matrix formalisms. The transmission properties of the system are numerically studied using a statistical procedure yielding various physical magnitudes such the transmission coefficient, the localization length and critical exponents. In particular, it is shown that the presence of correlation in disorder restores a large number of extended Bloch-like modes in contradiction with the general conclusion of the localization phenomenon in one-dimensional systems with correlated disorder.

Keywords: Disorder, One-dimensional system, Correlation, localization

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3- RESULTS AND DISCUSSION.

A 1D disordered system is simulated in this section. The propagating wire media is characterized by $\rho=5\text{kg/m}$, $T_0 = 10\text{N}$, $d v_\phi=1,4\text{m/s}$. and $d=5,10^{-2}\text{m}$. We have studied large systems up to $N = 1000$ unit cells, i.e. $L = 50\text{m}$.

Disorder is introduced by assuming that the masses are given by a binary alloy distribution:

$$P(M)=c\delta(M-M_H)-(1-c)\delta(M-M_A) \quad (15)$$

where M_A and M_H represent the host and the impurity unit cell masses respectively. The

propagating properties are described within a statistical procedure. A satisfactory convergence has been checked up to 10^4 random samples getting less than 1% for $T(\omega)$.

3.1 Random binary alloys

Here, the effects of the RDM on the nature of the vibration modes are examined in random binary alloys $A_{1-c}B_c$. Correlations in disorder are investigated by introducing at random with pairs a number of clusters of type B into the host lattice $AA...AAA...AAA$. The random

dimer unit [BB] is parameterized with the couple mass-stiffness string (M_d, k_d) while (M_A, k_A) characterizes the host unit one. Dimers appear with the concentration $c_d=0,5c$, where c is the concentration of impurity cells B. The dimer free frequency Ω_d is defined with respect to the host free frequency Ω_A . The dimers can be considered by pairing identical springs or/and identical masses and springs simultaneously. The corresponding $T(\omega)$ and $\frac{L}{\xi(\omega)}$ are investigated for different binary disorder cases and presented in Figs. 1 and 2.

3.2.1 Mass-spring binary disorders.

In this part, we deal with the mass-stiffness binary disorder when the impurity and host unit cells are built with different masses $M_B \neq M_A$ and different springs $k_B \neq k_A$ leading to different free frequencies $\Omega_B \neq \Omega_A$.

The impurity unit cell is characterized with $\Omega_B=10,75\text{rds}^{-1}$ and $M_B=0,06\text{kg}$ unit cell while the host lattice is defined with the unit cell $\Omega_A=40,0\text{rds}^{-1}$ and $M_A=0,1\text{kg}$. Lower and higher impurity concentration are considered : $c=0,1$ and $c=0,4$.

a. The free dimer resonance Ω_A .

Correlation in such binary disorder is considered. As well known from the electronic case, transparent dimer unit cells can be realized by choosing the fundamental frequency of the dimer ω_B within to the host allowed band [1].

An interesting case is the free dimer effects when the impurity fundamental frequency coincides with the host free frequency $\omega_B=\Omega_A$. For the above parameters, $T(\omega)$ is reported in Fig.1.

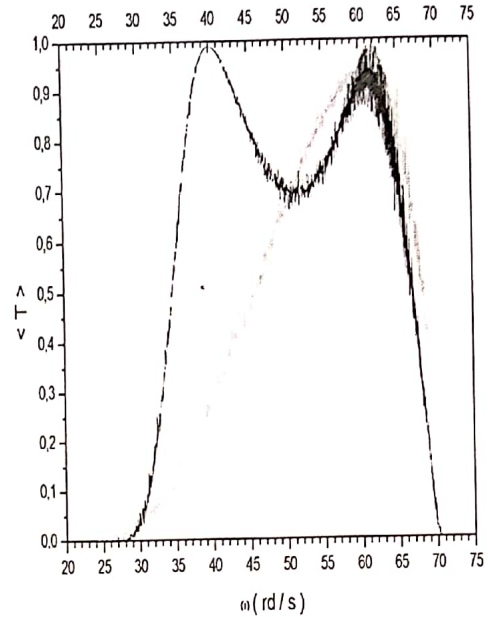


Figure 1: Correlated and uncorrelated transmission coefficient $T(\omega)$ versus frequency for different dimer concentrations

For such pairing of mass-stiffness, two different resonances ω_B and $\omega_c \approx 62\text{rds}^{-1}$ appear simultaneously. This yields attractive various ways to propagate incident waves through the wire media since the two delocalization processes appear within the same allowed band, as shown in Fig.2.

The free process, $\omega=\Omega_A=\omega_B=40\text{rds}^{-1}$

provides a challenging advantage since it extends the localization length, i.e. $\xi(\omega_B) \approx 200\xi(\omega_c)$. The comparison between the two types of resonances is shown in Figs.1 and 2. The nature of the amplitude is described with the transmission coefficient behavior versus the system size $T(N)$ close to the two particular resonances Ω_A and ω_c .

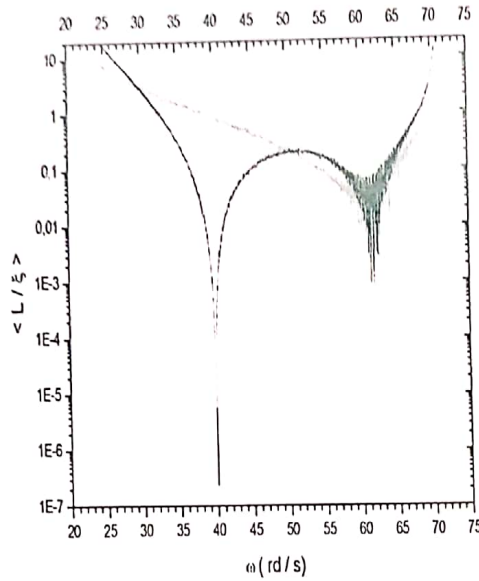


Figure 2: Correlated and uncorrelated Lyapunov exponent $\frac{L}{\xi}(\omega)$ versus frequency for different dimer concentrations

At the free frequency Ω_A , the ballistic regime abruptly settles down, showing clearly that Bloch-like waves character persist in such RDM configuration. This is in contrast with a recent literature which gives some controversies about the nature of states [2, 3].

Close to this particular free frequency, the envelope functions are sensitive to the disorder effects since the amplitude exponentially decreases near the free frequency $T(L) \propto \exp(-\frac{L}{\xi})$ with a negligible normalized

Lyapunov coefficient $\frac{L}{\xi}$.

b. The commute resonance ω_c

In contrast to the electronic model [4], the classical uncorrelated delta peak binary disorder presents constructive effects near the frequency $\omega_c \approx 62 \text{ rds}^{-1}$ with the existence of a

set of delocalized states for which: $\frac{L}{\xi} \approx 10^{-3}$

and $T \approx 1$. (see Fig.3)

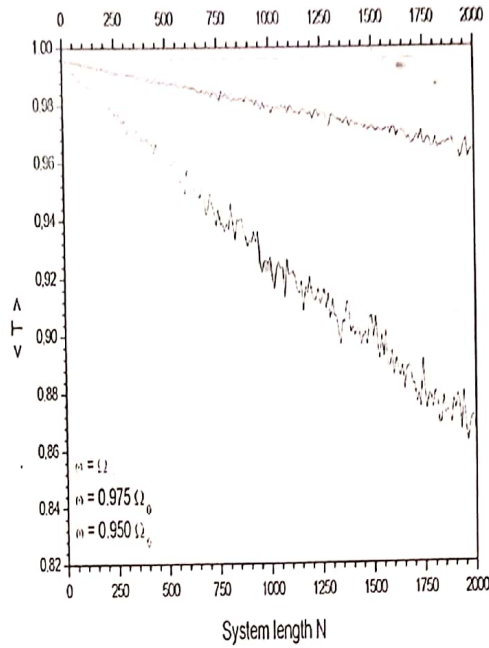


Figure 3: Coefficient transmission $T(N)$ versus system length, for different frequencies belonging to the allowed band : near the free resonance Ω_A ; $\omega = \Omega_A = 40 \text{ rds}$; $\omega = 0.975 \Omega_A$; $\omega = 0.950 \Omega_A$.

This statement may be understood from the on site description given by Eqs.12 and 13. The corresponding recursive transfer matrices associated to the host and impurity unit cells are defined by:

$$R_A = \begin{pmatrix} 2a_A(\omega)-1 & 1 \\ 1 & 0 \end{pmatrix}, \quad R_B = \begin{pmatrix} 2a_B(\omega)-1 & 1 \\ 1 & 0 \end{pmatrix} \quad (18)$$

In particular at ω_c , the corresponding KP formulas crossover, i.e. $a_A = a_B$. Hence, the resulting matrix elements become identical and consequently, R_A and R_B commute. Here, the incident propagating mode is not sensitive to the difference between the host unit cells from impurity ones since they displays the same local diffusive characters. The propagating media is felt as a perfect ordered lattice, with constant effective delta peak strength $\lambda_A(\omega_c) = \lambda_B(\omega_c)$. Such a characteristic frequency ω_c referred as the commuting frequency, can be determined from the conditions:

$$R_A(\omega_c) \equiv R_B(\omega_c)$$

$$a_A(\omega_c) = a_B(\omega_c) \Rightarrow \lambda_A(\omega_c) = \lambda_B(\omega_c) \quad (19)$$

$$\omega_c^2 = \frac{M_A \Omega_A^2 - M_B \Omega_B^2}{(M_A - M_B)}$$

At this commuting frequency $\omega_c = 61,86 \text{ rad/s}$, the two indiscernible unit cells present an additive behavior. This conclusion is in agreement with the electronic case recently reported on random dimer barrier heterostructures [5] which relate the origin of the resonance to the commuting properties of the binary individual unit cells. The existence of extended modes in a mini band around the discrete resonance ω_c is due to the non abrupt transition character since the recursive matrix elements get still very close together i.e. $a_A(\omega_c) \approx a_B(\omega_c)$.

Such feature is also preserved in the corresponding correlated disorder since there is no difference between the host and dimer unit cells. This was originally reported by Ishii [6] for random KP model on lattice. The commuting condition (Eq.21) leads to the same resonant statements, as previously demonstrated by various authors [3,5,7]. Finally, an interesting feature takes place on the commuting resonance with the presence of periodic amplitude at the commuting frequency ω_c , justifying the extended Bloch diffusive character of the corresponding propagating resonant mode. Moreover combined effects occur near this particular resonance since the vibration mode discerns between the two unit cells [A] and [B]. Such disorder localizes the Bloch-like extended wave functions within a mini band around ω_c , giving rise to a soft transition process. As depicted in Fig. 4, a simple way is presented to prescribe a clear description for the corresponding mixed modes with their exponentially decreasing periodic envelope amplitudes.

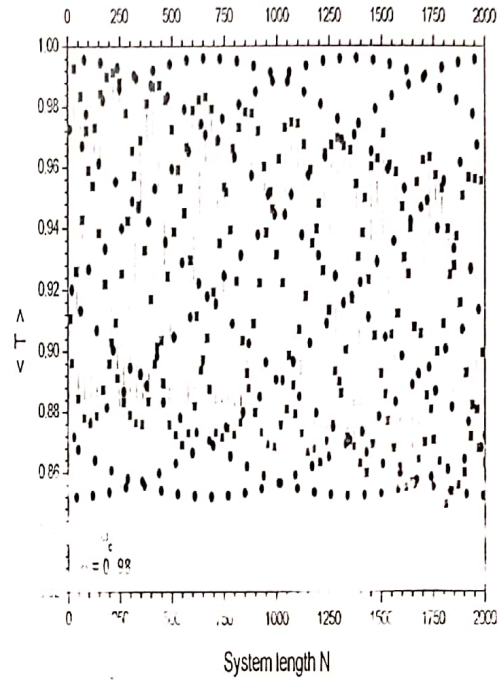


Figure 4: Coefficient transmission $T(N)$ versus system length, for different frequencies belonging to the allowed band :near the commute resonance $\omega_c, \omega = \omega_c = 61.86 \text{ rad/s}$ and $\omega = 0.98 \omega_c$

4. CONCLUSION

The propagation of classical waves in random media has been studied to observe the phenomenological aspects of the Anderson localization. In this description, two particular frequencies characterize the corresponding the ordered case: the fundamental frequency ω ($a(\omega)=0$) while the free frequency Ω_0 settles down the ballistic regime i.e. $\lambda_0=0$. Singular behavior happens around the free frequency Ω_0 since the spatial extent length diverges, $\frac{L}{\xi(\omega)} \propto |\omega - \Omega_0|^2$ suggesting strongly the Bloch-like wave character.

Dimers can be constructed with a new interesting way that preserves the ballistic regime even in presence of pairing configuration. The Bloch-like extended states are restored in controversies with the general belief that no periodic wave function exist in the well known random dimer model. Another resonance appears at the commuting frequency. This describes an additional



delocalization process since its corresponding extended states are fundamentally different.

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