

PHOTOELASTIC AND NUMERICAL ANALYSIS OF STRESS FIELDS IN A PLATE LOADED ON ITS EDGE AND IN TWO ORTHOGONAL CYLINDERS IN CONTACT

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Abstract

The purpose of this work is to develop numerical solutions in order to determine rapidly and as accurately as possible stress fields developed in mechanical components. Stress values and principal stresses directions can be determined easily and rapidly. An experimental solution using two dimensional photoelasticity was first conducted on a plate loaded on its edge by a rigid steel cylinder. Experimental photoelastic fringes, obtained on a polariscope, are compared to the simulated ones. Another comparison was made, along the vertical axis of symmetry, between stress values determined experimentally and stress values determined by the finite element method. A numerical solution for two orthogonal cylinders in contact was developed. The finite element program allowed us to determine the isochromatic and the isoclinics fringes as well as the stress values on any given slice isolated inside the model, particularly in the neighborhood of the contact zone. Simulated photoelastic fringes were determined on a six millimeter slice thickness in order to obtain enough photoelastic fringes for comparison purposes with experimental fringes that can be obtained either by the stress freezing method coupled with two dimensional photoelasticity or by the optical slicing method which uses laser plans to isolate optically slices inside a stressed model.

Keywords: birefringent / photoelasticity / isoclinic / isochromatic / fringe / contact / stress / simulation.

Résumé

Le but de ce travail est de développer une solution numérique permettant de déterminer rapidement et efficacement les champs de contraintes développés dans les composants mécaniques. Les valeurs des contraintes ainsi que les directions des contraintes principales peuvent être déterminées facilement et rapidement. Une solution expérimentale, en utilisant la photoélasticité bidimensionnelle, a été réalisée sur une plaque sollicitée par un effort de compression le long de son épaisseur via un cylindre rigide en acier. Les franges photoélastiques expérimentales obtenues sur un polariscope sont comparées aux franges simulées. Une autre comparaison est faite, le long de l'axe de symétrie vertical, entre les valeurs expérimentales et les valeurs simulées de la différence des contraintes principales. La solution numérique pour le cas de deux cylindres orthogonaux en contact a été développée; le programme permet de calculer les franges isochromes et les franges isoclines ainsi que les valeurs des contraintes sur une tranche isolée entre deux plans parallèles à l'intérieur du modèle particulièrement au voisinage de la surface de contact. Les franges photoélastiques simulées sont déterminées pour une tranche d'épaisseur 6mm pour avoir un nombre de frange suffisant pour permettre une comparaison avec des franges expérimentales qui peuvent être obtenues sur un polariscope après figeage et découpage mécanique du modèle ou en utilisant la méthode de découpage optique qui utilise deux plans laser pour isoler une tranche dans un modèle sollicité.

Mots clés: birefringence / photoélasticité / isocline / isochrome / frange / contact / contrainte / simulation.

Nomenclature :

α : angle characterizing the isoclinic, ϕ : the retardation angle, f : the fringe constant, C : the optical characteristic of the birefringent material, λ : the light wavelength, e : the model thickness.

1. INTRODUCTION

It is very important to determine the type and the amplitude of imposed stresses on mechanical parts in order to improve their design and durability. Theoretical studies of contact stresses are in some cases very complex. Experimental and numerical solutions have been used by various authors to tackle the problem [1-9]. In this paper we are interested, mainly, in developing a finite element solution so that problems with complicated geometry and boundary conditions can be numerically solved. A whole field analysis of the photoelastic fringes and a local analysis using the principal stresses difference are used for comparison between the experimental and the numerical solution.

2. EXPERIMENTAL ANALYSIS

Before dealing with the three dimensional stress problems let us start first with a two dimensional one for which the experimental solution is relatively easy to determine with two dimensional photoelasticity. The photoelastic model of thickness $e=10\text{mm}$, made of an epoxy resin (PLM-4R) mixed with a hardener, was loaded via a 14 mm diameter half steel cylinder, the load was set to $F=1000\text{N}$. The model in its loading frame equipped with two dynamometers (Fig. 1) was positioned in the polariscope for analysis. The model in the shape of a parallelepiped ($9.75 \times 40 \times 44 \text{ mm}$) was cut in the birefringent material. Poisson's ratio and Young's modulus, which are necessary to implement the numerical solution, are respectively equal to $\mu=0.37$ and $E_2=2435\text{MPa}$. These values were obtained with two samples of the birefringent material, a three point bending test and a bending specimen equipped with two strain gages, one mounted on the longitudinal direction and the other one mounted on the transversal direction. Two quarter Wheatstone bridges allowed us to record the transversal strain and the longitudinal strain measured by the strain gages for a given load. The ratio of the outputs of the two Wheatstone bridges gives directly the value of Poisson's ratio of the birefringent material. The light wave length used for analysis was equal to $\lambda=546\text{nm}$. Isochromatic fringes obtained on the analyzer can be used to determine the values of the principal stresses difference by using equation (1) and the principal stresses directions, particularly in the neighborhood of the contact zone.

$$\sigma_1 - \sigma_2 = \frac{N \left(\frac{\lambda}{C} \right)}{e} \quad (1)$$

This can only be done if the values of the fringe orders are completely determined. The values of the fringe order N can be determined either by the compensation technique or, whenever possible, by starting from a non stressed region on the model for which $N=0$. The fringe orders can then be easily deduced for the other fringes. The ratio $f=\lambda/C$ called the fringe constant depends on the light wave length λ used and the optical constant C of the model material. This value can be easily determined by loading a tensile specimen analyzed on a regular polariscope with circularly polarized light. The fringe constant can then be easily deduced from the graph of the applied stress versus fringe order. In our case the value was: $f=10.5 \text{ N/mm}$.

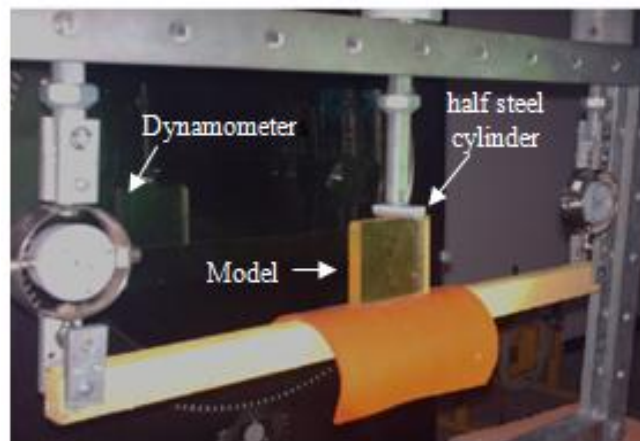


Figure 1: The model mounted on the loading frame.

To help the reader, a brief review of the experimental method is given bellow. Fig. 2 shows the well known photoelastic method based on the birefringent phenomenon. The light intensity given by equation (2) is obtained on the analyzer after traveling through the polarizer, the model and the analyzer [10].

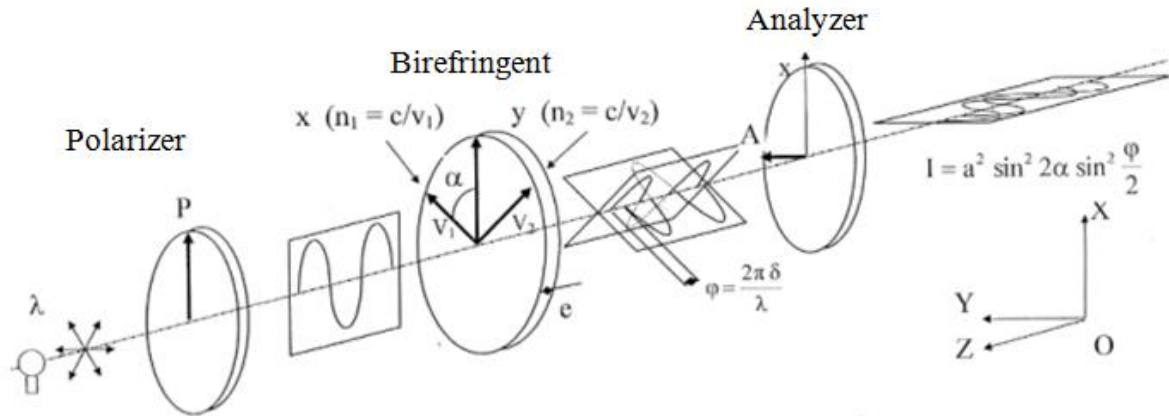


Figure 2: Light propagation through a photoelastic model [10].

$$I = \sin^2 2\alpha \sin^2 \varphi/2 \quad (2)$$

The terms $\sin^2 2\alpha$ and $\sin^2 \varphi/2$ give respectively the isoclinic fringes and the isochromatic fringes; α is the isoclinic parameter which is the angle between one of the principal stresses directions and the polarizer axis, φ is the retardation angle

2.1. Stress field developed in the model

The isochromatic fringes (Fig. 3) are obtained with circularly polarized light; the isoclinics are not visible as they are optically removed with the help of two quarter wave plates that change the light from plan polarized to circularly polarized light. These isochromatic fringes are used to determine the experimental values of the principal stress difference along the vertical axis of symmetry for comparison purposes with the numerical values. We can see, as expected, a concentration of stresses close to the contact zone. Away from the contact zone, at the lower part of the model, stresses are relatively much lower. The fringe orders are easy to determine; the value starts from the non stressed region, at the upper left, where the order is equal to zero. The fringe order increases then as we move toward the contact zone.

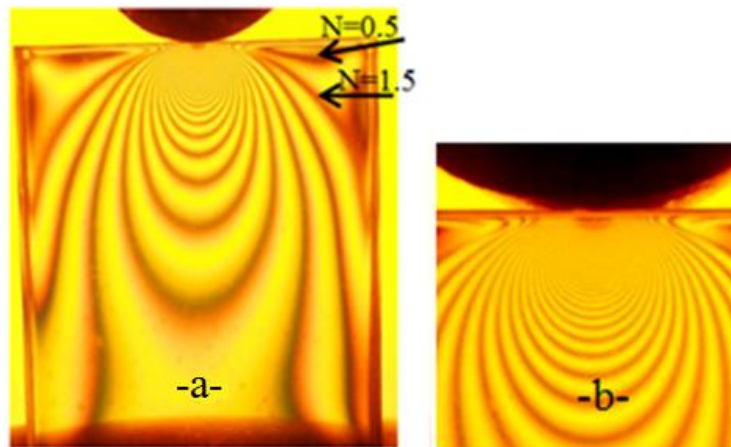


Figure 3: -a- Experimental isochromatic fringes
-b- Zoom of the contact zone.

Fig. 4 shows several images taken for different settings of the analyzer and the polarizer represented by the angle α relatively to a reference taken as the vertical axis. The experimental isochromatics are represented by the colored fringes. On the colored images we can see simultaneously the dark isoclinic fringes which are loci of points where the principal stresses directions are parallel to the polarizer and the analyzer axis.

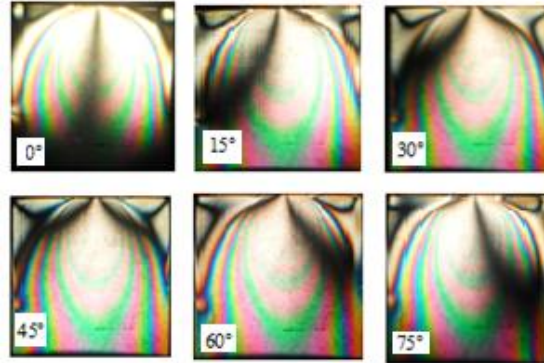


Figure 4: The photoelastic fringes at different angles.

3. NUMERICAL ANALYSIS

We consider that the material behaves everywhere as a purely elastic isotropic material. Fringe constant $f=10.5\text{N/mm}$, Young's modulus ($E_1=210000\text{ MPa}$, $E_2=2435\text{ MPa}$) and Poisson's ratios ($\mu_1=0.3$, $\mu_2=0.37$) respectively for the steel cylinder and the epoxy cylinder in contact were introduced in the finite element program. The mesh was refined in the neighborhood of the contact zone (Fig.5) in order to achieve better approximation of stresses (34908 nodes and 69234 elements of triangular type with 3 nodes were used to allow a good approximation of the model shape).

To achieve a better simulation of the applied load, an imposed displacement was applied to the model at the contact surface between the disc and the plan. The equivalent applied load was calculated then as the sum of the elementary vertical load components at the nodes located at the lower surface of the model. Iterations on displacements at the contact nodes are stopped when the calculated corresponding load is equal to the value of the applied load within an acceptable error (0.1%) set in the program.

The isochromatic fringes represented by $\sin^2 2\alpha$ were then easily calculated. The details of the calculation are shown here after.

3.1 Numerical calculation of the isochromatic fringes

Using equation (1), the following equation (3) can be obtained readily from Mohr's circle for stresses. This allows us to calculate the principal stresses difference at any point of a stressed model.

$$\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2} = \frac{N(\lambda/C)}{e} \quad (3)$$

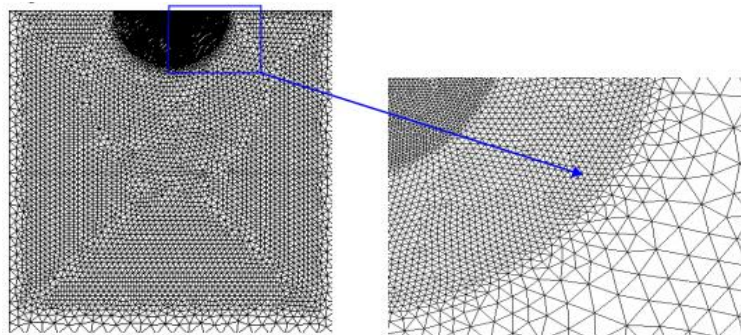


Figure 5: Finite element meshing.

Knowing the retardation angle $\varphi = 2\pi N$, the different values of φ can be calculated at any point on the model with the following relation: equation (4):

$$\varphi = (2\pi e / f) \sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2} \quad (4)$$

The different values of $\sin^2 \varphi / 2$ which represent the simulated isochromatic fringes can then be easily calculated and displayed (Fig. 6 left). A comparison can then be made with the isochromatic fringes obtained experimentally (Fig. 6 right). We can see relatively good agreement; however in the neighborhood of the contact zone we can see some discrepancies. Another comparison using the principal stresses difference along the vertical axis of symmetry (Fig. 7) shows relatively good agreement even though in the vicinity of the contact zone it was difficult to obtain the experimental values.

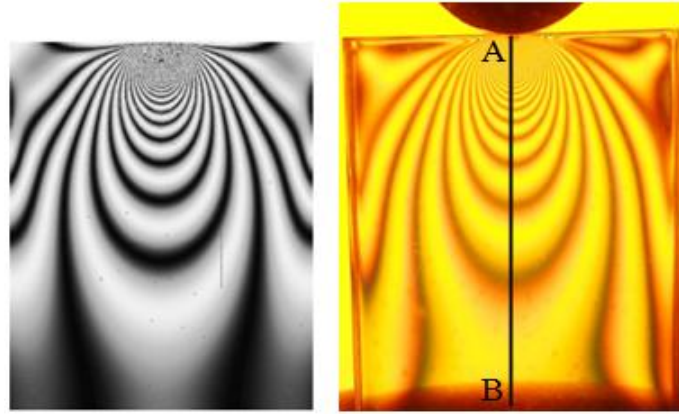


Figure 6: isochromatic fringes, simulated (left) and experimental (right)

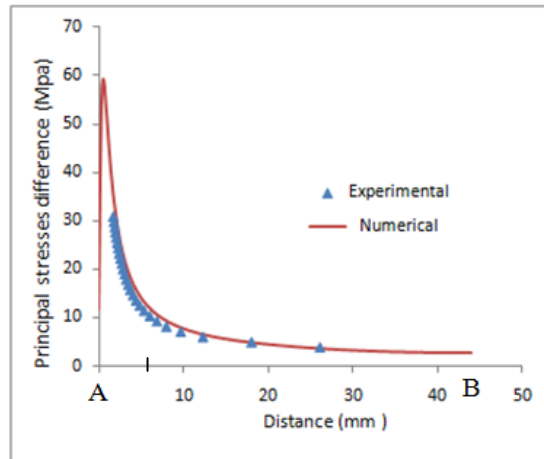


Figure 7: Principal stresses difference along the vertical axis of symmetry.

3.2 Numerical calculation of the isoclinic fringes

In the simulation program the different values of the isoclinic parameter α can be calculated with the following equation (5) which can be obtained readily from Mohr's circle of stresses:

$$\alpha = \arctan (2\tau_{xy} / (\sigma_x - \sigma_y)) \quad (5)$$

The term $\sin^2 2\alpha$ can therefore be calculated and displayed (black and white images of Fig. 8). The comparison is then possible with the experimental isoclinic fringes which are the dark fringes obtained experimentally.

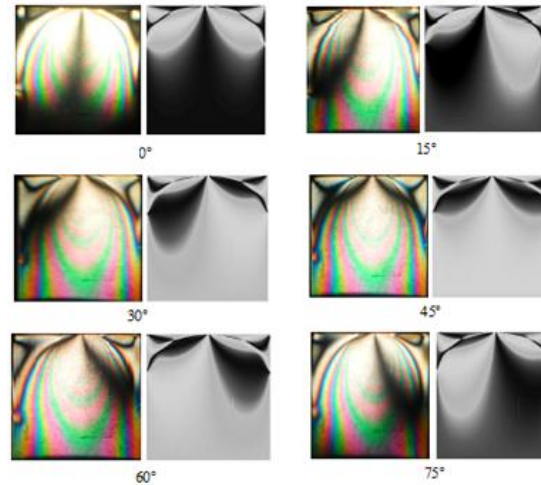


Figure 8: Comparison of experimental (left) and simulated (right) isoclinic fringes for different angles.

One should note that experimentally it is not possible to observe only the isoclinics whereas for the finite element solution this is possible. Good agreements are observed between the experimental and the simulated isoclinic dark fringes for similar values of the angle α .

The simulation was successfully applied to a two dimensional case; the next step is now to develop the solution for a three dimensional case for which stresses vary through the thickness of the model.

4. NUMERICAL SOLUTION FOR THE CASE OF A RIGID CYLINDER ON A DEFORMABLE ONE

To analyze stresses in three dimensional models different experimental techniques have been developed [3-6]. One of these methods is the stress freezing technique coupled with two dimensional photoelasticity which allows a complete analysis of stresses locked in a model. But, this method is time consuming. Another method which is more efficient is the optical slicing method [4,5,8] which uses, instead, laser plans to isolate optically a slice to be analyzed in the stressed model. The method requires, however, special equipment and good care is necessary to determine the stress field accurately.

The purpose here is to avoid these experimental procedures and determine stresses numerically in different slices along the z direction cut in the xoy plan (Fig. 9).

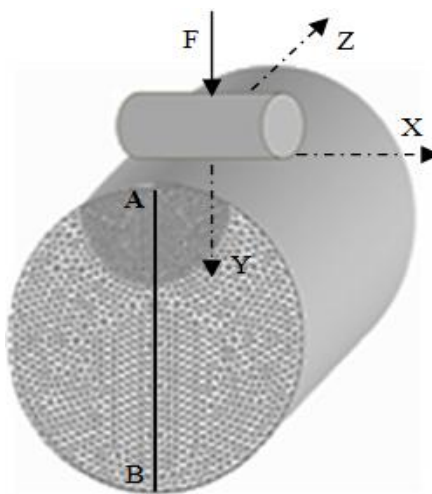


Figure 9: Two orthogonal cylinders with the finite element meshing.

4.1 Simulation of the mechanical slicing

Since for the stress freezing technique stresses should be locked inside the model at the stress freezing temperature, fringe constant and Young's modulus of the model material should be determined at the stress freezing temperature. In our simulation the values were taken from reference [8] to implement the numerical solution: ($f=0,44$ N/m, $E=15,9$ N/mm²); these values were obtained experimentally inside an oven at the stress freezing temperature. In order to achieve better approximation of stresses, we used 10575 nodes and 17544 triangular elements with 3 nodes which allow a good approximation of the model shape. In the meshing process a translation of the elements was done in the z direction in order to obtain the meshing of the whole cylinder. This gives volume elements of prismatic shapes. For the fringe reconstruction, we used the same method as in the two dimensional case. This procedure is repeated along the length of the cylinder in order to determine the variation of stresses in the whole volume. We can see, clearly, the decrease of the isochromatic fringes in the upper part of the cylinder, as we move away from the contact zone along the longitudinal axis z (Fig. 10). At the lower part of the cylinder the fringes remain the same. The reason is that the load is uniformly distributed along the cylinder. For a slice located at $z=0$ mm which corresponds to the upper left image (Fig. 10 a) we can see, as one can expect, a concentration of stresses close to the contact zone. Stresses decrease as we move away from the contact zone along the z direction; the successive slices are shown in alphabetic order.

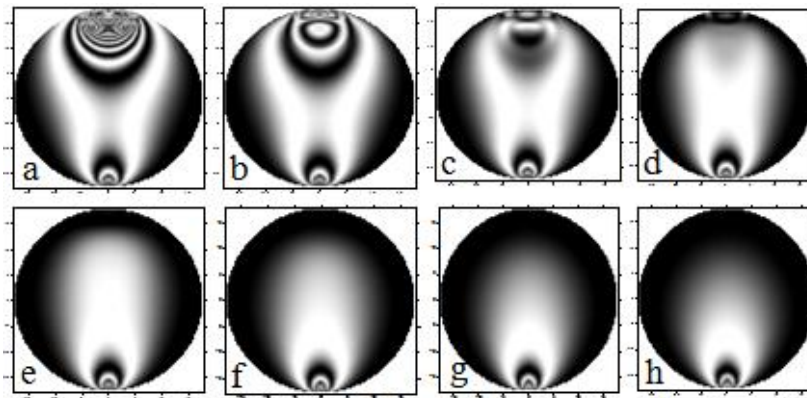


Figure 10: Simulated isochromatic fringes for different slices along the longitudinal axis.

For all slices, the principal stresses difference decreases significantly (Fig. 11) as we move down from the upper contact zone, vertically along the y direction, to the same stress value of approximately 0.03 MPa. As we move down to the lower part, stresses increase again to a value of approximately 0.22 MPa. This value remains the same for the different slices located at different z values, from $z=0$ mm, which corresponds to the direction of the applied load, to $z=10$ mm which correspond to a plan located 10 mm away along the longitudinal axis z of the cylinder. This explains clearly why in the lower part of the cylinder close to the contact zone fringes are similar for all plans along the longitudinal axis z of the cylinder.

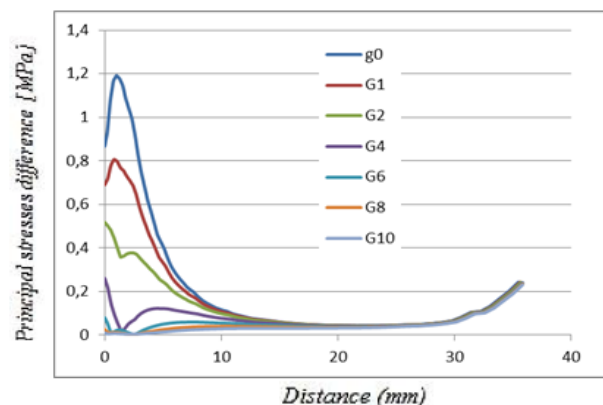


Figure 11: Variation of the principal stresses difference along segment AB for different slices.

Figure 12 shows the simulated isoclinics for a slice located at $z=0\text{mm}$ in the birefringent cylinder. One can obtain easily isoclinics for different values of the angle α by simulating a rotation of the polarizer and the analyzer axes in order to obtain the principal stresses direction in any chosen plan of the stressed model.

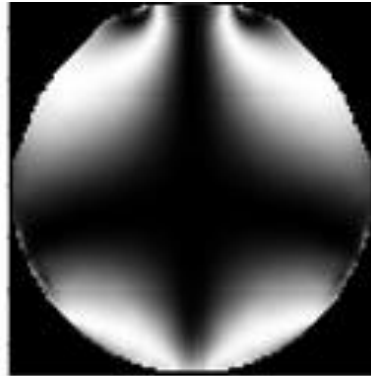


Figure 12: Simulated isoclinics for a slice located at $z=0\text{ mm}$, along the direction of the applied load, in the birefringent cylinder.

5. CONCLUSION

The study of a two dimensional model has shown that the simulation of stresses gives relatively good agreements with the experimental ones. The simulated photoelastic fringes are similar to the photoelastic fringes obtained on a regular polariscope. A solution for a three dimensional problem was developed for the case of a rigid cylinder on a deformable one. Isochromatic and isoclinic fringes were obtained for various slices along the longitudinal axis of the cylinder. This allows us to locate the zones of stress concentration which is of great importance in the design of mechanical components. An experimental solution either by the stress freezing method or the optical slicing method can be done for validation.

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