

ISSN: 2392-5442 ESSN : 2602-540X		<i>Sport system journal</i>
V/13 N/01 Year/2026		<i>International scientific journal published by: Ziane Achour –Djelfa- Algeria</i>
P 87 - 106		<i>Received: 2025-10-13 Accepted: 2025-11-24</i>

D- prime Energy Expenditure and Recovery Dynamics: Validation of the Mathematical Skiba Model During Continuous Effort

Mohamed Dayaa Eddine Guermat^{1*}, Abderrahmen Hadji², Ahmed Kabes³

¹ Université de Bejaia (ALGERIA) mohameddayaaeddine.guermat@univ-bejaia.dz

² Université de Bejaia (ALGERIA) abderrahmen.hadji@univ-bejaia.dz

³ Université de Bejaia (ALGERIA) ahmed.kabes@univ-bejaia.dz

¹²³laboratoire des sciences de l'expertise et de la technologie des activités physique et sports (c1671700) université Alger 3

Abstract:

This study tested how well the Skiba models—linear, hyperbolic, and inverse time continuous—can predict time to exhaustion (TTE) in athletes. Twenty participants (age = 17.4 ± 0.5 years; $\text{VO}_{2\text{max}} = 50.7\text{--}52.9 \text{ ml}\cdot\text{kg}^{-1}\cdot\text{min}^{-1}$) performed exercise tests, and their predicted TTE values were compared with actual results using Jamovi 2.3.28. On average, all models underestimated performance (predicted ≈ 220 s vs. actual ≈ 284 s). Linear regression analyses showed weak relationships between predicted and actual TTE ($R \approx 0.29$, $R^2 \approx 0.08$), a large error (RMSE ≈ 36 s), and no statistical significance ($p > 0.05$). The hyperbolic and inverse-time models performed slightly better, but not enough to be reliable. These findings imply that while the Skiba framework utilizes valid critical power theory, it does not provide a valid prediction of real athletic performance without further calibration. Future studies will need to modify the model.

Keywords: D- prime Energy, Recovery Dynamics, Skiba Model, Continuous Effort

*Corresponding author / mohameddayaaeddine.guermat@univ-bejaia.dz

1. INTRODUCTION

Quantifying and predicting athletic performance are key concerns in exercise physiology. Reliable predictive models are useful for coaches and sport scientists to manage training loads, monitor fatigue, and predict performance in both laboratory and field settings. In recent decades, many mathematical techniques have been proposed to characterize the relationship between power output and duration of exercise, with the ‘critical power’ (CP) concept being the most widely used (Jones et al., 2010). This model states that when athletes perform exercise above their CP, the finite work capacity above this threshold- W' (W' -prime) will progressively be depleted until exhaustion occurs. The CP- W' relationship has been demonstrated to describe limits to endurance during continuous severe-intensity exercise with physiologically interpretable parameters related to aerobic and anaerobic metabolism, respectively (Poole et al., 2016 ; Vanhatalo et al., 2011).

Building on this foundation, Philip Skiba and colleagues created an expanded set of mathematical models to predict time to exhaustion (TTE) and manage work–recovery during intermittent or variable-intensity work. The so-called Skiba models (linear, hyperbolic, and inverse-time continuous) give an explanation of W' depletion and reconstitution kinetics to predict the moment of task failure (P. F. Skiba et al., 2012 ; P. Skiba & Clarke, 2021). In reality, these types of models have been used to develop pacing strategies for cycling and running, prescribe interval training sessions, race simulation software, among other applications. While these models are elegant and theoretically based, it does not answer how accurately and widely these models will predict athletic performance in the field. The Skiba framework is based on the premise that the finite anaerobic energy resource (W'), is consumed exponentially when exercise will greater than CP, and replenished as exercise intensity are reduced below CP The model thus predicts exhaustion when W' is reduced to zero, at which point the athlete is unable to sustain exercise. This dynamic equilibrium can be expressed mathematically in the form of able to are differential or integral equation that updates the athlete’s remaining W' in real time (P. F. Skiba et al., 2012).

Although theoretically justified, this method rests upon a variety of simplifying assumptions. First, it assumes a constant level of metabolic efficiency, neglecting important intra-individual variation, neuromuscular fatigue, and environmental effects such as temperature or hydration. Second, it assumes W'



D- prime Energy Expenditure and Recovery Dynamics: Validation of the Mathematical Skiba Model During Continuous Effort

is a single, homogeneous energy store, yet recent physiology research suggests that there are multiple mechanisms, both central and peripheral, that contribute to fatigue and do so differently across endurance of exercise (Jones et al., 2019). Lastly, the estimation of the parameters—which is especially critical for the right selection of CP and W' with the performance trials—is important: small estimation errors can lead to disproportionately large errors from TTE model predictions (Mattioni Maturana et al., 2018).

Several studies have evaluated the predictive validity of CP and W'-balance models. Most notably, Jones et al (2010) established that CP and W' are sufficiently reliable to predict performance in exercise tolerance tests conducted in the laboratory. However, more recent studies demonstrated that predicted TTE values derived from the CP relationship could often provide substantial differences from observed performance in real-world and intermittent contexts (Mattioni Maturana et al., 2018). Investigations into W' reconstitution during variable-intensity exercise demonstrate significant inter-individual heterogeneity in recovery time constants ($\tau_{W'}$), hence constraining the model's generalisability (Welburn et al., 2025).

Recent validation studies have shown that Skiba's W'-balance equations are often an underestimate of actual performance in intermittent cycling and running exercises when recovery periods are short or intensity fluctuates rapidly (Weigend et al., 2019). This corresponds to the results seen with field data, where model root-mean-square error (RMSE) remains relatively high, even when utilizing individual calibrated parameters (Galán-Rioja et al., 2023). In summary, these results suggest that while some of the predictions along the physiological slope discussed by Skiba are meaningful, insufficient predictive accuracy for performance will remain as the needs for the models to be recalibrated.

From a theoretical perspective, testing the reliability of the Skiba model expands our knowledge of performance modeling approaches in sports science. A mathematical model, such as CP-W' or W'-BAL, offers a mechanistic explanation of fatigue; however, its usefulness for purposes of application is determined only by the empirical reliability of the data. If the Skiba models do

not account for the actual TTE, then the assumptions of constant recovery kinetics, W' fixed, and linearly depleting W' are not an accurate representation of the multifaceted endurance performance physiology. If reliability can be established to support the models, then they can be justified as efficient and non-invasive mechanisms for predicting performance and optimizing training at an individual level. Practically, accurate prediction models can revolutionize training prescription and race pacing strategies. Coaches may utilize them to gauge sustainable work rates, schedule optimal recovery times, and track change over time (Barroso et al., 2022). However, if predictive error exceed meaningful performance metrics, such as low R^2 values and large RMSEs, model-based prescriptions may be misleading about the capacity of the athlete, and potentially lead to some level of over- or under-training. It is critical to examine whether the validity of the Skiba model is unchanged outside the conditions by which it was derived, before treating it as a decision-making tool in preparation for sport.

Given the available data of model variation, and the increasing use of predictive analytics in sport, the current study asks the question: Is the Skiba model valid for predicting performance in sport in a specifically time to exhaustion (TTE) setting? In examining the actual TTE values and the model-predicted TTE values of the linear, hyperbolic, and the inverse time continuous versions of the model, we determined accuracy (systematic bias) and precision (variance of the error) of the TTE values. In this study, we will evaluate the statistical validity (correlation (R), explained variance (R^2), and RMSE), as well as the direction and magnitude of the prediction error to determine whether the model is under estimating, or over estimating performance.

This method adds to the existing discussion concerning the applicability of critical-power-based modeling to field performance and represents an important bridging of the gap between theoretical modeling and real-world results. The results, narrated in the following sections, are meant to provide clarity around whether the Skiba framework can reliably predict athletic performance and whether the underlying rationale for training and competition purposes remains scientifically valid.



D- prime Energy Expenditure and Recovery Dynamics: Validation of the Mathematical Skiba Model During Continuous Effort

2. Method and Materials

2.1 Participants

Twenty amateur football players, all male and in good health, belonging to the under-19 category of the Sidi Moussa Sports Club (CSSM), which competes in the regional division of the Algiers Wilaya Football League in Algeria, took part in this study. The team's coach served as the principal researcher. The participants were informed of the study protocol, and their participation was conditional upon signing an informed consent form. This form was also signed by their parents and validated with the official stamp of the club president. The participants had an average age of 17 ± 4 years (mean $\pm$ standard deviation), an average height of 173 ± 2 cm, and an average weight of 65 ± 7 kg. Their average BMI was 21.8 ± 1.29 , with an average MAS (maximum aerobic speed) of 14 ± 47 for session 1 (S1) and 15 ± 12 for session 3 (S3). Their average VO₂max was 50.7 ± 3 for S1 and 52.9 ± 2.37 for S3.

2.2 Materials

The trial took place on a 200-meter outdoor running track that was created using cones placed on a soccer field and dimensions were taken using a measuring tape. The Kalenji BT was used to collect the speed and heart rate data, and to determine critical speed (CS) or critical power (CP) and critical speed capacity (D') or (W') using the EXphysLab software, using three modeling methods: linear, quadratic, and inverse time (s^{-1}). Following a two day break, a continuous test was performed at 110% CS. After the data was recorded, the Python program was used to provide the time to exhaustion for the SKIBA model .

2.3 Design and Procedure

The study lasted three months. The first session was held from September 17 to 21, 2024, and the second from November 16 to 20, 2024. In each session, participants performed three running tests lasting 3, 6, and 12 minutes each. They also did a continuous effort test until exhaustion. The tests were done on a 200-meter track that was set up beforehand. During these tests, the athletes were told to run either until they were exhausted or until the time was up. For each session, the distance covered and the athletes' heart rates at the end of the effort were recorded.

2.4 Statistical Analysis

The data were analyzed using Jamovi version 2.3.28. As previously mentioned, descriptive statistics (mean $\pm$ SD, median, min, max) were computed for the predicted time to exhaustion (TTE) and the actual TTE. Normality was tested using the Shapiro–Wilk test in addition to Q–Q plots. Simple linear regression was used for analysis all three of the Skiba models (linear, hyperbolic, and inverse time) to evaluate the relationship between predicted and actual TTE. Each model reported the correlation coefficient (R), coefficient of determination (R^2), root mean square error (RMSE), t -statistic, and p -value. The statistical significance threshold was for $p < 0.05$ and effect size was interpreted based on recommendations by Cohen (1988). The models were reviewed for performance via R^2 and RMSE values, in consideration of confirming predictive validity.



D- prime Energy Expenditure and Recovery Dynamics: Validation of the Mathematical Skiba Model During Continuous Effort

3 Results

Sample

	N	Mean $\pm$ SD (m/s)	Max - Min
Age	20	17.4 $\pm$ 0.50 (y)	17 $\pm$ 18
Weight	20	65.7 $\pm$ 4.27 (	58 $\pm$ 72.5
Height	20	1.73 $\pm$ 0.05	1.65 $\pm$ 1.84
BMI	20	21.8 $\pm$ 1.29	19 $\pm$ 23
VO2max S01	20	50.7 $\pm$ 3	45.5 $\pm$ 56.3
VO2max S03	20	52.9 $\pm$ 2.37	49 $\pm$ 57.4

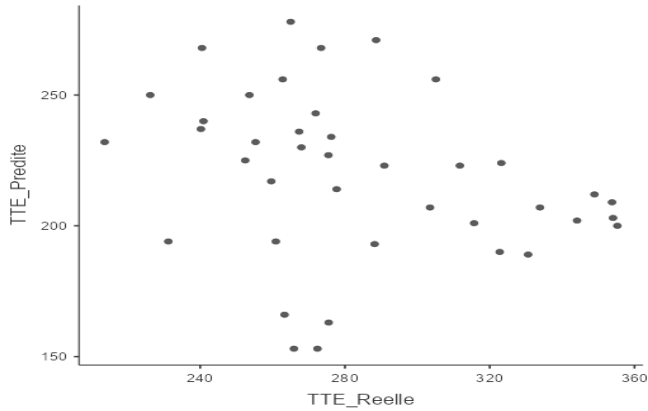
3.1 Skiba Linear Continuous Model

Table 1: Comparative Analysis of Predicted and Actual Performances (TTE in seconds)

Statistique	TTE Prédit (s)	TTE Réel (s)
N	40	40
Moyenne	219	284
Médiane	223	274
Écart-type	31.3	38.0
Minimum	153	214
Maximum	278	355

The table contrasts the predicted TTE values versus the actual TTE values. The actual TTE values (Mean = 284 s, SD = 38.0) are consistently higher than the predicted TTE values (Mean = 219 s, SD = 31.3). The median (274 s vs. 223 s) and the range (214-355 s vs. 153-278 s) help to confirm this finding. In conclusion, the model tends to underestimate performance as the predicted TTE is lower than the actual TTE results.

Figure 1: Scatter Plot of Predicted TTE as a Function of Actual TTE – Skiba Linear Continuous Model



The scatterplot compares TTE values predicted on the y-axis to TTE values obtained on the x-axis. Most points fall below the line of equality (where real = predicted), suggesting that the predicted TTE value is lower than real TTE value in general. There is a clear spread, indicating there is variability, but the action here indicates that the model undervalues performance systematically not over.

Table 2: Summary of the Linear Regression Model of Actual TTE as a Function of Predicted TTE – Skiba Linear Continuous Model

Modèle	R	R ²	RMSE	t	p
1	0.285	0.0811	36.0	-1.83	0.075

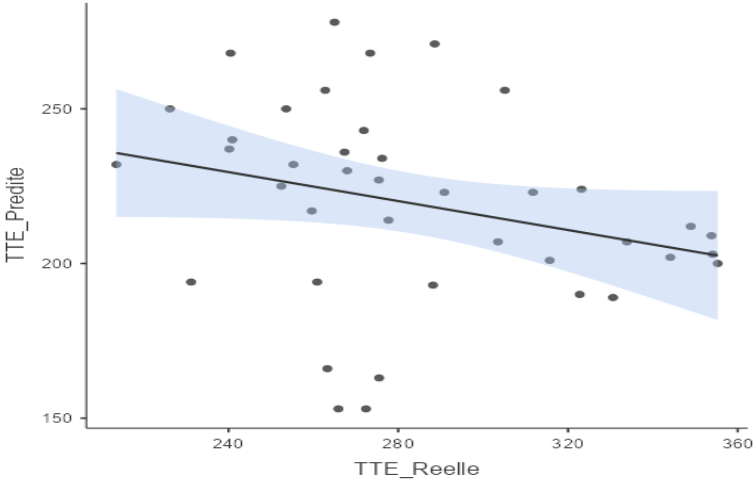
The regression analysis indicates quite weak association between computed values of time to exhaustion (TTE) from the Skiba linear model and TTE values measured in the field, according to its correlation coefficient ($R = 0.285$), indicating only a small positive association, and its coefficient of determination ($R^2 = 0.0811$), indicates only 8.1% of the variance of the actual TTE values can be accounted for by the computations of the model. This means the majority of the variance in performance by the participants is not captured in the predictions of the model. In addition, the root mean square error ($RMSE = 36.0$) suggests an appreciable mean difference of TTE values predicted by the Skiba model and actual TTE values, demonstrating a relatively large prediction error. The t statistic (-1.83) combined



D- prime Energy Expenditure and Recovery Dynamics: Validation of the Mathematical Skiba Model During Continuous Effort

with the p value (0.075) indicates that the slope of the regression is not statistically significant ($p > 0.05$), meaning the observed relationship may be due to chance variables and not a significant linear pattern. In summation, these results suggest that the current specifications of the Skiba linear model are not accurately or reliably predicting real time to exhaustion values, and that it requires revision, or at least additional variance, in order to properly predict time to exhaustion values.

Figure 2: Linear Regression Between Predicted TTE and Actual TTE – Skiba Linear Continuous Model



The scatter plot shows the relationship between real time to exhaustion (TTE_Réelle) and time to exhaustion (TTE_Predite) as determined by the Skiba linear model. The regression line is small and sloping downward, which indicates that there is a very weak downward correlation for TTE functions, whereby when one has an increase in actual TTE predite TTE decreases slightly. However, there is a very significant amount of point dispersion around the regression line, while the blue shaded area indicating a large confidence interval demonstrating a wide range of uncertainty of the model predicted. Visibly, the point dispersion correlates with the statistical findings, as the correlation coefficient and R^2 value suggest a weak linear relationship and poor explanatory powers. Overall, the plot confirms

my point that the model is not modeling TTE reasonably, meaning that the linear Skiba model is a not reasonable estimate of real performance outcomes in relation to the performance variable time to exhaustion.

3.2 Skiba Hyperbolic Continuous Model

Table 3: Comparative Analysis of Predicted and Actual Performances (TTE in Seconds) – Skiba Hyperbolic Continuous Model

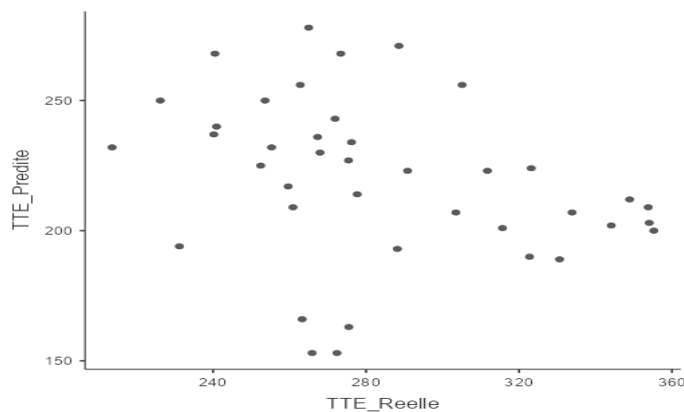
Statistique	TTE Prédit (s)	TTE Réel (s)
N	40	40
Moyenne	220	284
Médiane	223	274
Écart-type	31.0	38.0
Minimum	153	214
Maximum	278	355

The descriptive statistics suggest a clear gap between predicted-TTE and real TTE. The mean predicted TTE (220 s) is lower than the mean real TTE (284 s). Both the median (223 vs. 274) and range (153–278 vs. 214–355) measures support a systematic underestimation. Standard deviations in TTE measures is similar (31.0 vs. 38.0), indicating there is comparable variability, but overall, the model consistently predicts lower TTE across the sample. Thus, as it was with the linear model, the hyperbolic continuous approach examining TTE using the method of Skiba (2005) similarly erred on the side of underestimating TTE, providing evidence of a persistent bias.



D- prime Energy Expenditure and Recovery Dynamics: Validation of the Mathematical Skiba Model During Continuous Effort

Figure 3: Scatter Plot of Predicted TTE as a Function of Actual TTE – Skiba Hyperbolic Continuous Model



The scatterplot depicts the relationship between predicted TTE (y-axis) and actual TTE (x-axis) for the Skiba hyperbolic continuous model. Similar to the earlier linear model, the majority of values fall underneath the line of equality, suggesting that the model tends to systematically underestimate observed performance. The degree of spread across the values is wide, reflecting some variability and not a strong predictive relationship between predicted and actual values. In summary, the hyperbolic version of the Skiba model also does not adequately capture the actual TTE, with predictions continuously underestimating actual outcomes.

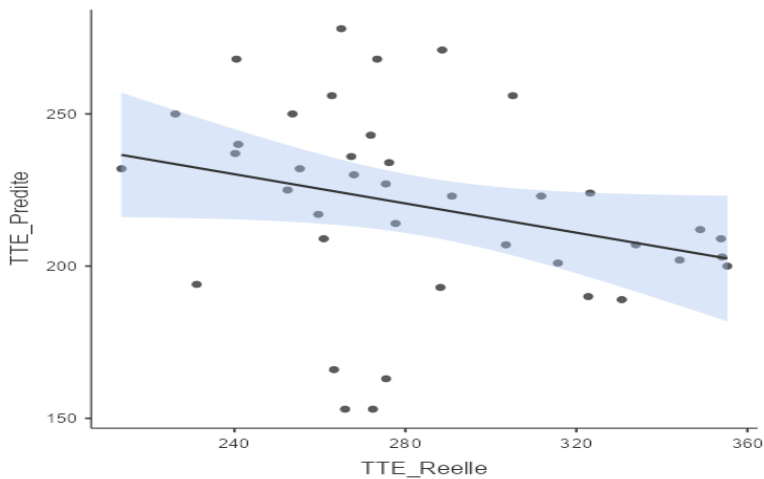
Table 4: Summary of the Linear Regression Model of Actual TTE as a Function of Predicted TTE – Skiba Hyperbolic Continuous Model

Modèle	R	R ²	RMSE	t	p
1	0.294	0.0867	35.9	-1.90	0.065

Regression analysis using the Skiba hyperbolic continuous model found a small and not significant relationship between predicted time to exhaustion (TTE) and actual TTE. The correlation coefficient ($R = 0.294$) indicates a very weak positive relationship where larger predicted times were slightly related to larger actual times

(i.e., larger predicted TTEs were positively related to larger actual TTEs). The coefficient of determination ($R^2 = 0.0867$) shows the model predicts only 8.7% of the variance for actual TTE, which indicates a very low level of predictive ability. The root mean square error (RMSE = 35.9) was somewhat high indicating a substantial amount of variance remains between predicted and actual. The t-stat (-1.90) and p-value suggests that this finding is not significant and is almost significant at the 0.05 level. In conclusion, while the hyperbolic model performed somewhat better than the linear model, it is not sufficiently explanatory, which indicates that ultimately it is not accurately predicting TTE as a model and neither model supported the reliability or utility to lead to the consideration of TTE as a factor of interest for future study.

Figure 4: Linear Regression Between Predicted TTE and Actual TTE – Skiba Hyperbolic Continuous Model



The scatterplot illustrates the link between actual Time to Exhaustion (TTE_Réelle) and TTE as estimated using the Skiba hyperbolic continuous model (TTE_Predite). The regression line calculated from within the sample indicates slight negative trajectory suggesting that less TTE is expected as TTE_Réelle increases. As compared with the regression slope, the data points contained in the blue confidence intervals (shaded blue area) suggest there was considerable variation throughout the data. Overall, the data does lead to the observation that there was a minor muscle fatigue relationship between TTE_Réelle and TTE_Predite contradictory to the



D- prime Energy Expenditure and Recovery Dynamics: Validation of the Mathematical Skiba Model During Continuous Effort

finding of a low R^2 (0.0867) and non-significant p value (0.065) from the regression analysis, thereby suggesting with regard to the relevance of findings that the model will only have limited predictive accuracy and reliability about TTE_Réelle.

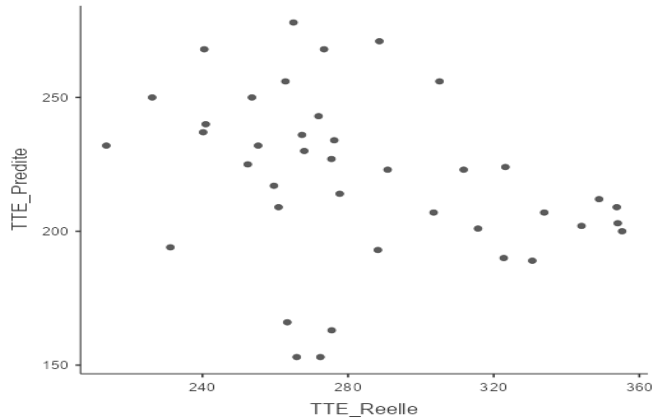
3.3 Skiba Inverse Time Continuous Model

Table 5: Comparative Analysis of Predicted and Actual Performances (TTE in Seconds) – Skiba Inverse Time Continuous Model

Statistique	TTE Prédit (s)	TTE Réel (s)
N	40	40
Moyenne	220	284
Médiane	223	274
Écart-type	31.0	38.0
Minimum	153	214
Maximum	278	355

Once again, the descriptive statistics reveal an underestimation of TTE. The mean predicted (220 s) was significantly lower than the actual mean (284 s). The inferior (223 vs. 274) and maximum-minimum (153-278 vs. 214-355) ranges support this systematic bias. The variability ($SD = 31.0$ vs. 38.0), while comparable, also shows the model does not fit actual outcomes. In summary: the inverse-time Skiba continuous model, as with the linear and hyperbolic models, systematically underestimates actual TTE.

Figure 5: Scatter Plot of Predicted TTE as a Function of Actual TTE – Skiba Inverse Time Continuous Model



The scatter plot shows the relationship between real time to exhaustion (TTE_Réelle) and predicted time to exhaustion (TTE_Predite). The points are widely scattered with no visible linear relationship showing weak relationship. There is a potential for a small downwards trending, however the lack of any strong association in any direction shows the predicted TTE do not follow the actual TTE closely.

Table 6: Summary of the Linear Regression Model of Actual TTE as a Function of Predicted TTE – Skiba Inverse Time Continuous Model

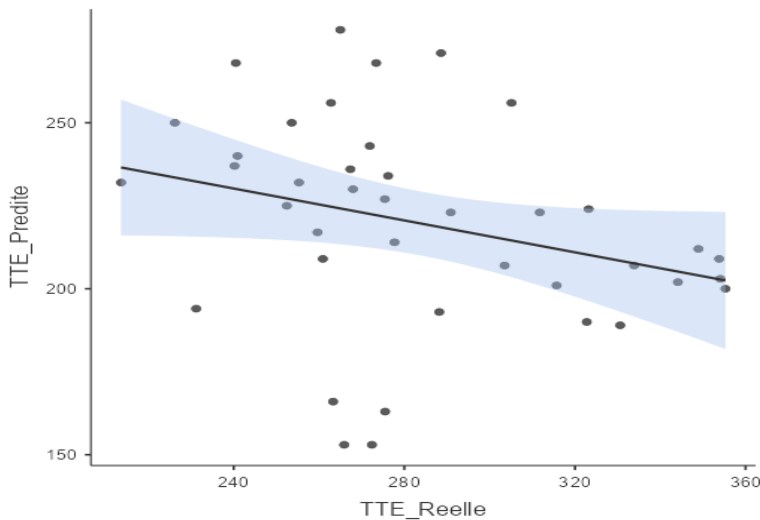
Modèle	R	R ²	RMSE	t	p
1	0.294	0.0867	35.9	-1.90	0.065

The Skiba inverse time model presents a weak relationship as with the previous models, between predicted and actual TTE. The intention of this model, which is based on an inverse time transformation ($1/TTE$), is to better account for the nonlinear relationship between power and exercise duration. However, given the data we obtained ($R \approx 0.29$, $R^2 \approx 0.087$, $RMSE \approx 35.9$, $p \approx 0.065$), this was not able to significantly enhance the predictive quality. The correlation coefficient suggests that the predicted TTE reflects the actual TTE, but not well. The value of the coefficient of determination indicates that less than 9% of the variance in actual TTE is accounted for, while the "all tells" RMSE indicates high variability. Finally, the p-value, slightly above 0.05, confirms that the relationship was not statistically significant. In sum, like the continuous linear and hyperbolic models, the Skiba

D- prime Energy Expenditure and Recovery Dynamics: Validation of the Mathematical Skiba Model During Continuous Effort

inverse time model does not improve predictive accuracy. The model resulted in the same weak consistency between prediction and actual values; suggesting the regression form does not appropriately account for the actual dynamics of TTE performance.

Figure 6: Linear Regression Between Predicted TTE and Actual TTE – Skiba Inverse Time Continuous Model



The scatter plot depicts the relationship between the actual (TTE_Réelle) and predicted (TTE_Predite) time to exhaustion based on the Skiba inverse time model. The regression line denotes a negative slope indicating as the TTE_Réelle variable increases the TTE_Predite variable decreases slightly. Clearly the data are widely spread away from the regression line and the confidence band (blue band) is fairly wide (with wide variation) which indicates uncertainty in the small handful of findings from the model. Overall, the scatter plot provides evidence of a weak association between predicted and actual time to exhaustion which was also reflected in the regression outputs where the R (≈ 0.29) and R^2 values (≈ 0.0867) values were low. Overall, the analysis determined that the Skiba inverse time model does a poor job at predicting performance in a TTE_Réelle, has low reliability, and suggests a poor fit to the data of this study.

4 Discussion

Overall, the three implementations (linear, hyperbolic, and inverse-time) yielded Skiba-based predictions which under-predicted actual TTE and explained little variance ($R \approx 0.29$, $R^2 \approx 0.08$; $RMSE \approx 36$ s; $p > 0.05$). That is to say, at face value, these provided weak predictive validity in the cohort. These are not surprising considering what existing literature reports, such as (i) the sensitivity of critical-power/Skiba-type models to protocol and model choice more broadly, and (ii) the assumptions behind W' expenditure/reconstitution during continuous work. The critical power (CP) model that Skiba's models are derived from describes the hyperbolic power--duration (or speed--duration) relationship well, predicting tolerance for continuous severe power efforts, if the parameters are estimated well. However, the practical conversion of that relationship into valid TTE predictions in the field relies on the testing protocols of the modeled relationships, and the model forms used to fit CP and W' . For example, reviews and comparative studies demonstrate how CP and W' can differ between protocols (constant-work rate TTE trials, 3-min all-out, ramp) and fit forms (linear, 2- or 3-parameter models, $1/t$) that could affect predicted downstream performance. This may lead to over- or under-estimating TTE outcomes, as varied TTE testing protocols are expected to produce different results: 3-min all-out would likely demonstrate a longer TTE than a constant power test of the same or similar power output. (Jones et al., 2010).

The Skiba W' -balance (W' _BAL) formulations (integral and differential) are developed for continuous exercise to model the depletion and curvilinear recovery of W' . Narrative and empirical work concludes the models are conceptually useful but demonstrate inconsistent accuracy across protocols and individuals; in particular, parameterizations of $\tau_{W'}$ and the choice of 2- vs. 3-parameter CP are shown to impact estimates of exhaustion, with recent work demonstrating potential for improvement (e.g., more accurate estimates of W' and context-modulated $\tau_{W'}$). (P. Skiba & Clarke, 2021).

Lastly, large convenience samples of performance models raise a caution that intra-individual variability, day-to-day noise, and the specific functional form of regression introduce additional error—a support for validating in the populations and tasks of interest (Sreedhara et al., 2019).

ongoing under-prediction and low R^2 suggest poor calibration and discrimination for our sample/task. Possible factors may be: (1) parameter estimation choices (e.g. model form & estimation protocol), (2) population specificity (e.g. young adults with narrow dispersion in population characteristics), (3) intermittent vs. continuous



D- prime Energy Expenditure and Recovery Dynamics: Validation of the Mathematical Skiba Model During Continuous Effort

(e.g. TTE tasks, training history) for sample match with the model's assumptions, and (4) unmodeled covariates (e.g. day-to-day variability, motivation, environment).

Provisionally, Skiba/CP modeling is not physiologically sound and cannot be justified by either continuous severe-domain predictions or pacing applications. However, even with these restrictions, the implementation of the Skiba variants will, without recalibration, yield a significantly weakened accuracy predicting TTE with actionable accuracy in the cohort. They could be considered trial-support (trend/monitoring) rather than what they actually are predicting TTE without recalibration. When you: Re-estimate CP/W' with a protocol-appropriate and robust modeling approach (e.g. compare 2- vs 3-parameter and linear vs $1/t$ fits – with at least a 2 min, up to 15 min TTE, repeat trials to quantify intra-individual variability) (Mattioni Maturana et al., 2018). For predicted intermittent, test both W'BAL integral and ODE variants and use $\tau_{W'}$ parameterizations; sometimes it may help the W' predictions to see whether a 3-parameter CP is beneficial (P. Skiba & Clarke, 2021). Perform out-of-sample validation (k-fold CV or a hold-out day) and calibration checks (Bland-Altman, calibration slope/intercept) as per prediction-model best practice (Debray et al., 2017).

5. CONCLUSION

All three models of the Skiba fitted poorly for time to exhaustion (TTE) with respect to the actual TTE during this study showing systematic underestimation of athletic performance with mean predicted TTE values across all models lower than the measured TTE. The weak correlation coefficients ($R \approx 0.29$) and low determination coefficients ($R^2 \approx 0.08$) indicated less than 9% of real TTE variance was explained by the models; RMSE values were large (~ 36 s) and indicated poor model fit with non-significant p-values (> 0.05), further reiterating the limited validity of the Skiba equations against time to exhaustion in this sample.

The hyperbolic and inverse-time formulations offer slight improvements in correlations as compared to the linear model. Differences between hyperbolic and inverse-time formulations and the linear model were small and statistically trivial. The results appear consistent with past studies of Skiba-type- and critical-power-

based models sensitivity to the protocol selected, individual variability, and the conceptual idea of linearity between power and duration (Jones et al., 2010 ; Mattioni Maturana et al., 2018). In addition, more contemporary studies examining the W' balance concept (Skiba & Clarke, 2021 ; Welburn et al., 2025), showed that while the models are conceptually useful to predict energetic availability and dynamics of fatiguing, practical application to field- or intermittent-based performance prediction is limited without careful calibration and individualized aspects of the variable parameter. To summarize, findings indicate the Skiba models (linear, hyperbolic, or inverse time) were not sufficient in validity and reliability for predicting time to exhaustion in this cohort of athletes. Future work should refine the models to include individualised parameters, non-linear adjustments to the models, and external factors (e.g., motivation, environmental conditions, states of fatigue). In addition, it is essential to validate the findings by examining larger sample sizes of unique cohorts of participants, to determine whether recalibrating the model, enhanced with successful validation, would result in stronger predictive validity and practical utility for performance monitoring and training prescription.

6. Bibliography List :

Barroso, R., Carmo, E. C. do, Foster, C., Skiba, P., & Barbosa, A. C. (2022).

Longitudinal analysis of the 800-m performances of the world's best female long-distance pool swimmer : A case study using critical speed and D'. *International Journal of Sports Science & Coaching*.

<https://doi.org/10.1177/17479541221104721>

Debray, T. P. A., Damen, J. A. A. G., Snell, K. I. E., Ensor, J., Hooft, L., Reitsma, J. B., Riley, R. D., & Moons, K. G. M. (2017). A guide to systematic review and meta-analysis of prediction model performance. *BMJ*, 356, i6460. <https://doi.org/10.1136/bmj.i6460>

Galán-Rioja, M. Á., González-Mohino, F., Skiba, P. F., & González-Ravé, J. M. (2023). Utility of the W'BAL model in training programme design for masters cyclists. *European Journal of Sport Science*, 23(7), 1259-1268. <https://doi.org/10.1080/17461391.2022.2142675>



D- prime Energy Expenditure and Recovery Dynamics: Validation of the Mathematical Skiba Model During Continuous Effort

- Jones, A. M., Burnley, M., Black, M. I., Poole, D. C., & Vanhatalo, A. (2019). The maximal metabolic steady state : Redefining the « gold standard ». *Physiological Reports*, 7(10), e14098.
<https://doi.org/10.14814/phy2.14098>
- Jones, A. M., Vanhatalo, A., Burnley, M., Morton, R. H., & Poole, D. C. (2010). Critical power : Implications for determination of $\dot{V}O_{2\max}$ and exercise tolerance. *Medicine and Science in Sports and Exercise*, 42(10), 1876-1890. <https://doi.org/10.1249/mss.0b013e3181d9cf7f>
- Mattioni Maturana, F., Fontana, F. Y., Pogliaghi, S., Passfield, L., & Murias, J. M. (2018). Critical power : How different protocols and models affect its determination. *Journal of Science and Medicine in Sport*, 21(7), 742-747.
<https://doi.org/10.1016/j.jsams.2017.11.015>
- Poole, D. C., Burnley, M., Vanhatalo, A., Rossiter, H. B., & Jones, A. M. (2016). Critical Power : An Important Fatigue Threshold in Exercise Physiology. *Medicine and Science in Sports and Exercise*, 48(11), 2320-2334.
<https://doi.org/10.1249/MSS.0000000000000939>
- Skiba, P., & Clarke, C., David C. (2021). The W' Balance Model : Mathematical and Methodological Considerations. *International Journal of Sports Physiology and Performance*, 16(11). <https://doi.org/10.1123/ijsp.2021-0205>
- Skiba, P. F., Chidnok, W., Vanhatalo, A., & Jones, A. M. (2012). Modeling the expenditure and reconstitution of work capacity above critical power. *Medicine and Science in Sports and Exercise*, 44(8), 1526-1532.
<https://doi.org/10.1249/MSS.0b013e3182517a80>

- Sreedhara, V. S. M., Mocko, G. M., & Hutchison, R. E. (2019). A survey of mathematical models of human performance using power and energy. *Sports Medicine - Open*, 5(1), 54. <https://doi.org/10.1186/s40798-019-0230-z>
- Vanhatalo, A., Jones, A. M., & Burnley, M. (2011). Application of critical power in sport. *International Journal of Sports Physiology and Performance*, 6(1), 128-136. <https://doi.org/10.1123/ijsp.6.1.128>
- Weigend, F. C., Clarke, D. C., Obst, O., & Siegler, J. (2019). *A hydraulic model outperforms work-balance models for predicting recovery kinetics from intermittent exercise* | *Annals of Operations Research*.
<https://link.springer.com/article/10.1007/s10479-022-04947-2>
- Welburn, A. J., Pugh, Charles F., Bailey, S. J., & Ferguson, R. A. (2025). *W' reconstitution modelling during intermittent exercise performed to task failure* | *European Journal of Applied Physiology*.
<https://link.springer.com/article/10.1007/s00421-025-05912-0>