

Analysis of Pulse Width Modulation DC-DC Chopper -On/Off Control via a second order filter-

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Abstract

*This paper presents the analysis of a nonlinear on/off control system including a filter of a second order in the closed loop for compensating the fluctuation of the irradiation as well as the variations of the load. The proposed system is capable of generating a pulse width modulation which is used to design and built up a **PWM** (Pulse Width Modulation) chopper dedicated to regulate fluctuating power supplies (photovoltaic, wind, etc.). The study essentially focuses on determining the relationship between the pulse duration with respect to system parameters and technological requirements. The theoretical study is followed by a simulation of a **DC – DC** chopper.*

Keywords: on/off control, pulse width modulation, chopper design, optimal control

1-Introduction

In recent years, renewable energy regains serious interest in most developed countries. Indeed, these energies are renewable, non-polluting, distributed through space and can be collected and converted by means of converters. Among renewable energy sources that have experienced great development, include solar photovoltaic, wind energy and biomass. However, one of the major problems concerning the spread of renewable energy is to further reduce system cost while increasing performance.

But if the first challenge relates to the manufacturing technology, there is another problem concerning the adaptation of this energy for use in various applications such as pumping water, controlling electric motors, lighting, battery charging, etc. [1, 2]. This concerns in particular the regulating devices which are **DC / DC** converters (choppers) and **DC / AC** (inverters). The interest in this problem is that the performance of these devices directly influences the overall performance of the energetic system as a whole. Research in the field of **DC – DC** converters and **DC – AC** revealed for more than a decade the importance of on-off power supplies. Indeed, the use of power transistors (bipolar and **VMOS**) as fast static switches in the power amplifier reduces the consumption. Also, it enables sufficiently high cutting frequencies to

facilitate the filtering of harmonics and consequently reducing the weight, size and cost of the converter. In this respect, switching converters based on the technique of pulse width modulation (**PWM**) were imposed as a viable system for the case of low power [3, 4, 5, and 6].

In a previous work [7], we have designed and built up an experimental **DC / DC** converter based on an on/off element with a feedback via a first order filter. The load was outside the loop. As a consequence, the system was able to regulate the output voltage w.r.t to the fluctuations of the solar energy. But did not regulate the output voltage w.r.t the variation of the load.

The work proposed in this article is an extension of [7]. It concerns the study of an **on – off** control system using a feedback via a second order filter including the load. In principle; this system enables the compensation of both variations stemming from the irradiation and of the load. The proposed model can also be analyzed in terms of time optimal control and can therefore be studied under the theory of "maximum principle" as established by [5]. This system is operated in the context of power converters which are dedicated to renewable energy sources (photovoltaic, wind, etc.) [8, 9, and 10], especially for the design and realization of a chopper.

Such a system is capable of generating a Pulse Width Modulation (*PWM*) which is used for the design and realization of a chopper. This study essentially aims to determine the relationship between the switching frequency according to system settings and related technological parameters. The theoretical study led to the establishment of conditions that determine the system performance based on the parameters chosen. It is followed by the simulation and the realization of an experimental chopper.

2-System Modeling Overview

The proposed control system is shown schematically in Figure (1). It can be reduced to the study of an *on-off* control system. It comprises the direct forward path a nonlinear $+/-$ symmetric element

denoted N . This nonlinear element is characterized by its threshold width h and its *on-off* level which equals $+E$ and $-E$. It also comprises a linear second order filter denoted W . In the feedback system path includes a matching element denoted k of the output to the set point value. The quantities involved are:

- f_0 : the set point signal.
- $X(t)$: the control signal.
- $U(t)$: the controlled signal.
- $Z(t)$: the feedback signal.
- $V(t)$: the output signal.

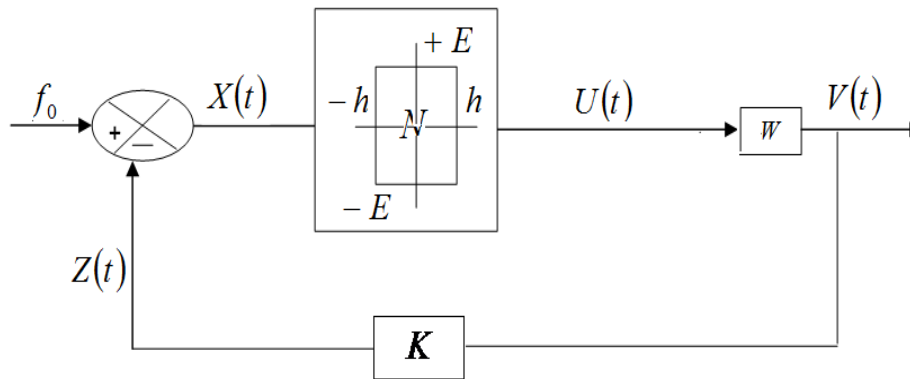


Figure (1): Scheme of the control system

The reference variable f_0 is compared with the controlled quantity $Z(t)$. The set point signal f_0 assumed constant in the case of a chopper. The resulting error is the control variable $X(t)$ which acts as input to the nonlinear N element. The signal $U(t)$ at the output N is converted by a linear second order W filter. This provides the output $V(t)$ to be used by the actual load. The inclusion of the filter W in the servo is used to regulate the output voltage vis-à-vis fluctuations (radiation, wind speed ...) and load variations. Note also that this system corresponds to the case of the optimal time control as developed by. [5]

The correspondence between the previous study and the envisioned *PWM DC-DC* is interpreted as follows. E Corresponds to the voltage provided by

the solar panel. It is thus, in general fluctuating and needs to be regulated before being able to feed the loads. $U(t)$ is the controlled voltage at the output of the *PWM* modulator. $V(t)$ is the output voltage which effectively supplies the load (engine; pump, battery, etc.)

2.1. the behavior of the non-linear element N

For simplicity, we assume that the input voltage $X(t)$ is triangular and can cross the threshold values $(+/-h)$ at specific instants. As a result, the output $U(t)$ will be a succession of constant pulses of amplitude E with alternating sign. The duration and the *on-off* intervals are variables. The instants during which the signal jumps are called the switching instants. These instants are determined by the following conditions

$$\begin{aligned}
 & \text{si } X(t) > +h \quad \text{et } \frac{dx(t)}{dt} > 0 \quad \Rightarrow U(t) = -E \\
 & \text{si } X(t) < -h \quad \text{et } \frac{dx(t)}{dt} < 0 \quad \Rightarrow U(t) = +E
 \end{aligned} \tag{1}$$

The evolution of the output signal in correspondence with the input signal is shown in **Figure (2)**

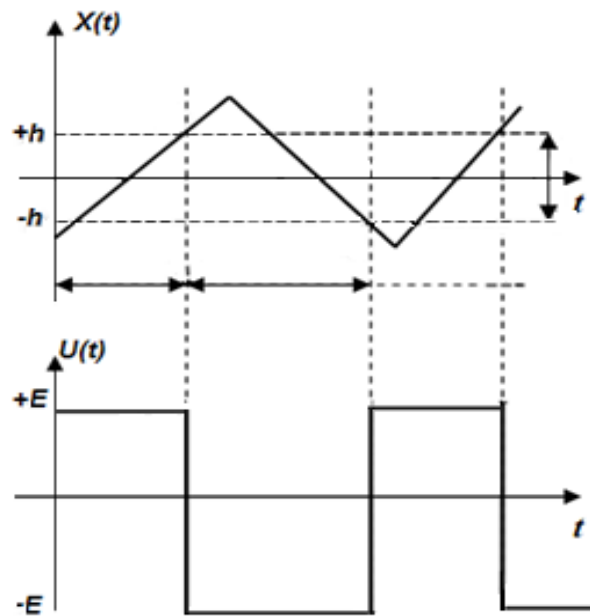


Figure (2): Input $X(t)$ and output $U(t)$ of the nonlinear element

II.2- Signal Analyze of the second order filter

The feedback is performed after the output of a linear element; the second filter order as show in Figure (3) consists of an inductor L , its internal resistance r ; a capacitor C , and the working load

R . The input V_{in} and output V_{out} of this filter.

When negligent the internal resistance r of the coil w.r.t the load resistance R of use, the input-output relation of the filter is given by the differential equation of second order

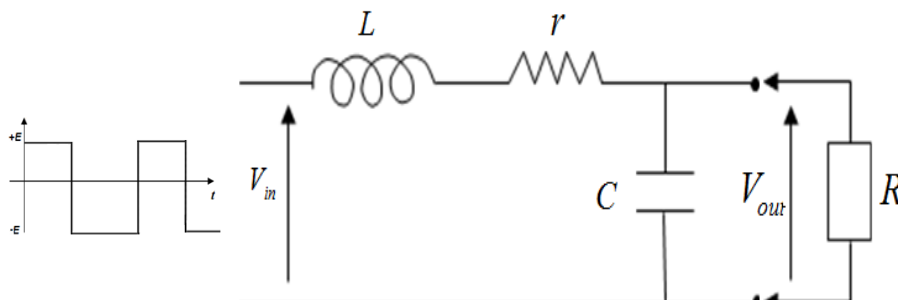


Figure (3): low pass filter of 2nd order

$$V_{in} = (-1)^k E = V_{out} + RC \frac{dV_{out}}{dt} + LC \frac{d^2 V_{out}}{dt^2} \quad (2)$$

The transfer function of this filter is:

$$\frac{V_{out}}{V_{in}} = F(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2} = \frac{\omega_n^2}{s_1 - s_2} * \left[\frac{1}{s - s_1} - \frac{1}{s - s_2} \right] \quad (3)$$

where

s : Is the Laplace variable.

ω_n, ξ : The angular frequency and dumping coefficient of the system

$$\omega_n = \frac{1}{\sqrt{LC}}$$

$$\xi = \frac{R}{2} * \sqrt{\frac{C}{L}}$$

3. Determination of Pulse Durations

3-1 General case

The determination of pulse duration is a fundamental problem in the analysis of *on-off* non linear systems. In the general case, obtaining an analytical solution is extremely, even impossible. And only graphical techniques or simulation can help to analyze this type of problems. The determination of analytical expressions of period of self-oscillation in case $f_0 \neq 0$ is difficult. Nevertheless, an analytical solution has been found for $f_0 = 0$ and is presented in this section.

For $f_0 = 0$, the input signal V_{in} of the second order filter is a square wave defined by

$$V_{in}(s) = X(s) = \frac{2E}{s} th \frac{Ts}{4} \quad (4)$$

By considering the characteristic equation (3) of the second order filter:

$$s^2 + 2\xi\omega_n s + \omega_n^2 = 0 \quad (5)$$

We can distinguish three cases with respect to the damping factor ξ , according to the two roots. s_1 and s_2 .

➤ for $\xi > 1 \Rightarrow$ we get two real roots which are ;

$$s_{1,2} = -\xi\omega_n \pm \omega_n \sqrt{\xi^2 - 1}$$

➤ for $\xi < 1 \Rightarrow$ we get two imaginary roots which are .

$$s_{1,2} = -\xi\omega_n \pm j\omega_n \sqrt{1 - \xi^2}$$

The system can generate oscillations,

➤ For $\xi = 1 \Rightarrow$ we get a double root:

$$s_{1,2} = -\omega_n$$

The output of the filter as an open loop

$$Y(s) = F(s) * X(s) \quad (6)$$

This can be rewritten.

$$Y(s) = \frac{2E}{s} * th \frac{Ts}{4} * \frac{\omega_n^2}{s_1 - s_2} * \left[\frac{1}{s - s_1} - \frac{1}{s - s_2} \right] \quad (7)$$

The switching condition according to the residue theorem is given by the closed loop condition:

$$Y(t_i) = 2E \oint \frac{1}{s} th \frac{Ts}{4} \left[\frac{\omega_n^2}{s_1 - s_2} \right] \left(\frac{1}{s - s_1} \right) e^{-st} ds + 2E \oint \frac{1}{s} th \frac{Ts}{4} \left[\frac{\omega_n^2}{s_1 - s_2} \right] \left(\frac{1}{s - s_2} \right) e^{-st} ds = h \quad (8)$$

It can be written as follows

$$\frac{1}{s_1} th \frac{Ts_1}{4} - \frac{1}{s_2} th \frac{Ts_2}{4} = \frac{h}{2E\omega_n^2} (s_1 - s_2) \quad (9)$$

This expression is the core of our analysis. We notice that the period of auto oscillation T depends on the parameters of the nonlinear element (E, h) as well as from the parameters of the second order filter (ω_n, ξ).

3-2 -Analyze of the auto-oscillation period w.r.t damping factor

a- First case:

In the first case where the two roots are real, the expression (9) becomes

$$\frac{th \frac{T}{4} (-\xi \omega_n + \omega_n \sqrt{\xi^2 - 1})}{-\xi \omega_n + \omega_n \sqrt{\xi^2 - 1}} - \frac{th \frac{T}{4} (-\xi \omega_n - \omega_n \sqrt{\xi^2 - 1})}{-\xi \omega_n - \omega_n \sqrt{\xi^2 - 1}} = \frac{h}{E\omega_n} \sqrt{\xi^2 - 1} \quad (10)$$

We notice that the period of auto oscillation depends from the parameters of the nonlinear element (E, h) as well as from the parameters of the second order filter (ω_n, ξ).

b- Second case:

In the second case $\xi \ll 1$, we imaginary conjugate roots, the expression (9) becomes:

$$\frac{-\xi \sin\left(\frac{T}{2} \omega_n \sqrt{\xi^2 - 1}\right) + \sqrt{\xi^2 - 1} sh\left(\frac{T}{2} \xi \omega_n\right)}{ch\left(\frac{T}{2} \xi \omega_n\right) + \cos\left(\frac{T}{2} \omega_n \sqrt{\xi^2 - 1}\right)} = \frac{h}{2E} (1 + 2\xi^2) \sqrt{\xi^2 - 1} \quad (11)$$

And the presence of oscillatory functions that complicates the solution of this equation

c- Third case:

This is a border line case which corresponds to a double root, the expression (9) becomes

$$\frac{-\omega_n T + 4sh \frac{\omega_n T}{2}}{2\omega_n \left(1 + ch \frac{T\omega_n}{2}\right)} = \frac{h}{E\omega_n^2} \quad (12)$$

4. Simulation Results

4-1 Simulation Scheme

Beside that we have found out the analytical expressions that links the auto-oscillation period w.r.t the damping factor, it remains difficult to found out the roots of such equations (10, 11, and 12). As a consequence, we proceed to determine these roots by simulation and to draw the relationship. We illustrate some cases by simulation with Simulink (Figure 4).

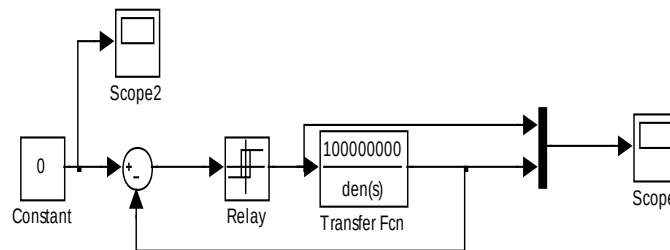


Figure (4): scheme of the 2nd order system with Simulink

The simulations cases have been performed with the following values; $f_0 = 0V$ $\omega_n = 10000 \text{ rd/s}$, $h = 0.1$ $E = 12$.

Somme examples of simulation for different values of ξ are presented below (Fig5). The period of self-oscillation is measured from the graphical representation of the output as provided by Simulink :

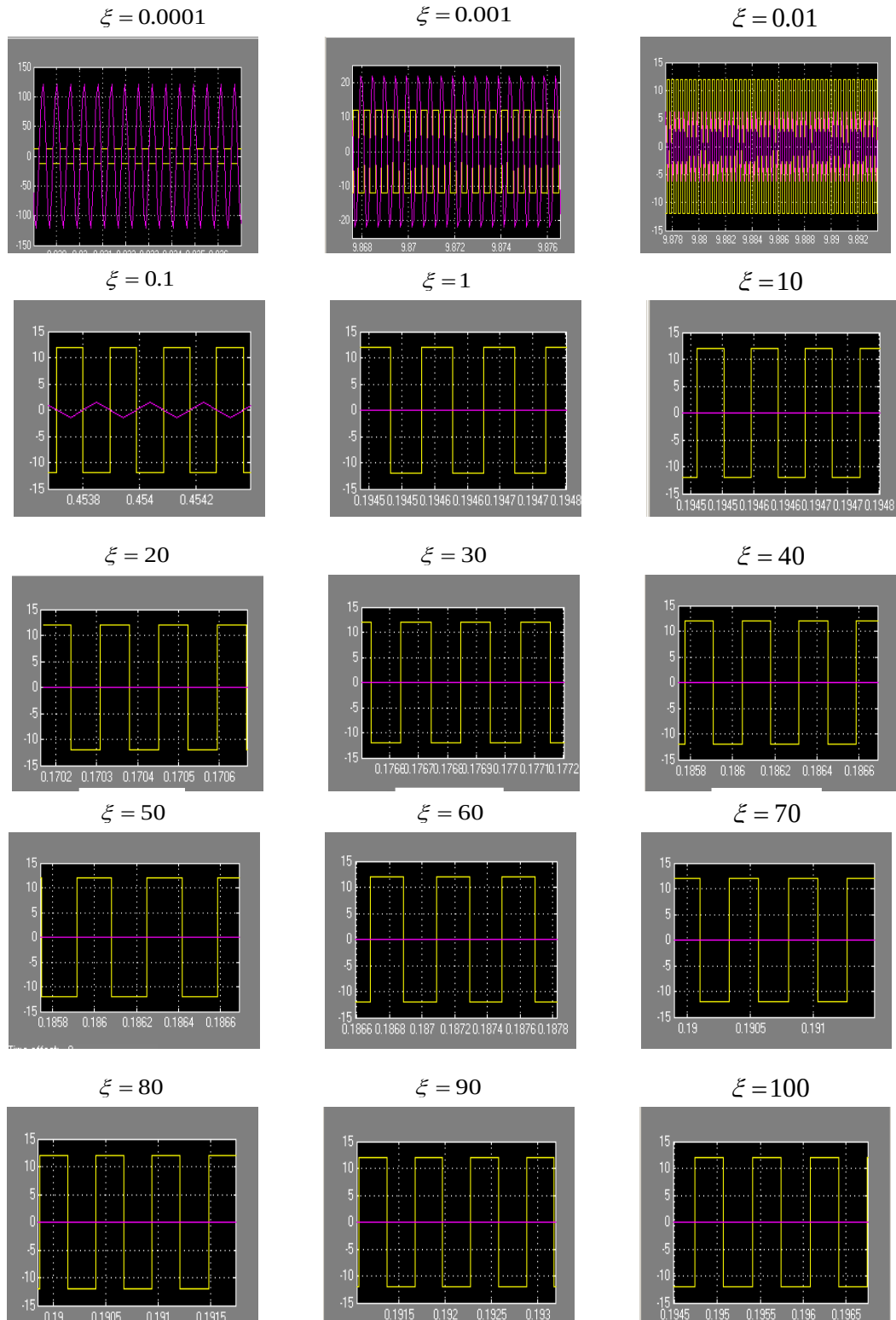
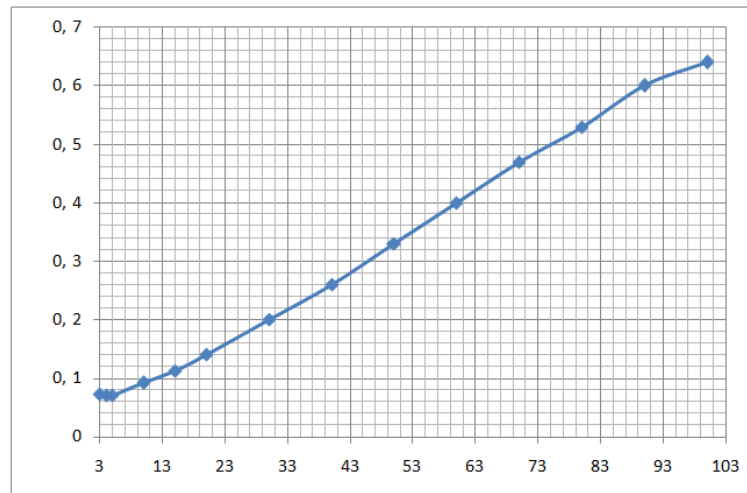


Figure.5. Input and output Signals for different values of ξ

4-2 The variation of T with respect to ξ

The variation of pulse durations (T) with respect to ξ have obtained through the simulation and the results are presented on **Figure (6)**.

$T \cdot 10^{-4}$ (s)



ξ

This curve shows that the variation of $T = f(\xi)$ increases linearly. We notice a minimal value of T ($T = 10 \mu s$) that corresponds to ξ approaching unity. For $\xi < 1$ the system becomes unexplainable. This relationship is the basic for dimensioning and optimizing the PWM choppers under consideration.

5. Conclusion

We have presented detailed study fan *on-off* feedback system via a second order filter. Such a system is capable of generating a *PWM* of varying duration and duty cycle and is used for the design and realization of choppers. This system is designed to enable the compensation of both variations stemming from the irradiation and of the load.

This theoretical study essentially aims to determine the relationship between the switching frequency according to system settings and related technological parameters. However, the determination of pulse duration is a fundamental problem in the analysis of *on-off* non linear systems. In the general case, obtaining an analytical solution is extremely difficult, even impossible. And only graphical techniques or simulation can help to analyze this type of problems. Nevertheless, an analytical solution of $T = f(\xi)$ is given for $f_0 = 0$ involving the parameters of the on-off system and of the second order filter.

Given the complexity of the expressions obtained for self oscillation ($f_0 = 0$), we have used graphical representation of simulation to determine graphically the relationship $T = f(\xi)$. Moreover; experiments to validate this analysis are under course.

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