

MATHEMATICAL MODELING AND HIGH PERFORMANCE PREDICTION FOR HYBRID STEPPING MOTOR

A. BENMOUNAH*

M. AYACHE*

P. K. SINHA**

Abstract

A mathematical model is developed for a hybrid stepping motor and then a numerical simulation is provided in view of studying the behaviour of the motor chiefly in dynamic start regime and in steady-state. The differential equations of the model are solved using an advanced continuous simulation language called ACSL. The motor is first tested by numerical simulation in dynamic start regime with constant frequency of input pulses and with different laws for varying this frequency determining the optimal law for motor acceleration, and second by improving the average torque of the motor in steady-state. The method presented here for this aim is generally valid for a wide range of hybrid stepping motors.

Key Words : Simulation, stepping motor, frequency control, and computer programming.

1. Introduction

In recent years there has been growing interest in the application of stepping motors in control systems. This motor has many apparent advantages in positioning control applications. For instance, the digital nature of its driving sequence permits it to interface with many types of digital transducers, networks, and computers with a minimum of interface hardware. However, there has not been a great deal of publication on the subject of stepping motors, especially in the area of mathematical modelling, simulation and optimal control of the motor. O'Donahue and Kiebert [1] developed some linear transfer functions for the stepping motor. These linearized representations are inadequate when the motor is used for precision positioning. Chang [2] recently published some interesting results on the theoretical aspects of type of variable reluctance stepping motor.

Stepping motor is a rotary electromechanical energy converter. It delivers an output in the form of discrete angular displacements when excited with prescribed electrical signals. In common terminology it is a rotating machine which rotates in steps of fixed angles. The input to

such a machine is in the form of voltage steps. A part from the well known stepping motors, variable-reluctance and permanent-magnet, another type of stepping motor having a permanent magnet in its rotor is the hybrid motor. The term 'hybrid' derives from the fact that motor is operated under the combined principles of the permanent-magnet and the variable-reluctance motors. Generally, hybrid motors are used for high resolution high-torque applications, and reluctance motors for high-speed low-inertia application, and permanent-magnet motor for large-step low-torque applications [2].

2. Description of the motor

The specific motor used here for illustration (table 1) is a hybrid stepping motor see [3] reference 34 (344-621), it has 4-phase that are capable of delivering much higher working torque and stepping rates than other types of stepping motor, whilst maintaining a high detent work even when not energised (Fig 1).

3. Development of the mathematical model

The model developed here is for a two phase bifilar wound stepping motor, by supposing the motor as a machine with four independent phases [4]. The mathematical model of an open loop stepping motor consists of a system of differential equations which describe the equations of electrical equilibrium of the motor phases and the torque equations[2].

Although the physical configuration of the motor contains four independent phases (Fig 1), for mathematical convenience, a bifilar winding is considered here. Phases a and c are the two components of one bifilar winding and phases b and d are two components of the other winding. Because of this arrangement of the four windings, full coupling exists between phases a and c and between b and d. In addition, the phase currents i_a and i_c oppose each other magnetically, as do phase currents i_b and i_d . Since the pair of windings a and c is in quadrature with the pair b and d, there is no common flux between these pairs of windings.

These lead to the following flux linkage conditions :

$$\begin{aligned}\lambda_{ab} &= \lambda_{ba} = 0 \\ \lambda_{ad} &= \lambda_{da} = 0 \\ \lambda_{bc} &= \lambda_{cb} = 0\end{aligned}\quad (1)$$

$$\begin{aligned}\lambda_{cd} &= \lambda_{dc} = 0 \\ \lambda_{ac} &= -\lambda_{cc} \\ \lambda_{ca} &= -\lambda_{aa} \\ \lambda_{bd} &= -\lambda_{dd} \\ \lambda_{db} &= -\lambda_{bb}\end{aligned}\quad (2)$$

With reference to Fig 1, the voltage equations for the four phases may be expressed as

$$V_k = r_k * i_k + \frac{d\lambda_k}{dt} \quad (3)$$

Where : V_k - voltage applied to phase K

i_k - current in phase k

r_k resistance of phase k

λ_k - flux linkages of phase k

k - winding phase = a, b, c, d

The flux linkages of phase k, have components from all phase currents and the permanent of the rotor. Thus, λ_k may be expressed as

$$\lambda_k = \lambda_{ka} + \lambda_{kb} + \lambda_{kc} + \lambda_{kd} + \lambda_{kf} \quad (4)$$

where

λ_{kj} = flux linking phase k due to current flowing in phase j, j = a, b, c, d

λ_{kf} = flux linking phase k due to the permanent magnet.

Using equation (4), the flux linkages of each phase can be detailed as follows :

$$\begin{aligned}\lambda_a &= \lambda_{aa} + \lambda_{ab} + \lambda_{ac} + \lambda_{ad} + \lambda_{af} \\ \lambda_b &= \lambda_{ba} + \lambda_{bb} + \lambda_{bc} + \lambda_{bd} + \lambda_{bf} \\ \lambda_c &= \lambda_{ca} + \lambda_{cb} + \lambda_{cc} + \lambda_{cd} + \lambda_{cf} \\ \lambda_d &= \lambda_{da} + \lambda_{db} + \lambda_{dc} + \lambda_{dd} + \lambda_{df}\end{aligned}\quad (5)$$

Substituting equations (1) into equation (5), the flux linkages are simplified to the following form :

$$\begin{aligned}\lambda_a &= \lambda_{aa} + \lambda_{ac} + \lambda_{af} \\ \lambda_b &= \lambda_{bb} + \lambda_{bd} + \lambda_{bf} \\ \lambda_c &= \lambda_{ca} + \lambda_{cc} + \lambda_{cf} \\ \lambda_d &= \lambda_{db} + \lambda_{dd} + \lambda_{df}\end{aligned}\quad (6)$$

Substituting equation (2) into equation (6), the flux linkages are simplified to the following from :

$$\begin{aligned}\lambda_a &= \lambda_{aa} - \lambda_{cc} + \lambda_{af} \\ \lambda_b &= \lambda_{bb} - \lambda_{dd} + \lambda_{bf} \\ \lambda_c &= \lambda_{cc} - \lambda_{aa} + \lambda_{cf} \\ \lambda_d &= \lambda_{dd} - \lambda_{bb} + \lambda_{df}\end{aligned}\quad (7)$$

Since the objective here is to derive a mathematical model for dynamic motion, there is a need to relate the shaft angle θ and its derivative to the input voltage through generated shaft torques. The first stage of this derivation plus needs to relate input voltage to phase currents. These phase currents are then to be derive the torque in terms of the angular position. Using the general concept of self and mutual inductances, the total flux linkages are expressed as.

$$\lambda_a = l_{ka} * i_a + l_{kb} * i_b + l_{kc} * i_c + l_{kd} * i_d + \lambda_{kf} \quad (8)$$

$k = a, b, c, d$

l_{kj} = average self inductance of phase k when $k = j$

= average mutual inductance between phases k and j when $k \neq j$ (≈ 0)

Therefore for a bifilar winding hydrid stepping motor, the flux linkages of phase may be writtem as

$$\begin{aligned} \lambda_a &= l_{aa} * i_a - l_{cc} * i_c + \lambda_{af} \\ \lambda_b &= l_{bb} * i_b - l_{dd} * i_d + \lambda_{bf} \\ \lambda_c &= l_{cc} * i_a - l_{aa} * i_c + \lambda_{cf} \\ \lambda_d &= l_{dd} * i_b - l_{bb} * i_d + \lambda_{df} \end{aligned} \quad (9)$$

The inductances of this stepping motor (phases) are not depending on the rotor position θ . However, when the magnetic circuits are saturated, the inductance will depend on the phase currents. A number of further assumptions are made here for mathematical convenience: since the four windings of the motor are likely to be manufactured in volume quantity, it may reasonably be assured that they have the same number of turns, same amount of resistance and same inductance (equal to a constant value).

From the last assumption with the inductance

$$l_{aa} = l_{bb} = l_{cc} = l_{dd} = l$$

and the resistance

$$r_a = r_b = r_c = r_d = r$$

equation (9) becomes equal to

$$\begin{aligned} \lambda_a &= l(i_a - i_c) + \lambda_{af} \\ \lambda_b &= l(i_b - i_d) + \lambda_{bf} \\ \lambda_c &= l(i_a - i_c) + \lambda_{cf} \\ \lambda_d &= l(i_b - i_d) + \lambda_{df} \end{aligned} \quad (10)$$

From equation (3) the phase currents can be expressed as

$$i_k = (v_k - \frac{d\lambda_k}{dt}) \frac{1}{r_k} \quad (11)$$

Equations (10) and (11) need now to be combined to obtain the input output relationship between the phase voltages v_a, v_b, v_c , and their current i_a, i_b, i_c and i_d . Each phase current is expressed as

$$\begin{aligned} i_a &= (v_a - l_1(di_a - di_c) + d\lambda_{af}) \frac{1}{r} \\ i_b &= (v_b - l_1(di_b - di_d) + d\lambda_{bf}) \frac{1}{r} \\ i_c &= (v_c - l_1(di_a - di_c) + d\lambda_{cf}) \frac{1}{r} \\ i_d &= (v_d - l_1(di_b - di_d) + d\lambda_{df}) \frac{1}{r} \end{aligned} \quad (12)$$

The total torque developed by the unsaturated motor may be written as

$$t_{tot} = t_a + t_b + t_c + t_d \quad (13)$$

and each phase torque is expressed as

$$\begin{aligned} t_a &= -k_2 * i_a * \sin(n_r \theta) \\ t_b &= -k_2 * i_b * \sin(n_r \theta - \frac{\pi}{2}) \\ t_c &= -k_2 * i_c * \sin(n_r \theta - \pi) \\ t_d &= -k_2 * i_d * \sin(n_r \theta - \frac{3\pi}{2}) \end{aligned} \quad (14)$$

At low-speed, the e . m . f effect is neglected and the current phases build up and decay instantaneously, because the excitation of one phase is greater than the winding time constant $\xi = l_1 / r$. The optimal instant of switching is the intersection of torques curves.

At high-speed, static torque curves are not any more sinusoidal [4], because the current in the phase donot build up or decay instantaneously and thus, the torque build up time is finite . In general, the average torque depends on the electromagnetic time constant and the e . m . f . Another control technique commonly referred to as " Ignition advance " is used for improving the average torque of the stepping motor used for higher speed [4,6] .

For better understanding the above technique, it is necessary to introduce the concept of lead-angle β and the switching angle α . β indicates an advance in comparison with the instant of detent α which is defined by $\alpha = 2\pi r_e - \beta$, $r_e = \pi/2$ the motor resolution, can allow generation of a command signal when the rotor in position before detent [4,6]

The variation of switching angle or the lead angle has direct effect on the motor acceleration .

A good choice of lead angle versus the frequency of control allow us maximising the average torque. This technique justified by the fact that current phase must get enough time to build up until its nominal value and this before the end of excitation interval .

As the average torque is equal to the average value of the total electromagnetic torque developed by the motor.

$$T_m = \frac{1}{2\pi} \int_0^{2\pi} t_{tot} d\theta \quad (15)$$

This can simplified when average developed torques by the phases are supposed to be identical, so

$$T_m = -\frac{2k_2}{\pi} \int_0^{\frac{\pi}{2}} i_a(\theta) \sin(n_r \theta) d\theta \quad (16)$$

In order to optimise the average torque, the lead angle β is used in the last equation as follows

$$T_m = -\frac{2k_2}{\pi} \int_{\beta}^{\beta + \frac{\pi}{2}} i_a(\theta) \sin(n_r \theta) d\theta \quad (17)$$

After transformation and integration we can get the following average torque: [8, 9] :

$$T_m = -\frac{2k_2}{l_1^2 \omega^1 + r} [V_{max}(r \cos(\beta) - l_1 \omega \sin(\beta)) + k_1 r \omega] \quad (18)$$

where k_2 [Nm/A] motor constant

k_1 [V/(rad/s)] motor constant

l_1 [H] phase inductance

r [Ω] total phase resistor

$V_{max} = 4v / \pi$: max value of the voltage supply

β [rad]: Lead angle

ω [rad/s] angular velocity

It is clearly see that T_m depends on the lead angle β and the angular velocity ω . T_m is at its maximal value when commutation is done at an optimal lead angle .

In order to evaluate this optimal angle, the maximum of $T_m = f(\beta)$ must be found . This means calculate $dT_m/d\beta$ and put it equal to zero, and at the end we have .

$$\beta = -\pi - \arctan \frac{l_1 \omega}{r} \quad (19)$$

Where

$$-\frac{3\pi}{2} < \beta < -\pi \quad (20)$$

The variation of the optimal T_m versus frequency of the hybrid stepping motor will be then the envelope of all torque versus velocity curves .

The rotor dynamics are given by :

$$\omega = \frac{d\theta}{dt} \quad (21)$$

$$\frac{d\omega}{dt} = (t_{tot} - bb * \omega - t_{td}) \frac{1}{j} \quad (22)$$

t_{tot} = developed torque

t_{td} = frictional load torque

bb = viscous friction coefficient

j = total rotor and load inertia

$$\theta = \int \omega dt \quad (23)$$

In summary, modelling any kind of stepping motors requires four relations [2] These are :

- (1) - the voltage equations of the electrical circuits
- (2) - the flux linkage relations
- (3) - the expressions for developed torque
- (4) - the dynamic equation of the rotor

The induced e.m.f due to the permanent-magnet in the phase as supplied by the Astrosyn International Technology PLC may be expressed as

$$e_k = \phi n_r \omega_s n_s \quad (24)$$

with

$$\phi = \frac{h_i}{(n_s n_r i)} \quad (25)$$

ϕ - magnet flux ; in Wb

h_i - holding torque at the current phase i with one phase on

From the RS-data sheet the following is taken

n_r - rotor teeth number = 50

n_s winding turns = 176

ω shaft speed ; in RAD/SEC

The flux linking phase k due to the permanent magnet may be written as

$$\begin{aligned} \frac{d\lambda_{af}}{dt} &= -e \sin(n_r \theta) \\ \frac{d\lambda_{bf}}{dt} &= -e \sin(n_r \theta - \frac{\pi}{2}) \\ \frac{d\lambda_{cf}}{dt} &= -e \sin(n_r \theta - \pi) \\ \frac{d\lambda_{df}}{dt} &= -e \sin(n_r \theta - \frac{3\pi}{2}) \end{aligned} \quad (26)$$

4. Simulation Language

Regardless of the stepping motor type, its dynamic behaviour is most of the time non-linear. Because of analytical complexities, the most convenient method of obtaining dynamic solutions to the equation of motion is to develop a computer simulation. The main advantage of a

digital computer simulation is that, in addition to studying the existing system parameters, the worst-case parameters can also be investigated without additional modelling effort. The dynamic of stepping motors is usually described by differential equations, in most of the cases **FORTRAN** or a simulation language like the **ADVANCED CONTINUOUS SIMULATION LANGUAGE** (known as **A.C.S.L**) can be used. **A.B.S.L** is particularly suitable for dependent, non-linear differential equations and/or transfer functions [5]. The user can write **A.C.S.L** statements based on an equation that describes the system under investigation.

A.C.S.L processor is able to sort equations in such way that no values are used before they have been calculated, this is valid only on the derivative section statements describing the model, the equations need not to be ordered.

5. Simulation Results

The model derived above, is simulated using the flow-chart of Fig 2. This was implemented by using the advanced continuous simulation language **ACSL** installed on a 386/87 PC.

The simulation language results of the angular position, angular velocity and torque with constant frequency input pulses are plotted on the first set of Fig 3. Because of the low frequency of the pulse train created for this simulation (first set 20Hz), it is very clearly seen from these plots that the rotor stops at the end of each step operation is particularly suited to applications where large amounts of load inertia is usually combined with a viscous damper in order to obtain the required transient response. If the frequency of the pulse train created at the input controller is increased, see the second, third and fourth set of plots on Fig 4, 5 and 6 corresponding respectively to the frequencies of 40Hz, 80Hz, and 160Hz, the rotor speed will increase, and beyond certain frequency the rotor will move from single-step to a continuous mode. In this mode, the rotor does not come to a stop at the end of a step, and thus its behaviour is similar to a conventional rotary machine.

However, the variation of the angular position, angular velocity and torque of Fig 7 at $F = 100\text{Hz}$ is done for $t_{\max} = 0.1\text{s}$ and Fig 8 for $t_{\max} = 0.2\text{s}$. Certain lows (Fig 9 linear frequency, Fig 10 hyperbolic frequency profile, Fig 11 exponential frequency profile) were tested for motor start with variable frequency input pulses.

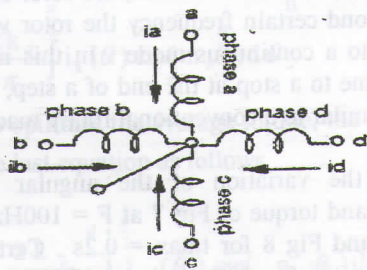
The variation of frequency (frequency profile), of its slope and also of the angle and speed are plotted for each case, as indicated in Fig 9, 10, 11 ; Examining these last plots, the following conclusions can be drawn : hyperbolic frequency profile provides the minimal starting time without losing steps, and the slope of frequency profile must be high at the beginning of the start and then it must decrease. In Fig 12 are the plots of the torques versus velocity for three values of β .

Références

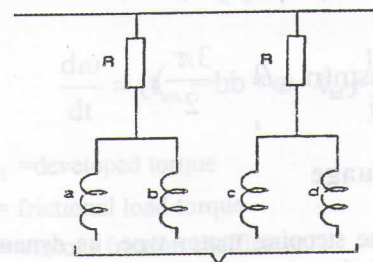
- [1] B . C Kuo, G . Singh, and R . Yackel, "Modelling and simulation of a stepping motor", IEEE Transaction on Automatic Control, Dec69 .
- [2] - A. Benmounah, "ransputer control of an AGV : design, construction, and testing of a mobile platform", Aug 1991.
- [3] RS data (7017, "Stepping motors", issued Nov 1986.
- [4] B. C. Kuo, "Incremental motion control step motors and control systems", University of Illinois at Urbana-Champaign, 1979.
- [5] Mitchell and Gauthier associates, "A. C. S. L référence manual", fourth edition, 1986 .
- [6] - P. P. Acarnley, " Stepping motors, a guide to modern theory and practice ", P. Peregrinus TLD on behalf IEE, 1984 . controls ", Clarendon Press . Oxford 1984 .
- [7] T. Kenjo, "Stepping motors and their microprocessor controls", Clarendon Press . Oxford 1984 .
- [8] M. Kant, "Les actionneurs électriques pas à pas", Edition Hermes, Paris, 1989 .
- [9] - R. H. Brown and M. Jaroudi, "Torque prediction and maximisation strategy for bifilar wound hybrid step motor", IEEE Transactions on Power Electronics, Vol 7, N° 3, July 1992 .

STEP ANGLE	1.8	DEGREE
NOMINAL PHASE PARAMETER & RATING VOL	3.0	V dc
RESISTANCE AT 20°C	1.8	OHM
CURRENT	1.7	A
INDUCTANCE	5	mH
HOLDING TORQUE	1100	mNm
DETENT TORQUE	100	mNm
WORKING TORQUE	650	mNm
ROTOR INERTIA	640	g cm ²
PULL-IN RATE (MAX)	800	step/s
AMBIENT TEMPERATURE	- 40 to + 65	°C
TEMPERATURE RISE (max above ambient)	-	°C
INSULATION CLASS	B	-
WEIGHT	1350	g
PHASE NUMBER	4	-

Table 1 Motor technical parameters



a) axial view of a two phase-bifilar wound hybrid stepping motor V motor (12 V)



to the respective pins of the driver
b) 1 Schematic of the hybrid stepping motor winding

INITIAL

PROGRAM

- statements performed before the run begins .
- state variable do not contain the initial conditions yet .

END

DYNAMIC

DERIVATIVE

- statements needed to calculate derivatives of each INTEG statement ;

END

statement executed every communication interval

END

TERMINAL

- statement executed when the termination condition TEMP become true .

END

END

Table 2 Program structure

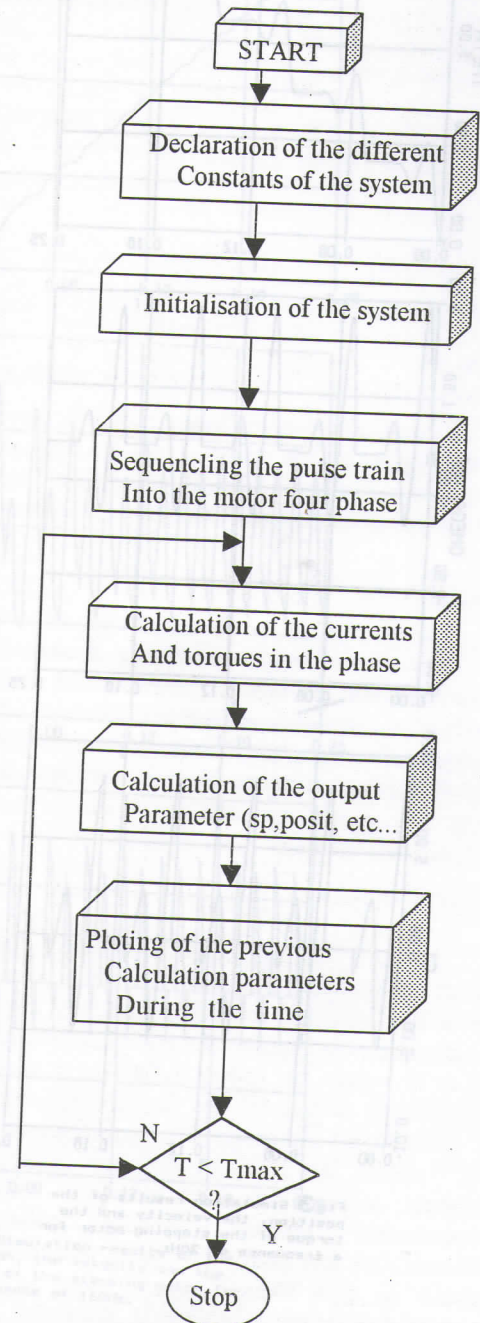


Fig 2 System simulation flow-chart

* Electrical Engineering Universite of Annaba - Algéria
** University of Reading, England.

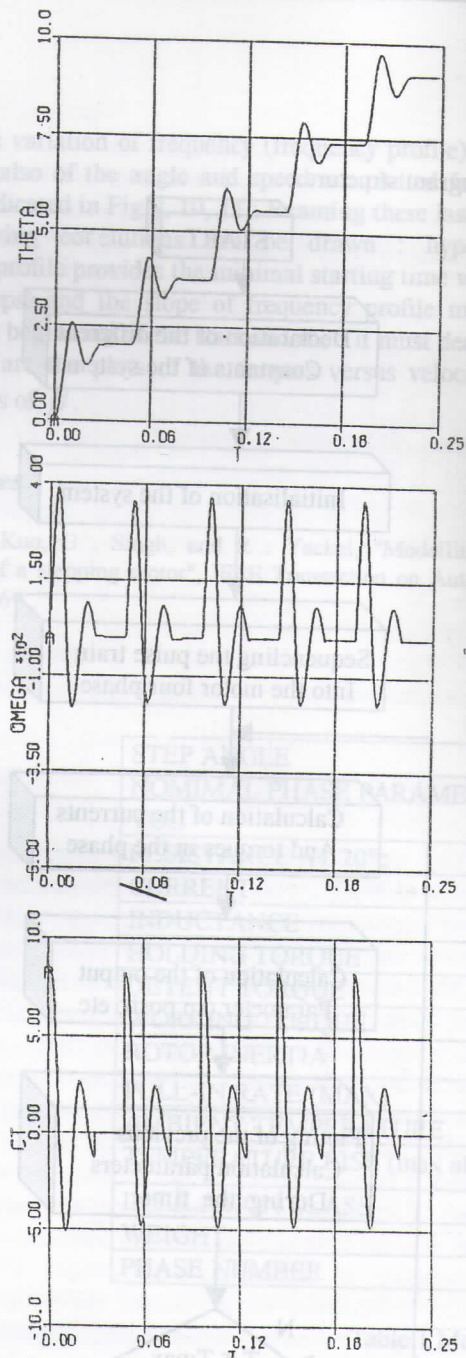


Fig 3 Simulation results of the position, the velocity and the torque of the stepping motor for a frequency of 20Hz.

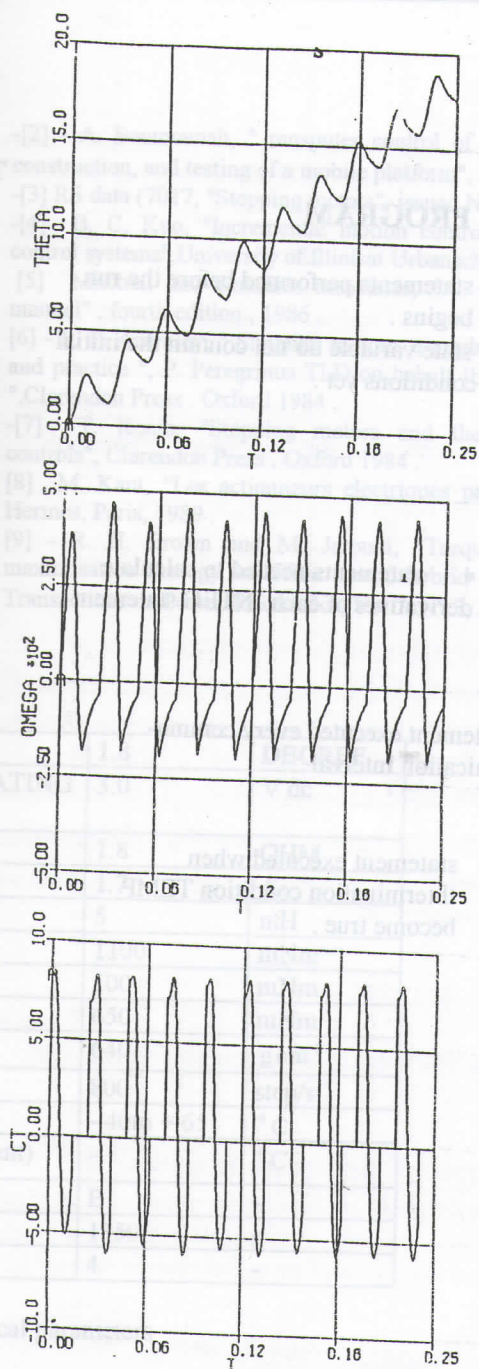


Fig 4 Simulation results of the position, the velocity and the torque of the stepping motor for a frequency of 40Hz.

II. Paramètres principaux du système

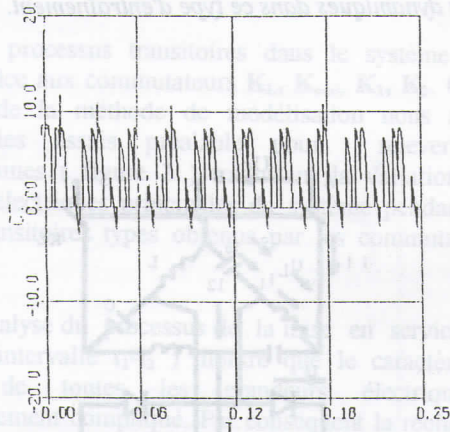
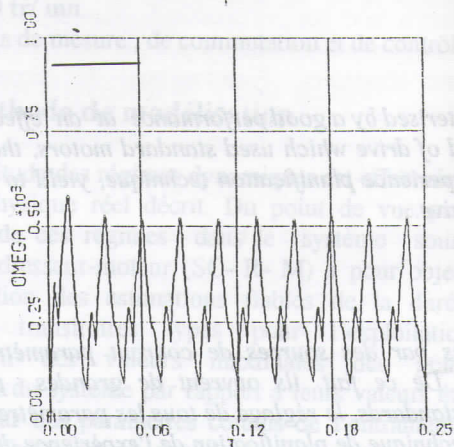
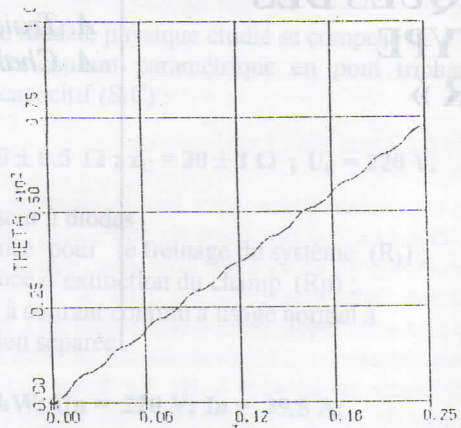


Fig 5 Simulation results of the position, the velocity and the torque of the stepping motor for a frequency of 80Hz.

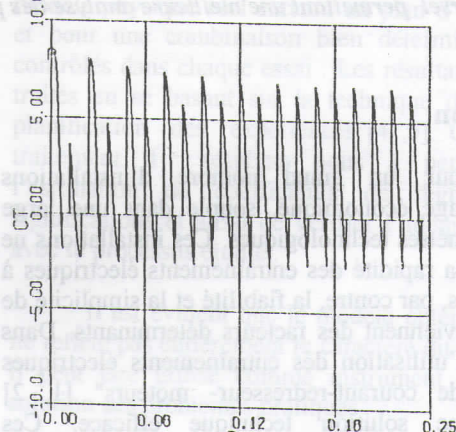
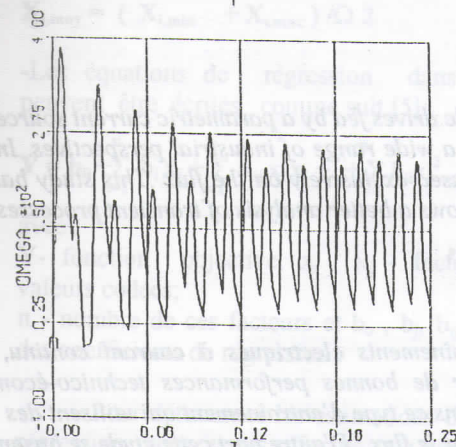
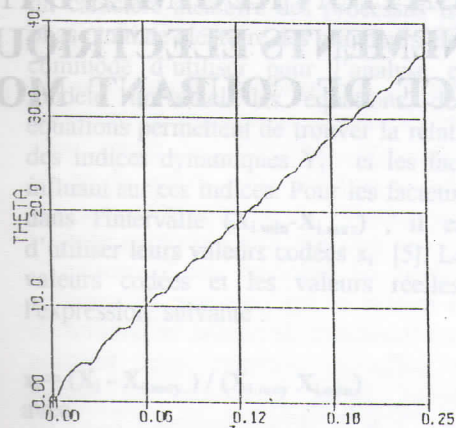


Fig 6 Simulation results of the position, the velocity and the torque of the stepping motor for a frequency of 160Hz.