A study of the gradient descent method for shale gas reservoirs

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ABSTRACT

In shale gas reservoir development, determination of hydraulic fracture geometry for horizontal wells is still a demanding challenging task. One of the approaches for hydraulic fracture optimization is based on reservoir simulation, and to improve efficiency and accuracy, an automatic and robust procedure integrating the gradient descent method with gas reservoir simulation has been developed by many authors, where some proposed an automatic and robust optimization procedure based on the gradient descent method, that aims to enhance both the efficiency and accuracy. However, we have to make sure which gradient descent method to use for optimizing hydraulic shale gas reservoirs fractures, as this requires several factors and variations to be taken into account. The wise choice is to consider the nature of the problem (convex or non-convex), the size of the dataset, and the specific requirements of the reservoir simulation. Advanced variants and multiple initial points can enhance the effectiveness of the optimization process. This method can achieve significant computational reduction and high prediction accuracy for various shale gas reservoir cases. Here we analyse numerically this method for different cases and present some numerical results.

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1. INTRODUCTION

The gradient descent method (GDM) is integrated with a gas reservoir simulator to optimize hydraulic fracture parameters such as spacing, half-length, and conductivity. This method is a fundamental optimization technique that iteratively adjusts parameters based on gradient information, making it a valuable tool in various applications, including hydraulic fracturing optimization in shale gas reservoirs. Its related derived algorithm is used to minimize an objective function. It works by updating parameters in the direction of the negative gradient of the function at the current point, effectively seeking the lowest point in the function's landscape.

2. RELATED WORKS

It has been recommended by many authors, for example [1], [2] and [3] and references therein, to incorporate the Armijo rule in the GDM as part of the optimization process. This rule helps in determining the appropriate step size for each iteration, ensuring that the algorithm makes sufficient progress towards the optimal solution while maintaining stability. in the updates.

While not a direct variant of the GDM, the paper suggests running the algorithm with multiple initial points to address the limitations of GDM in nonconvex problems. This approach helps in exploring different regions of the solution space, increasing the likelihood of finding better local optima and reducing errors in the optimization results. By exploring different starting points, the algorithm can potentially find better local optima, thus improving the overall optimization results. This approach is particularly useful in complex landscapes where the risk of getting trapped in local minima is high.

The standard GDM is a first-order iterative optimization algorithm that updates parameters by moving in the direction opposite to the gradient of the objective function. It is straightforward and effective for convex problems, but it can struggle with nonconvex functions due to ifs reliance on the initial point, which may lead to different local optima

The Armijo rule is used in conjunction with GDM to adaptively determine the step size for each iteration. This approach ensures that the algorithm makes sufficient progress towards the minimum while maintaining stability. By adjusting the step size based on the function's behavior, the Armijo rule can enhance convergence rates compared to a fixed step size used in basic GDM. In the context of this work, we combine these two techniques to show some associated efficient results

3 PROPOSED PROCEDURE

In the paper by Chen and al. [3] an automatic fracture optimization procedure is developed that integrates the gradient descent method with reservoir simulation for shale gas. This method can be compared with various gradient methods that

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utilize different step strategies, including fixed, variable, random, and optimal step sizes.

3.1 Gradient descent method

Gradient descent is a first order iterative optimization algorithm for finding the minimum of an unconstrained problem. It descends by taking steps in the opposite direction of the function gradient at a given point. The Taylor series of a continuously differentiable function is:

$$f(x_k + \alpha_k d_k) = f(x_k) + \alpha_k g_k^T d_k + \sigma(\alpha_k), \alpha_k > 0 \quad (3.1)$$

With

$$g_k^T = \| g_k \| \| d_k \| \cos\overline{\theta} \quad (3.2)$$

where x_k is an independent optimization parameter at k^{th} point d_k is the search direction at k^{th} point, step size α_k is the amount of advance of x_k in each iteration, g_k^T is gradient of f at x_k, higher-order terms of α_k is $\sigma(\alpha_k)$ which can be ignored, $\overline{\theta}$ is the angle between g_k and d_k. Thus, if we want to obtain the minimum value of f, $g_k^T Td_k$ should be minimized. That is, $\cos\overline{\theta} = -1$, $\overline{\theta} = \pi$, so d_k should be opposite to g_k^T. It can be proved that under the condition that $f(x)$ is differentiable and g_k^T is negative. Ensuring the inequality by choosing the proper step is the guarantee of searching in the right direction for the optimal objective from previous iteration.

GDM is powerful for solving convex problems; however, it has limitations for nonconvex problem since it relies on the initial point, i.e. different initial points might lead to different local optimal values.

$$f(xk + \alpha_k d_k) \leq f(x_k) + \alpha_k g_k^T d_k$$

Gradient Descent with Armijo Rule

The method studied in the article employs the Armijo rule to determine the step size dynamically during the optimization process. This ensures that each step taken results in a sufficient decrease in the objective function, enhancing convergence efficiency.

Fixed Step Size Gradient Descent

This method uses a constant step size throughout the optimization process.

Variable Step Size Gradient Descent

In this case, the step size changes during the optimization process, but without a structured approach like the Armijo rule.

Random size Gradient Descent

This variant selects the step size randomly at each iteration.

Optimal Step Size Gradient Descent

The procedure aims to determine the best possible step size for each iteration, often through sophisticated techniques. To mitigate the limitations of basic gradient descent in non-convex problems, it is recommended to run the algorithm with multiple initial points. This approach increases the likelihood of finding a better local minimum and reduces the risk of being trapped in suboptimal solutions.

3.2 Model

The function $f = 3x^2 + 4y^2$ is an elliptic paraboloid with a minimum value of 0 at (0, 0) in 3D space. It was coded using the gradient descent optimization procedure with different initial values.

The gradient descent method is the focus of this study, we suggest that the optimization procedure could be adapted to incorporate other gradient-based methods for further exploration and comparison.

There are several variants of gradient descent that can be considered here: Fixed Step Size Gradient Descent, Variable Step Size Gradient Descent, Variable Step Size Gradient Descent and Optimal Step Size Gradient Descent.

3.3 Results and discussion

In this section we present and comment on the obtained results of optimization using different gradient descent methods cited in the previous section.

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Table 3.1: Fixed Step Size Gradient Descent

initial point	Number of iteration	objectif function value
(-1, 1)	9	1.2885164031999967e-06
(-10, 10)	12	5.27765749104638e-07
(-20.80, 30.42)	13	3.6533206387325256e-07
(-150, 300)	15	4.86388769419689e-07
(1000, 3000)	17	5.534023237574726e-07

- For all starting points, the final value of the objective function is very close to zero (of order $10-7$). This indicates that the algorithm has indeed converged to the global minimum of the function, which is (0, 0) for this particular quadratic function.
  - The number of iterations varies depending on the starting point. Initial points further from the origin generally require more iterations to converge.
  - The final values of the objective function are very close to zero.
  - A step that is too small can slow down convergence, while a step that is too large can prevent the algorithm from converging or cause it to converge to a local minimum. a step of 0.1 seems suitable for all the used initial points.

Table 3.2: Variable Step Size Gradient Descent

initial point	Number of iteration	objectif function value
(-1, 1)	10	7.490129545384946e-07
(-10, 10)	13	7.667371906116573e-07
(-20.80, 30.42)	14	7.579296183911785e-07
(-150, 300)	16	2.2170109988106287e-06
(1000, 3000)	19	1.5764587959097451e-06

From these provided results, it can be observed that the gradient descent algorithm with linear decreasing step converges to an approximate minimum of the elliptic paraboloid function for all tested starting point.
  - The number of iterations necessary for convergence increases with the distance from the star- ting point to the real minimum.
  - The approximate minimum value of the objective function remains very low for all tested star- ting points, indicating that the algorithm finds good quality approximations of the true minimum

Table 3.3: Random size Gradient Descent

initial point	Number of iteration	objectif function value
(-1, 1)	19	1.8917132908196657e-06
(-10, 10)	20	1.4722794042591394e-05
(-20.80, 30.42)	23	2.545217316245968e-05
(-150, 300)	23	2.299381203198477 7e-06
(1000, 3000)	24	6.212519237802194e-06

The results indicate that the algorithm converges to the minimum of the objective function for some initial points. However, for initial points far from the minimum, the algorithm may take more iterations and encounter convergence difficulties. It is important to choose a judicious initial point for effective convergence.

Table 3.4: Optimal Step Size Gradient Descent

initial point	Number of iteration	objectif function value
(-1, 1)	511	1.6489405053802872e+307
(-10, 10)	508	2.5764695396567e+307
(-20.80, 30.42)	506	2.764539545945754e+307
(-150, 300)	501	1.4717701203252608e+307
(1000, 3000)	499	2.2153274900047977e+307

The objective function values are extremely large and the number of iterations is high (500 or more)

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Table 3.5: Gradient Descent with Armijo Rule

initial point	Number of iteration	objectif function value
(-1, 1)	17	3.0161364967786066e-12]
(-10, 10)	20	2.188603053161877e-12
(-20.80, 30.42)	21	3.9209456571502955e-12
(-150, 300)	5	2.1428724256451733e-12
(1000, 3000)	27	2.00792289176775e-12]

Sususbsection Concluding remarks from the results resumed in Tables 3.1-3.5, we can deduct that:
Gradient descent method is effective for convex optimization problems, where it consistently finds the global minimum. However, in non-convex scenarios, it may converge to local minima depending on the initial chosen point.

To improve the robustness of the algorithm, we recommend testing multiple initial points. This approach helps in exploring different local minima and can lead to better overall optimization results. The findings suggest that using various starting points can reduce errors and enhance the accuracy of the optimization process.
It can be beneficial in scenarios with large datasets or complex models, as it updates parameters based on a random subset of data, which can lead to faster convergence. However, the results may be noisier compared to batch methods, requiring careful tuning of learning rates.

The Random Gradient Descent can be beneficial in scenarios with large datasets or complex models. It updates parameters based on a random subset of data, which can lead to faster convergence. However, the results may be noisier compared to batch methods, requiring careful tuning of learning rates.

4 CONCLUSION

For the sake of comparison, we used the same example as in [1]. The choice of gradient descent method for hydraulic fracture optimization should consider very particular critiria such as, the nature of the problem from the point of view of the convexity, the size of the dataset, and the specific requirements of the reservoir simulation. Advanced variants and multiple initial points can enhance the effectiveness of the optimization process. The obtained results in Tables 3.1-3.5 can guide us to which GDM one can use.

REFERENCES

[1] Asadollahi, M., Naevdal, G. Waterflooding optimization using gradient based methods. Paper SPE 125331 Presented at SPE/EAGE Reservoir Characterization & Simulation Conference, Abu Dhabi, UAE, 19-21 October, 2009.

[2] Brake, A. C. Fracture optimization in a giant gas field. Paper SPE 164029 Presented at SPE Unconventional Gas Conference and Exhibition, Muscat, Oman, 28-30 January, 2013.

[3] Jiaheng Chen, Lei Wang, Cong Wang, Bowen Yao, Ye Tian, Yu Shu Wu., Automatic fracture optimization for shale gas reservoirs based on gradient descent method and reservoir simulation. Colorado School of Mines, Chengdu University of Technology. 25 June 2021. (Yandy Scientific Press). Vol. 5, Iss.: 2, pp 191-201.

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