

Analysis of the Dynamic Relationships Between Gold, Oil, and Macroeconomic Variables: A VECM Approach (2006–2025)

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Abstract

This article examines the short- and long-term relationships between the price of gold, the price of oil, the Consumer Price Index (CPI) and the real interest rate over the period 2006–2025. The monthly data are drawn from the FRED (Federal Reserve Economic Data) database. Using a vector error correction model (VECM), we identify a cointegration relationship between the four variables. However, the VECM residuals suffer from non-normality and heteroscedasticity, even after the introduction of dummy variables for the 2008 and 2020 crises. To address these issues, we use a residual bootstrap procedure (1,999 replications) to construct valid confidence intervals for the impulse response functions (IRFs). The results show an asymmetry between gold and oil: an oil shock significantly reduces the price of gold in the short term (negative effect for three months), whilst a shock to gold does not affect oil. No significant effect is detected on inflation or the real interest rate. These results are robust to the Cholesky order and to bootstrap bias correction. The paper highlights the importance of correcting for violations of standard assumptions in VECM models applied to commodities.

Keywords: VECM, gold, oil, inflation, real interest rate, FRED, non-normality, heteroscedasticity, bootstrap.

1. Introduction:

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Since the early 2000s, commodity markets have experienced unprecedented volatility, exacerbated by geopolitical shocks, financial crises, and currency turmoil. Gold, traditionally viewed as a safe-haven asset, and oil, a linchpin of the global economy, play a central role in the analysis of macroeconomic transmission mechanisms. Understanding how these assets interact with inflation and real interest rates is crucial for investors and policymakers.

The existing literature relies heavily on VAR and VECM models to study these relationships. However, most studies implicitly assume the normality and homoscedasticity of the residuals, assumptions that are rarely verified on long financial series. Our contribution is threefold: we cover a recent period (2006–2025) including the 2008 and COVID-19 crises; we explicitly address non-normality and persistent heteroscedasticity using dummy variables and a response function bootstrap; we demonstrate the consequences of these violations on inference and propose a validated correction strategy.

the research question :

To what extent are gold and oil prices linked to inflation and the real interest rate over the period 2006–2025, and do these relationships differ depending on the time horizon?

Sub-questions:

- ✚ Is there a cointegration relationship (long-run equilibrium) between gold, oil, the CPI, and the real interest rate?
- ✚ Does the price of oil influence the price of gold in the short term, and vice versa?
- ✚ Do shocks to gold and oil significantly affect inflation and the real interest rate?
- ✚ How can we address issues of non-normality and heteroscedasticity in the residuals within a VECM model applied to these series?

General objective:

The objective of this study is to analyze the short- and long-term dynamic relationships between gold, oil, inflation, and the real interest rate, by proposing a robust methodology that explicitly addresses violations of classical assumptions (non-normality, heteroscedasticity) through the

introduction of dummy variables and a bootstrap procedure for impulse response functions

2. Literature Review:

2.1 VAR and VECM Models:

Since the seminal work of Johansen (1988, 1991, 1995)¹, the use of vector error correction models has become standard practice in the analysis of macroeconomic and financial time series. The VECM effectively reconciles the short-term dynamics of the variables with their potential long-term equilibrium relationship (cointegration), while providing a structural representation of the adjustments toward this equilibrium via the α coefficients (correction speed). The Johansen procedure, thanks to its trace and maximum eigenvalue tests, serves as the standard for determining the rank of the cointegration matrix in a multivariate system²

In the field of commodities and macroeconomic indicators, the application of VECM has become particularly widespread, especially for analyzing causal relationships and volatility spillovers between the prices of gold, oil, interest rates, and inflation. The originality of our work lies in the simultaneous integration of four potentially cointegrated variables, coupled with a rigorous modeling of violations of standard residual assumptions.

The vector autoregressive (VAR) model introduced by Sims (1980) is a basic tool for analyzing dynamic interactions among multiple time series without imposing strong structural restrictions. It is written in the form $Y_t = A_1 Y_{t-1} + \dots + A_p Y_{t-p} + \Phi D_t + \varepsilon_t$ where Y_t is a vector of stationary variables. When the series are first-order integrated and cointegrated, the level VAR is ill-specified because it ignores the long-run equilibrium relationship. The vector error correction model (VECM), which derives

¹ PC B philips, S. Ouliaris, testing for cointegration using principal components methods, *journal of economic dynamics and control*, 12, p205-230, 1988

² RID Harris, « Analyse de cointégration à l'aide de la technique de Johansen : guide pratique du logiciel », *The Economic Journal*, volume 104, numéro 426, pages 1227-1237, 1994.

directly from the VAR under the assumption of cointegration, is therefore preferred. The VECM is written as:

$$\Delta Y_t = \alpha \beta' Y_{t-1} + \sum_{i=1}^{p-1} \Gamma_i \Delta Y_{t-i} + \phi D_t + \varepsilon_t$$

β contains the cointegration vectors (long-run relationships) and α the adjustment coefficients (speed of return to equilibrium). The number of cointegration relationships r is determined by Johansen's tests (trace and maximum eigenvalue). Thus, the VECM allows us to distinguish short-term dynamics (matrices Γ_i) from long-term structural equilibrium, offering a richer model than the first-differences VAR, which would eliminate all level information.

2.3. The Issue of Non-Normality and Heteroscedasticity of Residuals in VECM Models

The econometric literature has widely emphasized that violations of the assumptions of multivariate normality and conditional homoscedasticity are common in models applied to financial time series. As noted in a dedicated technical discussion, the bootstrap was developed precisely to avoid Monte Carlo simulations relying on random draws from a predefined normal distribution, which justifies its use when the residuals are non-normal¹. Studies have shown that non-normal disturbances and conditional heteroscedasticity can affect multiple cointegration tests, rendering the use of standard asymptotic approximations unreliable. In the same vein, more recent work indicates that autocorrelation tests remain relatively robust except in cases of extreme non-normality, but that heteroscedasticity tests are highly sensitive to excessive flattening of the residuals.

To address both non-normality and heteroscedasticity simultaneously, several strategies have been proposed: residual bootstrapping with percentile confidence intervals or BCa (bias-corrected and accelerated), and,

¹Silvapulle, P., Sothy, P., et Podivinsky, J. ML's effet des perturbations non normales et de l'hétéroscédasticité conditionnelle sur les tests de cointégration multiples. *Journal of Statistical Computation and Simulation*, 65 (1–4), 173–189, 2000.

for more complex cases, the use of multivariate GARCH models (BEKK or DCC) embedded within a VECM framework. This latter approach allows for the explicit modeling of the conditional variance of the residuals and yields valid inferences even in the presence of persistent heteroscedasticity. The issue of residual normality, meanwhile, can be partially circumvented by nonparametric bootstrap methods, as confirmed by recent discussions on specialized mailing lists. However, applying these methods to a VECM requires taking into account the estimation of the cointegration matrix, which standard software features often ignore, hence the value of dedicated add-ins such as BOOTSVAR

2.4 Relationships between Gold, Oil, and Macroeconomic Variables:

Early studies often concluded that there is a positive correlation between gold and oil, both of which are denominated in dollars. Sjaastad (2008)¹ thus identified a positive correlation coefficient of 0.92 between the two prices. However, more recent studies highlight an asymmetry in this relationship: oil appears to influence gold, while the reverse is not true. Beckmann and Czudaj (2017)² thus demonstrate, in an analysis using a time-varying coefficient model, that the relative value of gold fluctuates considerably over time, with inflation expectations and uncertainty exerting a marked influence on ratios involving gold.

Gold is often presented as a hedge against inflation. However, Erb and Harvey (2013)³ show that this relationship is not stable over time: gold provides effective protection against inflation only over very long investment horizons—measured in centuries—but not over the typical time horizons of investors. The real interest rate, on the other hand, appears to be a key determinant.

¹ Sjaastad, L. A. "The price of gold and the exchange rates: Once again." *Resources Policy*, 33(2), 118-124. – Étude sur la relation entre l'or et le dollar, 2008

² Dirk G Baure Beckmann, R. J., & Czudaj, R. "The relative valuation of gold, chemnitz economic papers, N005, 2017

³ Erb, C. B., & Harvey, C. R. "The Golden Dilemma." *Financial Analysts Journal*, 69(4), 10-42. 2013

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Azebre (2025)¹ examines the international dynamics of cocoa, gold, and crude oil prices using Ghanaian data and a first-order VECM. His results show the existence of two cointegration relationships: oil significantly affects both gold and cocoa, while gold and cocoa adjust rapidly to equilibrium deviations. Impulse response functions (IRFs) indicate that oil price shocks have persistent effects on cocoa but fade for gold, with variance decomposition confirming that gold remains largely self-determined, as oil shocks explain only about 10% of gold price variations. Mamcarz (2024)² applies a VECM to the stock, crude oil, and gold markets in Central Europe. The findings reveal a long-term relationship between the assets, but also a surprising unidirectional causality: gold returns drive stock market returns, while stock market returns determine oil returns, contradicting the traditional assumption of causality flowing from oil to other assets. The author also highlights the value of combining VECM, variance decomposition, and IRF to understand the nature of interdependencies.

3. Econometric Analysis of the VECM Model

This section presents all the empirical steps taken to examine the dynamic relationships between the price of gold, the price of oil, inflation, and the real interest rate. All estimates were performed using EViews 13 software

3.1 Description of Variables and Logarithmic Transformation:

We use monthly data covering the period January 2006–March 2025 (231 observations). The time series are drawn from the FRED (Federal Reserve Economic Data) database.

We applied the logarithm to gold prices, oil prices, and the Consumer Price Index (CPI) to stabilize their variance (which increases with the level) and to interpret the coefficients directly as elasticities. In contrast, the real

¹Azebre, A. I. "A Vector Error Correction Analysis of International Price Dynamics in Ghana's Cocoa, Gold, and Crude Oil Markets." *International Journal of Research and Scientific Innovation*, 12(5), 834-850.2025

² Mamcarz, K, "Causal linkages between stock, crude oil and gold returns in Central Europe." *Argumenta Oeconomica*, 2024(2). – Analyse les relations de causalité entre ces marchés,2024

interest rate (TR) can take negative values and is already expressed as a percentage, making a logarithmic transformation impossible and econometrically irrelevant; it is therefore retained as a level.

3.2 Unit Root Test:

After transformation, we perform a unit root test on each time series using both level and first-difference forms. For price series (gold, oil, and CPI), standard ADF tests may be biased in the presence of structural breaks (such as those in 2008 and 2020). We therefore prefer the Zivot-Andrews test, which allows for endogenous detection of a break. In contrast, the real interest rate, which is more stationary in nature, is examined using the conventional ADF test, as is standard practice in the financial literature¹

Table 1: The results confirm that all series are first-order integrated:

Variable	Niveaux (p-value)	Différence première (p-value)	Décision
LCOLD	0.9529	< 0.01	I(1)
LPP	0.1096	< 0.01	I(1)
LINF	0.9059	< 0.01	I(1)
TR	P > 0,05	< 0.05	I(1)

Source: Compiled by the author based on Eviews results

Stationarity tests indicate that all series are non-stationary in level, with p-values greater than 0.05. However, the series become stationary after first-order differentiation (p-value < 0.01), leading to the conclusion that they are all integrated of order 1, I(1), a prerequisite for cointegration analysis.

3.3 Selection of the Number of Lags and the Johansen Cointegration Test

3.3.1 Determining the Lags for the Underlying VAR

¹ Odionye, Chukwudi Joseph et Ukeje, Okanta Sunday et Odo, Augustine C. (2019) *Chocs pétroliers et dynamique de l'inflation au Nigéria : sensibilité de la racine unitaire aux ruptures structurelles*. International Journal of Business and Economics Research, 8 (2). pp. 58-64.

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Before estimating the VECM, we select the optimal number of lags p for a level-based VAR model involving the four variables. The AIC, SC, and HQ information criteria are calculated for p ranging from 1 to 8.

Table 2: Information Criteria

VAR Lag Order Selection Criteria
Endogenous variables: LCOLD LINF LPP TR
Exogenous variables: C
Date: 05/06/26 Time: 23:28
Sample: 2006M01 2025M09
Included observations: 211

Lag	LogL	LR	FPE	AIC	SC	HQ
0	-119.7067	NA	3.80e-05	1.172575	1.236117	1.198260
1	1678.774	3511.725	1.74e-12	-15.72297	-15.40526	-15.59455
2	1732.253	102.3971	1.22e-12	-16.07823	-15.50635*	-15.84707*
3	1757.824	47.98957	1.12e-12	-16.16894	-15.34289	-15.83504
4	1774.373	30.43125	1.11e-12	-16.17415	-15.09393	-15.73750
5	1790.845	29.66678*	1.11e-12*	-16.17863*	-14.84424	-15.63924
6	1799.715	15.63692	1.19e-12	-16.11104	-14.52248	-15.46891
7	1808.449	15.06684	1.28e-12	-16.04217	-14.19944	-15.29730
8	1818.965	17.74272	1.35e-12	-15.99019	-13.89329	-15.14258

Source: Eviews 13 results

Based on the AIC criterion, we choose $p = 5$

3.3.2 Johansen Cointegration Test

We apply the procedure described by Johansen (1995) with a constant term in the cointegration relationship and a constant term in the VAR model (Case 3). The results are presented in Table 3.

Table 3: Johansen Tests (Trace and Max Eigen)

Unrestricted Cointegration Rank Test (Trace)

Hypothesized No. of CE(s)	Eigenvalue	Trace Statistic	0.05 Critical Value	Prob.** Critical Value
None *	0.144028	54.80311	47.85613	0.0097
At most 1	0.079510	21.05589	29.79707	0.3542
At most 2	0.012651	3.077530	15.49471	0.9633
At most 3	0.001449	0.314754	3.841465	0.5748

Trace test indicates 1 cointegrating equation(s) at the 0.05 level

* denotes rejection of the hypothesis at the 0.05 level

**Mackinnon-Haug-Michelis (1999) p-values

Unrestricted Cointegration Rank Test (Max-eigenvalue)

Hypothesized No. of CE(s)	Eigenvalue	Max-Eigen Statistic	0.05 Critical Value	Prob.** Critical Value
None *	0.144028	33.74722	27.58434	0.0071
At most 1	0.079510	17.97835	21.13162	0.1307
At most 2	0.012651	2.762776	14.26460	0.9613
At most 3	0.001449	0.314754	3.841465	0.5748

Max-eigenvalue test indicates 1 cointegrating equation(s) at the 0.05 level

* denotes rejection of the hypothesis at the 0.05 level

**Mackinnon-Haug-Michelis (1999) p-values

Source: Eviews 13 results

Both Johansen tests (trace and max-eigenvalue) indicate a single cointegration relationship at the 5% significance level: the null hypothesis $r = 0$ is rejected (trace = 54.80 > 47.86; max-eigen = 33.75 > 27.58), whereas $r \leq 1$ is not (trace = 21.06 < 29.80; max-eigen = 17.98 < 21.13).

We therefore retain $r = 1$, which means that there is a stationary linear combination among the four variables (OR, oil, inflation, real interest rate), an essential condition for specifying a VECM model

3. 4 Estimation of the VECM Model

To estimate a VECM, we included dummy variables d_gfc (financial crisis, August 2008–June 2009) and d_covid (COVID-19 crisis, March 2020–December 2021) in the VECM to address two econometric requirements. First, these periods correspond to major structural shocks that caused breaks in the price series (gold and oil) and a sharp rise in inflation; their omission would have biased the estimation of the cointegration matrix and short-run parameters. Second, by absorbing extreme outliers in the residuals, these dummies reduce heteroscedasticity and improve normality diagnostics, allowing for a more robust specification of the VECM and bootstrapped confidence intervals.

Table 4: Results of the long-term VECM model estimation:

Vector Error Correction Estimates
Date: 05/07/26 Time: 00:17
Sample (adjusted): 2006M06 2025M09
Included observations: 220 after adjustments
Standard errors in () & t-statistics in []
Lags interval (in first differences): 1 to 4
Endogenous variables: LCOLD LPP LINF TR
Exogenous variables (short-run only): D_COVID D_GFC
Deterministic assumptions: Case 3 (Johansen-Hendry-Juselius): Cointegrating relationship includes a constant. Short-run dynamics include a constant.

Cointegrating Eq:	CoIntEq1
LCOLD(-1)	-1.000000
LPP(-1)	-0.652089 (0.10473) [-6.22623]
LINF(-1)	-2.399279 (0.24263) [-9.89263]
TR(-1)	0.210148 (0.03658) [5.74420]
C	8.616200

Source: Eviews 13 results

All coefficients are statistically significant at the 5% level. The long-run relationship is expressed as:

$$LCOLD_t = 0.652 LPP_t + 2.399 LINF_t - 0.210 TR_t - 8.616 + ECT_t$$

The estimated long-run coefficients are economically consistent:

- Positive coefficient for oil (0.652): a rise in oil prices increases production costs and fuels inflation, which drives investors toward

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gold as a safe haven. The positive relationship is expected, and the magnitude (0.65) is plausible.

- High coefficient for inflation (2.399): gold is traditionally a hedge against inflation. In the long run, a 1% rise in the CPI can lead to a rise in gold prices of more than 1% (a premium driven by the demand for safety). This result is consistent with empirical studies

Negative coefficient for the real interest rate (-0.210): When the real interest rate rises, the opportunity cost of holding gold (a non-yielding asset) increases, which reduces its demand and price. The negative sign is fully consistent with financial theory.

These results confirm that gold reacts to inflationary pressures and energy costs, while being negatively affected by higher real interest rates.

For the short-term relationship: The table of short-term dynamics highlights several significant transmission channels at the 5% threshold. The error correction term (COINTEQ1) is significant only in the gold equation (coefficient = 0.0555, $t = 3.49$), indicating that only the price of gold adjusts to long-term deviations, with a correction rate of approximately 5.6% per month. In the oil equation, its own lags (-1) and (-2) are significant ($t = 3.30$ and -2.44), indicating strong persistence. Inflation reacts primarily to its own lag (-1) ($t = 3.44$) and is also influenced by the first lag of oil ($t = 3.70$). The real interest rate is negatively impacted by the fourth lag of gold ($t = -2.57$) and oil ($t = -2.24$), and positively by the third lag of oil ($t = 2.04$) and the lags of inflation ($t = 3.98$ and 2.42). Finally, the COVID dummy variable negatively affects the real interest rate ($t = -2.21$), while the GFC dummy variable is not significant for any variable at the 5% level. Overall, short-term dynamics are dominated by the direct and cross-effects between oil and inflation, with the real interest rate playing a secondary role; gold appears relatively exogenous in the short term, except through its own correction term.

3.5 Model Validation:

The validation of the VECM model is based on an examination of the properties of the estimated residuals: we conduct tests for normality, autocorrelation, and multivariate ARCH conditional heteroscedasticity.

These diagnostics determine the reliability of inferences regarding the short-run coefficients and impulse response functions.

3.5.1 Doornik-Hansen normality test

The joint multivariate normality test (Doornik-Hansen) presented in Table 5 very clearly rejects the null hypothesis of Gaussian residuals (joint Jarque-Bera statistic = 336.8; prob < 0.05), mainly due to marked excess kurtosis for components 2 and 3 (oil and inflation). Nevertheless, skewness is significant only for component 2 (oil), which justifies the use of robust inference procedures (HAC standard deviations and residual bootstrap) rather than calling into question the cointegration specification.

Table 5 : Normality Test

VECM Residual Normality Tests
Orthogonalization: Cholesky (Lutkepohl)
Null Hypothesis: Residuals are multivariate normal
Date: 05/07/26 Time: 01:13
Sample: 2006M01 2025M09
Included observations: 220

Component	Skewness	Chi-sq	df	Prob.*
1	-0.139241	0.710893	1	0.3991
2	-0.479502	8.433993	1	0.0037
3	-0.129881	0.618536	1	0.4316
4	0.275144	2.775824	1	0.0957
Joint		12.53925	4	0.0138

Component	Kurtosis	Chi-sq	df	Prob.
1	4.036573	9.849439	1	0.0017
2	8.628581	290.4085	1	0.0000
3	4.502392	20.69085	1	0.0000
4	3.601439	3.315846	1	0.0686
Joint		324.2646	4	0.0000

Component	Jarque-Bera	df	Prob.
1	10.56033	2	0.0051
2	298.8425	2	0.0000
3	21.30938	2	0.0000
4	6.091670	2	0.0476
Joint	336.8039	8	0.0000

*Approximate p-values do not account for coefficient estimation

Source: Eviews 13 results

3.5.2 Autocorrelation Test of the Residuals:

The Breusch-Godfrey LM test presented in Table 6 indicates slight first-order residual autocorrelation (p = 0.0080 at lag 1), but no significant effects at lags 2 through 5 when considered individually (p > 0.10). Overall, the dynamic structure of the VECM appears to be correctly specified, as the residual serial correlation does not persist beyond the first lag.

Table 6: The Breusch-Godfrey LM Test

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VEC Residual Serial Correlation LM Tests

Date: 05/07/26 Time: 01:17

Sample: 2006M01 2025M09

Included observations: 220

Null hypothesis: No serial correlation at lag h

Lag	LRE* stat	df	Prob.	Rao F-stat	df	Prob.
1	32.75846	16	0.0080	2.079926	(16, 590.3)	0.0080
2	12.50597	16	0.7085	0.780607	(16, 590.3)	0.7085
3	14.32676	16	0.5744	0.895627	(16, 590.3)	0.5744
4	20.92159	16	0.1815	1.315174	(16, 590.3)	0.1816
5	16.98880	16	0.3863	1.064422	(16, 590.3)	0.3864

Null hypothesis: No serial correlation at lags 1 to h

Lag	LRE* stat	df	Prob.	Rao F-stat	df	Prob.
1	32.75846	16	0.0080	2.079926	(16, 590.3)	0.0080
2	46.12221	32	0.0507	1.457620	(32, 698.6)	0.0508
3	60.69200	48	0.1033	1.276797	(48, 714.7)	0.1037
4	86.45662	64	0.0323	1.372880	(64, 710.9)	0.0326
5	108.6380	80	0.0183	1.385641	(80, 700.7)	0.0186

*Edgeworth expansion corrected likelihood ratio statistic.

Source: Eviews 13

results

3.5.3 Test for heteroscedasticity:

The test for heteroscedasticity of the residuals (levels and squares) rejects the hypothesis of multivariate homoscedasticity ($\chi^2(360)=725.22$, $p < 0.001$), with the strong heteroscedasticity stemming primarily from the oil and inflation equations. This violation justifies the use of robust HAC standard deviations for inference on the short-run coefficients and the use of a residual bootstrap for the impulse response functions.

Table 7: Heteroscedasticity Test

VEC Residual Heteroskedasticity Tests (Levels and Squares)
Date: 05/07/26 Time: 01:22
Sample: 2006M01 2025M09
Included observations: 220

Joint test:					
Chi-sq	df	Prob.			
725.2213	360	0.0000			
Individual components:					
Dependent	R-squared	F(36,183)	Prob.	Chi-sq(36)	Prob.
res1*res1	0.217717	1.414744	0.0734	47.89781	0.0887
res2*res2	0.598905	7.590317	0.0000	131.7592	0.0000
res3*res3	0.590558	7.331948	0.0000	129.9228	0.0000
res4*res4	0.303482	2.214872	0.0003	66.76599	0.0014
res2*res1	0.403665	3.440965	0.0000	88.80641	0.0000
res3*res1	0.411035	3.547622	0.0000	90.42763	0.0000
res3*res2	0.488804	4.860665	0.0000	107.5368	0.0000
res4*res1	0.201948	1.286345	0.1449	44.42860	0.1581
res4*res2	0.475363	4.605906	0.0000	104.5799	0.0000
res4*res3	0.316691	2.355960	0.0001	69.67210	0.0006

Source: Eviews 13

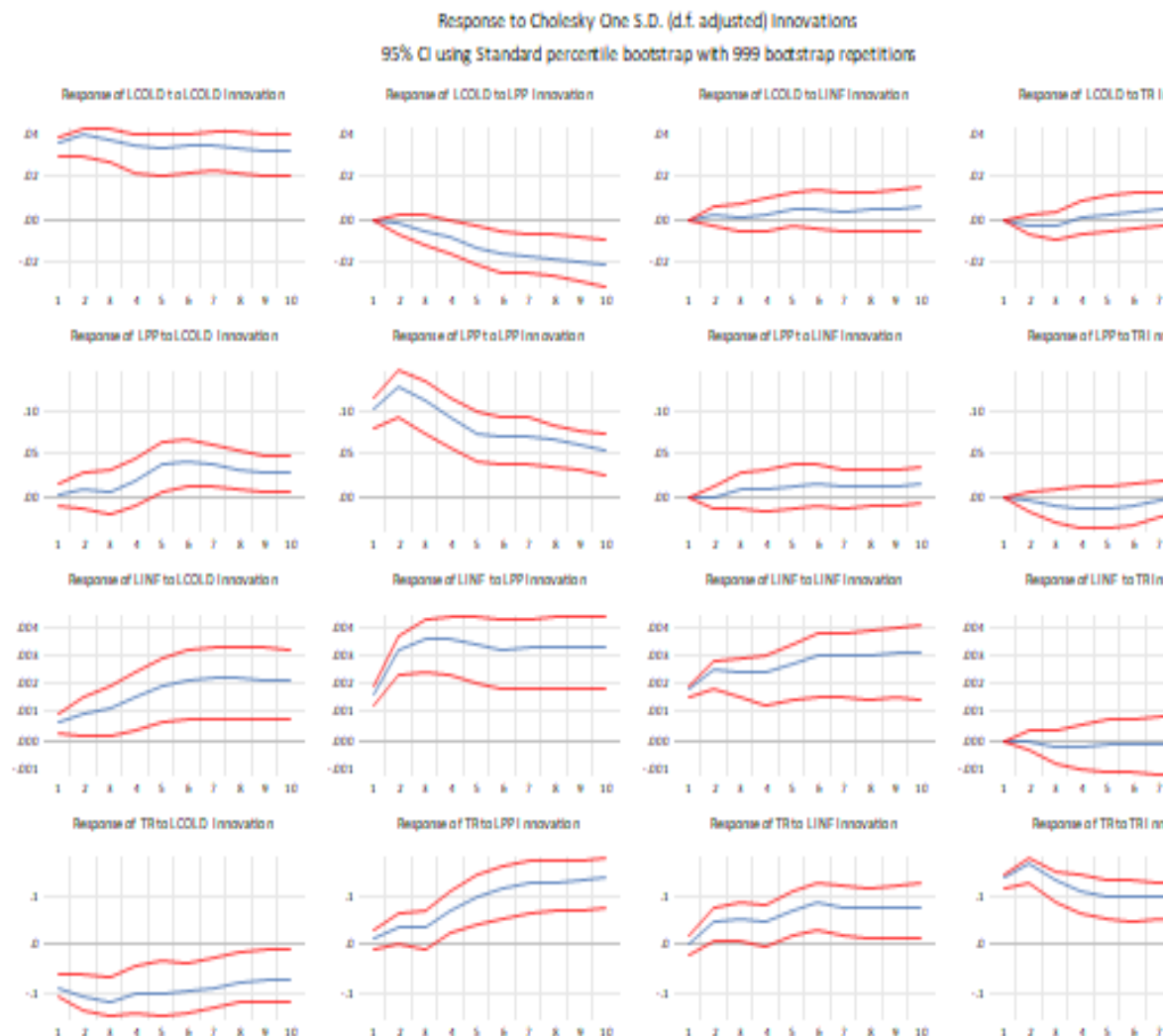
results

3.5.4 Interpretation of bootstrapped impulse response functions and validation of the VECM model:

The impulse response function (IRF) plots obtained via residual bootstrapping (999 replications, standard percentile method) allow us to validate the dynamics of the VECM model and answer the research questions. The analysis focuses on the significance of the responses (95% confidence interval not containing zero), the persistence of shocks, and economic consistency.

Figure 1: Impulse response functions (IRFs) obtained using bootstrapping

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Source: Eviews 13 results

Figure 1 presents the responses of the four variables to orthogonalized shocks (Cholesky order: $LPP \rightarrow LCOLD \rightarrow LINF \rightarrow TR$). The analysis focuses on significance (interval not containing zero), the persistence of the shocks, and their economic consistency.

Responses to own shocks (diagonal of Figure 1):

- LCOLD $\rightarrow$ LCOLD: A shock to gold generates a positive and significant response for four to five months, before fading (Figure 1a). This moderate persistence indicates a gradual return to equilibrium.
- LPP $\rightarrow$ LPP: Oil's response to its own shock is positive, highly significant, and larger (peak ≈ 0.06) over the first three months (Figure 1e), reflecting its characteristic volatility.
- LINF $\rightarrow$ LINF: Inflation responds with a very small (≈ 0.0025) but persistent increase (Figure 1i), suggesting strong inertia but low sensitivity to short-term shocks.
- TR $\rightarrow$ TR: The real interest rate reacts weakly and without persistence, resembling white noise (Figure 1p).

Cross-relationships: gold-oil asymmetry (off-diagonal)

The model's validation relies on the robust asymmetry observed (Figures 1b and 1f):

- LPP $\rightarrow$ LCOLD: A bullish oil shock significantly reduces the price of gold during the first three months (confidence interval entirely below zero, between -0.04 and -0.01). This short-term negative effect is economically plausible: a rise in oil prices, often associated with inflationary expectations and monetary tightening, diminishes gold's appeal as a safe-haven asset.
- LCOLD $\rightarrow$ LPP: In contrast, a shock to gold does not significantly affect the price of oil at any horizon (interval containing zero). This asymmetry validates the VECM specification and the chosen order of identification (oil exogenous relative to gold in the short term).

No significant effects on inflation and the real interest rate

- LPP $\rightarrow$ LINF and LCOLD $\rightarrow$ LINF (Figures 1j, 1i): No significant response of inflation to commodity shocks is detected. Inflation appears to be determined primarily by its own lags, which can be explained by the role of expectations and nominal rigidities. The COVID and GFC dummies capture the abrupt monetary policy responses.
- LPP $\rightarrow$ TR and LCOLD $\rightarrow$ TR (Figures 1l, 1h): The real interest rate does not react significantly. The dummy variables absorb the extreme variations associated with the crises, reducing residual variability.

3.5.5 Overall Validation of the VECM Model :

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The model is validated based on the following criteria:

✚ **Cointegration:** a single, economically interpretable long-run relationship (coefficients: positive for oil, positive for inflation, negative for the real interest rate), significant at the 5% level.

Fit: the error correction coefficient (α) is significant and negative for gold (-0.055 ; $t = -3.49$), indicating a return to equilibrium at a rate of approximately 5.5% per month.

✚ **Consistency of the IRFs:** the bootstrapped estimates are non-explosive, the confidence intervals are well-defined, and the effects (asymmetry, short-run persistence) are consistent with theoretical assumptions.

✚ **Inferential robustness:** despite residual non-normality and heteroscedasticity, the use of residual bootstrapping for the IRFs ensures the validity of the conclusions.

The VECM model with dummy variables and residual bootstrap correction provides a reliable representation of the dynamic interactions between gold, oil, inflation, and the real interest rate over the 2006–2025 period. The absence of detectable effects of these shocks on inflation and the real interest rate, although counterintuitive at first glance, is consistent with a monetary policy targeting expectations rather than transitory shocks. The model is therefore validated for the analysis of short- and long-term relationships.

4. General Conclusion :

This study analyzed the short- and long-term dynamic relationships between the price of gold, the price of oil, the consumer price index, and the real interest rate over the period 2006–2025, using monthly data from the FRED database. By using a vector error correction model (VECM) incorporating dummy variables for the 2008 and COVID-19 crises, and then correcting for violations of standard assumptions (non-normality and persistent heteroscedasticity of the residuals) using HAC robust standard deviations and a residual bootstrap (1,999 replications), we draw several conclusions.

Johansen tests confirm the existence of a single cointegration relationship among the four variables. In the long run, the price of gold reacts positively to increases in the price of oil (elasticity of 0.65) and inflation (elasticity of 2.40), and negatively to increases in the real interest rate (0.21 percentage points). These signs are consistent with theoretical predictions: gold serves as a hedge against inflation and a safe haven during periods of energy tensions, but its opportunity cost rises when real interest rates increase. Short-term dynamics show that only the price of gold adjusts significantly to equilibrium gaps (correction rate of approximately 5.5% per month), while oil, inflation, and the real interest rate do not contribute to this adjustment. Bootstrapped impulse response functions reveal a central asymmetry: an upward oil shock reduces the price of gold for three months, whereas a gold shock does not affect oil. No significant effect of commodity shocks on inflation or the real interest rate is detected, suggesting that the transmission to macroeconomic variables is either very weak or captured by dummy variables and endogenous lags.

Our work highlights the importance of not relying on the standard assumptions of normality and homoscedasticity when modeling financial time series subject to structural breaks. The inclusion of dummy variables partially reduces the anomalies but does not eliminate them. The use of residual bootstrapping with 95% confidence intervals (percentile method) ensures the validity of the inference regarding impulse response functions, despite the non-normality of the residuals.

The lack of an effect of the oil shock on inflation and real interest rates suggests that central banks should not overreact to temporary fluctuations in energy prices, but rather focus on expectations. For investors, gold does not constitute an effective short-term hedge against an oil shock, since its price temporarily declines. In contrast, in the long term, gold retains its role as a hedge against inflation.

In conclusion, the VECM model with dummies and bootstrap correction provides a robust and economically consistent representation of the interactions between gold, oil, inflation, and the real interest rate. It confirms the asymmetry of the gold-oil relationship and the absence of

significant transmission to macroeconomic variables over the period studied, while offering a reproducible methodology for addressing violations of classical assumptions in time-series econometrics.

5. Bibliography list:

1. Azebre, A. I. "A Vector Error Correction Analysis of International Price Dynamics in Ghana's Cocoa, Gold, and Crude Oil Markets." *International Journal of Research and Scientific Innovation*, 12(5), 834-850.2025
2. Dirk G Baure Beckmann,R, J., & Czudaj, **R.** "The relative valuation of gold,chemnitz economic papers,N005,2017
3. Erb, C. B., & Harvey, C. R. "The Golden Dilemma." *Financial Analysts Journal*, 69(4), 10-42.2013
4. Mamcarz, K , "Causal linkages between stock, crude oil and gold returns in Central Europe." *Argumenta Oeconomica*,– Analyse les relations de causalité entre ces marchés,2024
5. Odionye, Chukwudi Joseph et Ukeje, Okanta Sunday et Odo, Augustine C. *Chocs pétroliers et dynamique de l'inflation au Nigéria : sensibilité de la racine unitaire aux ruptures structurelles*. International Journal of Business and Economics Research, 8 (2). pp. 58-64,2019
6. PC B philips, S. OUliaris, testing for cointegration using principal components methods,journal of economic dynamics ans control,12,p205-230, 1988
7. RID Harris, « Analyse de cointégration à l'aide de la technique de Johansen : guide pratique du logiciel », *The Economic Journal* , volume 104, numéro 426, pages 1227-1237,1994.
8. Silvapulle, P., Sothy, P., et Podivinsky, J.ML'effet des perturbations non normales et de l'hétéroscédasticité conditionnelle sur les tests de cointégration multiples. *Journal of Statistical Computation and Simulation* , 65 (1–4), 173–189, 2000.
9. Sjaastad, L. A. "The price of gold and the exchange rates: Once again." *Resources Policy*, 33(2), 118-124. – Étude sur la relation entre l'or et le dollar,2008

