

Specifications of the "Min condition" in the simplest one market disequilibrium model

K. HADRI

Because of aggregation, market frictions ... it has been shown that the quantity transacted is likely to fall on the left of the short side of a market affected by quantity rationing. All previous econometric disequilibrium models do not allow for this. In this paper we suggest some specifications which cater for it. This is done essentially by adding a negative disturbance term (half-normal).

1 Introduction to the simplest one market disequilibrium model:

We shall consider in this paper only partial disequilibrium models. That is we analyse each market in isolation of the others.

The usual assumption in the equilibrium case is that economic agents can purchase and sell as much as they want at the proposed price and no one has the power to change the price variable by his sole intervention (perfect competition is assumed). The equilibrium price p^* is the price for which the aggregate demand equals the aggregate supply (fig. 1)

$$D(p^*) = S(p^*) = Q^* \quad (1)$$

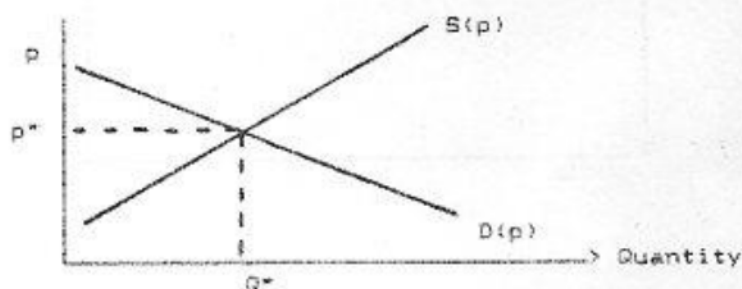


Fig. 1

It should be noted that the demand and the supply functions can be made function of other variables but this will not make any difference to the present analysis.

In market disequilibrium models with rigid prices, we suppose that the price is not a result of balancing a numerous demands and supplies in the market but, we assume that the price is set exogenously. In this case we have three possibilities:

1) The price p' (see fig. 2) is less than the equilibrium price p^* ($p' < p^*$)

in this case the aggregate demand exceeds the aggregate supply ($D(p') > S(p')$).

The demand side is rationed and the aggregate quantity transacted is equal to $S(p')$. Therefore, at least one buyer cannot purchase as much goods as he wants at the fixed price p' .

2) If on the other hand the price p'' is fixed at level higher than the equilibrium price p^* (see fig. 2). In this case the aggregate quantity supplied exceeds the aggregate quantity demanded. The aggregate quantity transacted is equal to $D(p'')$. This means that at least one supplier cannot sell as much goods as he wants at the given price p'' .

3) Last, if the fixed price is set to be equal to p^* we are in the equilibrium case where no economic agent is rationed. Each one sells or buys as much as he wants at the given equilibrium

price p^* .

In the market disequilibrium case, just described, it is said that we are on the "short" side of the market, that is, the aggregate quantity transacted is equal to the aggregate quantity to be sold or supplied whichever is the smallest quantity at the given fixed price (see fig. 2). Let Q_s be the quantity transacted

$$Q_s = \min(D(p), S(p)), \quad p \text{ fixed exogenously.} \quad (2)$$

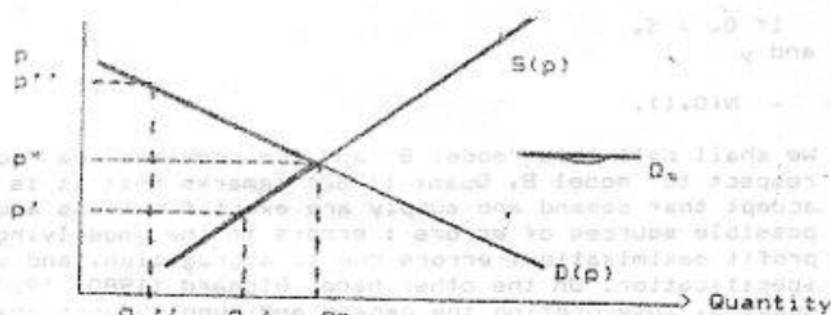


fig. 2

The model of market in disequilibrium described above can be used as a first approximation to describe a slow price adjustment in the short run or to analyse situations in which wage or price are fixed authoritatively.

11) The Econometric Representation of the Simplest One Market Disequilibrium Model

Fair and Jaffee (1972) were the first to attempt to estimate model of market in disequilibrium. The simplest one market disequilibrium model where the demand and supply equations are assumed to be linear, can be stated as follows :

$$\begin{aligned} D_s &= a'X_{1s} + \epsilon_{1s} \\ S_s &= b'X_{2s} + \epsilon_{2s} \\ Q_s &= \min(D_s, S_s) \end{aligned} \quad (3)$$

X_1 and X_2 are sets of exogenous variables (the price variable p being set exogenously, is included in these sets). D_s and S_s are unobserved, Q_s is the quantity transacted in the market and ϵ_{1s} and ϵ_{2s} are random variables.

This model cannot be estimated by OLS when ϵ_{1s} and ϵ_{2s} are assumed to be uncorrelated or by the SURE method in the correlation case, because their conditional expectations are always different from zero. In this case we can use the maximum likelihood method as shown by Maddala and Nelson (1974), Hartley and Mallela (1977), Amemiya and Sen (1977) and Goldfeld and Quant (1975) and (1978).

A second approach to this problem is adopted by Sneessens (1985). Following, in particular, Tishler and Zang (1979), Sneessens specifies the simplest one-market disequilibrium model as follows:

$$D_t = a'X_{1,t}$$

$$S_t = b'X_{2,t}$$

$$Q_t = \min(D_t, S_t) + w_t \quad (4)$$

where w_t is an error term which may have two different variances depending on which regime prevails i.e. :

$$w_t = \sigma_1 y_t$$

$$\text{if } D_t \leq S_t$$

$$= \sigma_2 y_t$$

$$\text{if } D_t > S_t$$

and y_t

$$= N(0,1).$$

We shall call this "model B" and the previous one "model A". With respect to model B, Quant (1982) remarks that it is difficult to accept that demand and supply are exact functions and lists three possible sources of errors : errors in the underlying utility or profit maximization, errors due to aggregation, and errors in specification. On the other hand, Richard (1980, 1982) prefers model B, interpreting the demand and supply functions as agents' contingent plans with respect to the unobserved D_t^* and S_t^* . These contingent plans are the expected quantities which optimize utility or profit conditional on the information available at time t . Hence, these quantities are fixed ex ante. The quantity transacted is given by :

$$Q_t = \min(D_t^*, S_t^*) + w_t \quad (5)$$

where w_t , the disturbance term, represents all the uncontrolled factors affecting the observed quantity Q_t .

It seems from the limited Monte-Carlo experiment conducted by Sneessens (1985) that model B is more robust than model A. This is due certainly to the fact that sample separation in model B is deterministic:

$$Q_t \in D_t \text{ if } a'X_{1,t} \leq b'X_{2,t}$$

$$Q_t \in S_t \text{ otherwise,}$$

whereas in model A the sample separation is stochastic:

$$Q_t \in D_t \text{ if } a'X_{1,t} + \epsilon_{1,t} \leq b'X_{2,t} + \epsilon_{2,t}$$

$$Q_t \in S_t \text{ otherwise.}$$

III) New Specifications of the Min Condition in the Simplest One Market Disequilibrium Model

The main feature of the econometric disequilibrium models is the so-called "Min Condition".

Many specifications have been suggested. To our knowledge, none of them incorporate the assumption that the transacted quantities may fall on the left of the short side of the market i.e.

$$Q_t \leq \min(D_t, S_t) \quad (6)$$

This specification is justified for aggregated markets Muellbauer (1978), inefficient market Benassy (1982) and stochastic manipulable perceived rationing Green (1980) and Weinrich (1984).

a) The aggregation problem : We are going to show in a simple example that two markets in disequilibrium of a same physical product with the same price when aggregated, in the resulting market, the aggregated quantity transacted is no longer equal to $\min(D_1, S_1)$ but it is strictly inferior to $\min(D_1, S_1)$ for some price range (see fig. 3).

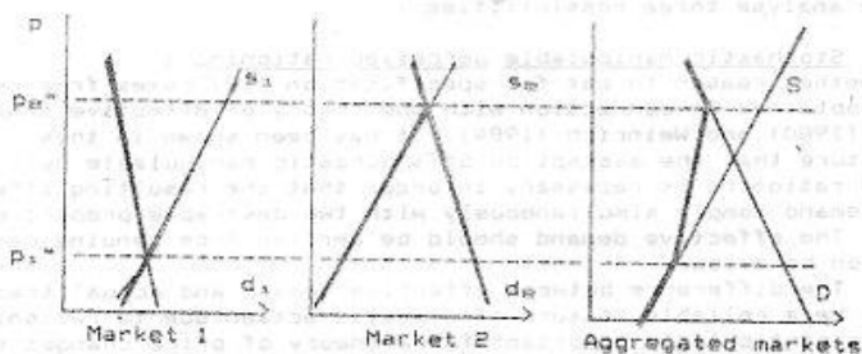


fig. 3

For a price above p_2^* in both markets suppliers are rationed and the quantity transacted is equal to the quantity demanded, therefore $p > p_2^*$

$$\min(d_1, s_1) + \min(d_2, s_2) = \min(d_1 + d_2, s_1 + s_2)$$

Where $q_1 = d_1$, $q_2 = d_2$ and $Q = d_1 + d_2 = D$
For a price below p_1^* , in both markets the demanders are rationed and the quantity transacted is equal to the quantity supplied, we get for $p < p_1^*$

$$\min(d_1, s_1) + \min(d_2, s_2) = \min(d_1 + d_2, s_1 + s_2)$$

Where $q_1 = s_1$, $q_2 = s_2$ and $Q = s_1 + s_2 = S$

However, when the value of p is comprised between p_1^* and p_2^* , in this case, in the first market the quantity transacted is equal to the quantity demanded which means that in this market the suppliers are rationed whereas, in the second market the quantity transacted matches the quantity supplied and therefore it is the demanders who are rationed here, hence :

$$q_1 = d_1 < s_1 \rightarrow q_1 < s_1 \quad (7)$$

$$q_2 = s_2 < d_2 \rightarrow q_2 < d_2 \quad (8)$$

adding q_2 to both side of of inequality 7 we obtain

$$q_1 + q_2 < s_1 + q_2 = s_1 + s_2 \quad (9)$$

adding q_1 to both side of inequality 8 we get

$$q_1 + q_2 < d_2 + q_1 = d_1 + d_2 \quad (10)$$

Therefore $Q = q_1 + q_2 < \min(S, D)$

for $p_1^* < p < p_2^*$

At an aggregated level a more accurate assumption is that

$$Q < \min(S, D) \quad (12)$$

b) Market inefficiency

The short side rule i.e. $Q = \min(S, D)$ means that demanders and suppliers are not rationed at the same time. This rule is a consequence of voluntary exchange and market efficiency assumptions. The voluntary exchange assumption means that no economic agent is forced to purchase more than what he wants or sell more than what he can supply. This is easily acceptable, but it is satisfied even by zero trade. However, market efficiency or frictionless market as it is also called seems to be less satisfactory

ry assumption for large and decentralized markets. In these last cases buyers and sellers might not always meet and therefore the mutually advantageous trade might not take place. For instance vacancies and demands for employment may exist simultaneously.

In order to incorporate this specification, we suggest to introduce a disturbance term following a normal distribution truncated by above at zero.

We analyse three possibilities :

c) Stochastic manipulable perceived rationing :

Another reason to opt for specification (12) comes from results obtained in connection with the theory of effective demand: Green (1980) and Weinrich (1984). It has been shown in this literature that the assumption of stochastic manipulable perceived rationing is necessary in order that the resulting effective demand comply simultaneously with two desirable properties :

- 1) The effective demand should be derived from genuine maximization behaviour.
- 2) The difference between effective demand and actual trade should be a reliable measure of dissatisfaction due to rationing. This is particularly important for a theory of price changes to be based on market excess demand.

If we accept the assumption that economic agents perceive rationing as stochastic and manipulable, it is clear that in this condition we do not know for certain which actual trade will be associated with a given demand. Therefore, it seems necessary to include a disturbance term in the min condition to take into account this stochastic feature. It should also be noted that, because of these assumptions, Q_t , the quantity transacted, is likely to be less than the corresponding demand which reinforces the choice of specification (12). In what follows we shall suggest some specifications which allow for the quantity transacted to fall on the left side of the short side of the market in the context of both models A and B.

Model I : We add to the Min Condition equation of model A an error term which follows a normal distribution truncated by above at zero. We obtain the following model:

$$\begin{aligned} D_t &= a'X_{1t} + u_{1t} \\ S_t &= b'X_{2t} + u_{2t} \\ Q_t &= \text{Min}(D_t, S_t) + u_{3t} \end{aligned} \quad (13)$$

The price variable being set exogenously, is included in X_{1t} and X_{2t} , the exogenous variables of the demand and supply equations respectively.

We assume that :

$$u_{1t} \sim N(0, \sigma_1^2), \quad u_{2t} \sim N(0, \sigma_2^2)$$

and u_{3t} follows a normal distribution truncated by above at zero :

1 For other definitions of effective demand see Drèze (1975) and Benassy (1975).

$$u_{2t} \sim f_2(u_{2t}) = \begin{cases} \frac{2}{\sigma_2} \phi\left(\frac{u_{2t}}{\sigma_2}\right) & u_{2t} \leq 0 \\ 0 & u_{2t} > 0 \end{cases} \quad (14)$$

Model 1 can be rewritten in the following way :

$$\begin{aligned} D_{1t} &= a'X_{1t} + \epsilon_{1t} \\ S_{2t} &= b'X_{2t} + \epsilon_{2t} \\ Q_t &= \min(D_{1t}, S_{2t}) \end{aligned} \quad (15)$$

where $\epsilon_{1t} = u_{1t} + u_{2t}$ and $\epsilon_{2t} = u_{2t} + u_{3t}$ (16)

ϵ_{1t} and ϵ_{2t} are the sum of a normal variable and a truncated normal. The corresponding p.d.f has been derived by Weinstein (1964). Using Aigner et AL (1977) notation we get :

$$\epsilon_{1t} \sim f_1(\epsilon_{1t}) = \frac{2}{\sigma_1} \phi\left(\frac{\epsilon_{1t}}{\sigma_1}\right) (1 - \Phi(\epsilon_{1t} \lambda_1 \sigma_1^{-1})) \quad -\infty \leq \epsilon_{1t} \leq +\infty \quad (17)$$

$$\epsilon_{2t} \sim f_2(\epsilon_{2t}) = \frac{2}{\sigma_2} \phi\left(\frac{\epsilon_{2t}}{\sigma_2}\right) (1 - \Phi(\epsilon_{2t} \lambda_2 \sigma_2^{-1})) \quad -\infty \leq \epsilon_{2t} \leq +\infty \quad (18)$$

$$\text{where } \sigma_1^2 = \sigma_1^2 + \sigma_2^2 \quad \lambda_1 = \sigma_2 / \sigma_1 \quad (19)$$

$$\sigma_2^2 = \sigma_2^2 + \sigma_3^2 \quad \lambda_2 = \sigma_3 / \sigma_2 \quad (20)$$

ϕ and Φ are the p.d.f and the c.d.f of the standard normal respectively.

Even, if we assume that u_{1t} , u_{2t} , u_{3t} are independent from one another, ϵ_{1t} and ϵ_{2t} are contemporaneously correlated.

A necessary condition that ϵ_{1t} and ϵ_{2t} are independent is that $E(\epsilon_{1t}, \epsilon_{2t}) = E(\epsilon_{1t})E(\epsilon_{2t})$.

$$\begin{aligned} E(\epsilon_{1t}, \epsilon_{2t}) &= E(u_{1t} + u_{2t})(u_{2t} + u_{3t}) \\ &= E(u_{2t}^2) = \sigma_2^2 E\left(\left(\frac{u_{2t}}{\sigma_2}\right)^2\right) \\ &= \sigma_2^2 \left(1 - c_1 \phi\left(\frac{c_1}{\sigma_2}\right)\right) \end{aligned} \quad (21)$$

where c_1 is the point of truncation. For $c_1 = 0$ we obtain :

$$E(\epsilon_{1t}, \epsilon_{2t}) = \sigma_2^2$$

$$E(\epsilon_{1it})E(\epsilon_{2it}) = E(u_{1it} + u_{2it})E(u_{2it} + u_{3it}) \\ = (E(u_{2it}))^2 = \sigma_u^2 \left(-\phi \frac{(0)}{\phi(0)} \right)^2 \quad (22)$$

$$E(\epsilon_{1it})E(\epsilon_{2it}) = \frac{2\sigma_u^2}{R} \quad (23)$$

Therefore : $E(\epsilon_{1it}, \epsilon_{2it}) \neq E(\epsilon_{1it})E(\epsilon_{2it})$ which means that ϵ_{1it} and ϵ_{2it} are not independent.

In order to derive the likelihood function of model I, we need the joint distribution of ϵ_{1it} and ϵ_{2it} . We did not find the mathematical expression of this bivariate distribution in the literature. We tried to find other specifications easier to handle.

Model II :

$$\begin{aligned} D_{it} &= a'X_{1it} + u_{1it} \\ S_{it} &= b'X_{2it} + u_{2it} \\ Q_{it} &= \min(D_{it}, S_{it}) \end{aligned} \quad (24)$$

Where u_{1it} and u_{2it} are normal distribution truncated by above at zero :

$$u_{1it} = f_1(u_{1it}) = \begin{cases} \frac{2}{\sigma_1} \phi\left(\frac{u_{1it}}{\sigma_1}\right) & u_{1it} \leq 0 \\ 0 & u_{1it} > 0 \end{cases} \quad (25)$$

$$u_{2it} = f_2(u_{2it}) = \begin{cases} \frac{2}{\sigma_2} \phi\left(\frac{u_{2it}}{\sigma_2}\right) & u_{2it} \leq 0 \\ 0 & u_{2it} > 0 \end{cases} \quad (26)$$

Derivation of the likelihood function :

a) u_{1it} and u_{2it} are independent

$$h_1(Q_{it}) = f_1(u_{1it}) \int_0^{a'x_{1it}} f_2(u_{2it}) dS_{it} + f_2(u_{2it}) \int_0^{b'x_{2it}} f_1(u_{1it}) dD_{it} \quad (27)$$

$$L = \prod_{i=1}^T (h_1(Q_i)) \quad (28)$$

b) u_{1t} and u_{2t} are contemporaneously correlated :

$$h_2(Q_i) = \int_{Q_i}^{a'X_{1i}} f(Q_i, S_i) dS_i + \int_{Q_i}^{a'X_{2i}} f(Q_i, D_i) dD_i \quad (29)$$

$$L = \prod_{i=1}^T (h_2(Q_i)) \quad (30)$$

One of the regularity condition of M.L. estimation (see Dhrymes page 115) that the range of the random variable under study here Q_i , should be independent of the parameters is violated.

The range of Q_i is $(-\infty, a'X_{1i})$ when $D_i \leq S_i$ and $(-\infty, b'X_{2i})$ when $D_i > S_i$.

In this case the properties of the maximum likelihood estimators are uncertain and have to be analysed for each particular case at hand.

We suggest a third model in order to overcome the difficulties encountered in model I and model II.

Model III :

$$D_i = a'X_{1i} + \epsilon_i$$

$$S_i = b'X_{2i} + \epsilon_i$$

$$Q_i = \min(D_i, S_i) + \epsilon_i$$

where $\epsilon_i = u_{1i} + u_{2i}$
 $u_{1i} \sim N(0, \sigma_1^2)$

$$f(u_{2i}) = \begin{cases} \frac{2}{\sigma_2} \phi\left(\frac{u_{2i}}{\sigma_2}\right) & u_{2i} \leq 0 \\ 0 & u_{2i} > 0 \end{cases} \quad (32)$$

As before :

$$\epsilon_i = (u_{1i} + u_{2i}) \sim f(\epsilon_i) = \frac{2}{\sigma} \phi\left(\frac{\epsilon_i}{\sigma}\right) (1 - \phi(\epsilon_i / \lambda \sigma)) \quad (33)$$

where $\sigma^2 = \sigma_1^2 + \sigma_2^2$, $\lambda = \sigma_2 / \sigma_1$ (34)

Estimation of model III :

The likelihood function of model III when we assume the sample separation known is :

$$L = \prod_{T_1} \frac{2}{\sigma} \phi\left(\frac{\epsilon_{1i}}{\sigma}\right) (1 - \phi(\epsilon_{1i} \lambda \sigma^{-1})) \prod_{T_2} \frac{2}{\sigma} \phi\left(\frac{\epsilon_{2i}}{\sigma}\right) (1 - \phi(\epsilon_{2i} \lambda \sigma^{-1})) \quad (35)$$

where $\epsilon_{1i} = Q_{1i} - a'X_{1i}$

$\epsilon_{2i} = Q_{2i} - b'X_{2i}$

T_1 are the observations for which $D_{1i} \leq S_0$

T_2 are the observations for which $D_{1i} > S_0$

$T = T_1 + T_2$

$$\begin{aligned} \ln L(Q, |a, b, \lambda, \sigma^2) &= T_1 \ln \left(\sqrt{\frac{2}{\pi}} \right) + T_1 \ln(\sigma^{-1}) \\ &+ \sum_{i=1}^{T_1} \ln(1 - \phi(\epsilon_{1i} \lambda \sigma^{-1})) - \frac{1}{2\sigma^2} \sum_{i=1}^{T_1} \epsilon_{1i}^2 + T_2 \ln \left(\sqrt{\frac{2}{\pi}} \right) \\ &+ T_2 \ln(\sigma^{-1}) - \frac{1}{2\sigma^2} \sum_{i=1}^{T_2} \epsilon_{2i}^2 + \sum_{i=1}^{T_2} \ln(1 - \phi(\epsilon_{2i} \lambda \sigma^{-1})) \end{aligned} \quad (36)$$

$$\begin{aligned} \ln L(Q, |a, b, \lambda, \sigma^2) &= T \ln \left(\sqrt{\frac{2}{\pi}} \right) + T \ln(\sigma^{-1}) + \sum_{i=1}^{T_1} \ln(1 - \phi(\epsilon_{1i} \lambda \sigma^{-1})) \\ &- \frac{1}{2\sigma^2} \left(\sum_{i=1}^{T_1} \epsilon_{1i}^2 + \sum_{i=1}^{T_2} \epsilon_{2i}^2 \right) + \sum_{i=1}^{T_2} \ln(1 - \phi(\epsilon_{2i} \lambda \sigma^{-1})) \end{aligned} \quad (37)$$

We take derivatives :

$$\begin{aligned} \frac{\partial \ln L}{\partial \sigma^2} &= -\frac{T}{2\sigma^2} + \frac{\lambda}{2\sigma^2} \sum_{T_1} \frac{\phi_{1i}}{1 - \phi_{1i}} (Q_{1i} - a'X_{1i}) \\ &+ \frac{\lambda}{2\sigma^2} \sum_{T_2} \frac{\phi_{2i}}{1 - \phi_{2i}} (Q_{2i} - b'X_{2i}) \\ &+ \frac{1}{2\sigma^4} \left(\sum_{T_1} (Q_{1i} - a'X_{1i})^2 + \sum_{T_2} (Q_{2i} - b'X_{2i})^2 \right) \end{aligned} \quad (38)$$

$$\frac{\partial \ln L}{\partial \lambda} = -\frac{1}{\sigma} \left(\sum_{T_1} \frac{\phi_{1i}}{1 - \phi_{1i}} (Q_{1i} - a'X_{1i}) + \sum_{T_2} \frac{\phi_{2i}}{1 - \phi_{2i}} (Q_{2i} - b'X_{2i}) \right) \quad (39)$$

$$\frac{\partial \ln L}{\partial a} = \frac{\lambda}{\sigma} \sum_{T_1} \frac{\phi_{1i}}{1 - \phi_{1i}} X_{1i} + \frac{1}{\sigma^2} \sum_{T_1} (Q_{1i} - a'X_{1i}) X_{1i} \quad (40)$$

$$\frac{\partial \ln L}{\partial b} = \frac{\lambda}{\sigma} \sum_{T_2} \frac{\phi_{2i}}{1 - \phi_{2i}} X_{2i} + \frac{1}{\sigma^2} \sum_{T_2} (Q_{2i} - b'X_{2i}) X_{2i} \quad (41)$$

ϕ_{1i} and Φ_{1i} are the p.d.f and the c.d.f of the standard normal respectively evaluated $(Q_{1i} - a'X_{1i})/\sigma$

ϕ_{2i} and Φ_{2i} are the p.d.f and the c.d.f of the standard normal respectively evaluated $(Q_{2i} - b'X_{2i})/\sigma$

At the optimum (39) gives :

$$\sum_{i=1}^T \frac{\phi_{1i}}{1-\phi_{1i}} (Q_i - a'X_{1i}) - \sum_{i=1}^T \frac{\phi_{2i}}{1-\phi_{2i}} (Q_i - b'X_{2i}) = 0 \quad (42)$$

Using this result (38) becomes :

$$-\frac{T}{2\sigma^2} + \frac{1}{2\sigma^4} \left(\sum_{i=1}^T (Q_i - a'X_{1i})^2 + \sum_{i=1}^T (Q_i - b'X_{2i})^2 \right) = 0 \quad (43)$$

Which gives :

$$\hat{\sigma}^2 = \frac{1}{T} \left(\sum_{i=1}^T (Q_i - a'X_{1i})^2 + \sum_{i=1}^T (Q_i - b'X_{2i})^2 \right) \quad (44)$$

The likelihood function of model III is similar to the likelihood function of the Tobit model and therefore it might share its properties. (The log likelihood function of the Tobit model is well behaved, see Maddala (1983)).

Many iterative methods exist for finding the optimizing values of a , b , λ and σ^2 .

If a Newton-Raphson algorithm is used to maximize $\ln L$, we need the second derivatives with respect to the parameters. The second derivatives can be derived easily.

An iterative method needs a starting values of the parameters to be estimated. One may try to apply a two-stage method like for the Tobit model to find consistent estimates which can be used as a starting values.

It should be noted that model III is similar to model I in which we assume that $\epsilon_{1i} = \epsilon_{2i} = \epsilon_i$.

Model IV :

Now, if we prefer to use model B as a basis for disequilibrium modelling instead of model A, we suggest the model below which caters for the fact that Q_i is likely to be on the left side of the short side of the market:

$$\begin{aligned} D_i &= a'X_{1i} \\ S_i &= b'X_{2i} \\ Q_i &= \min(D_i, S_i) + \epsilon_i \end{aligned} \quad (45)$$

where $\epsilon_i = u_i + \sigma_1 v_i$ if $D_i \leq S_i$
 $= u_i + \sigma_2 v_i$ otherwise

$u_i \sim N(0, \sigma)$ and v_i is the standard normal truncated by above at zero. Using the same notation as before we get :

$$f_1(\epsilon_i) = (2/\sigma_1) \phi(\epsilon_i/\sigma_1) [1 - \Phi(\epsilon_i/\sigma_1)] \text{ and}$$

$$f_2(\epsilon_i) = (2/\sigma_2) \phi(\epsilon_i/\sigma_2) [1 - \Phi(\epsilon_i/\sigma_2)]$$

where $\sigma_1 = \sigma^2 + \sigma_1^2(1 - 2/\pi)$ and

$$\sigma_2 = \sigma^2 + \sigma_2^2(1 - 2/\pi)$$

The loglikelihood of model III is given by :

$$L = \sum_i \log[(1-r_i)f_1(Q_i) + r_i f_2(Q_i)] \quad (46)$$

where $r_i = 0$ if $a'X_{1i} \leq b'X_{2i}$
 $= 1$ otherwise

As noted by Sneessens (1985), the binary variable r_e makes the loglikelihood function not continuously differentiable. This difficulty is overcome by using the Tishler and Zang (1979) twice continuously differentiable approximation to r_e :

$$r_e^* = \begin{cases} 0 & \text{if } E_e \geq \epsilon \\ 3/16(E_e/\epsilon)^6 - 5/8(E_e/\epsilon)^4 + 15/16(E_e/\epsilon)^2 + 1/2 & \text{if } -\epsilon < E_e < \epsilon \\ 1 & \text{if } E_e \leq -\epsilon \end{cases}$$

where $E_e = a'X_{1e} - b'X_{2e}$ is excess demand and ϵ is an arbitrarily small positive number. Sneessens suggests in his paper a two-step iterative procedure for optimizing the loglikelihood function of model B. The same procedure can be used to optimize the loglikelihood of model IV.

If we set $u_e = 0$ in our model IV we fall into the same difficulty as before, one of the regularity conditions of the M.L. estimation would be violated.

To allow for a different symmetrical disturbance term in the demand and the supply functions we can use the following specification for ϵ_e in model IV :

$$\begin{aligned} \epsilon_e &= u_{1e} + \sigma_1 v, \text{ if } D_e \geq 0 \\ &= u_{2e} + \sigma_2 v, \text{ otherwise.} \end{aligned}$$

Conclusions

We have suggested, in this paper, new specifications for the simplest one market disequilibrium model that take into account the fact that the quantity transacted may fall on the left of the short side of the market. This means that both, suppliers and buyers might be rationed simultaneously. This situation arises for aggregation, inefficiency in real markets and the assumption that the perceived rationing by agents is stochastic and manipulable. The new specifications have been achieved essentially by adding a normal distribution truncated by above at zero.

The next step could be to extend this specifications to multimarket disequilibrium models in order to introduce the spillover effect, that is the effect of rationing in one market on the others and reciprocally. The spillover effect is a central feature in the analysis of multimarket models in disequilibrium.

References

- Aigner, D.J., CA. Knox Lovell and P. Schmidt, 1977, Formulation and estimation of stochastic frontier production function models, Journal of Econometrics 6 : 21-37.
- Amemiya, Y. and Sen, G. 1977, The consistency of the maximum likelihood estimator in a disequilibrium model, Technical report n°238, Institute for Mathematical Studies in Social Sciences, Stanford University.
- Benassy J.P. 1982, The economics of market disequilibrium, Academic Press.
- Dhrymes, P.J., 1970, Econometrics: Statistical foundations and applications, Harper and Row.
- Fair, R.C. and Jaffee, D.M., 1972, Methods of estimation for markets in disequilibrium, Econometrica 40-3 : 495-514.

- Goldfeld, S.M. and Quant, R.E 1975, Estimation of disequilibrium model and the value of information, *Journal of econometrics* 3 : 325-348.
- Goldfeld, S.M. and Quant, R.E 1978, Some properties of the simple disequilibrium model with covariance, *Economics Letters* 1 : 343-346.
- Green, J. 1980, On the theory of effective demand, *The Economic Journal* 90 : 341-353.
- Hartley, M.J. and Mallela, P., 1977, The asymptotic properties of a maximum likelihood estimator for a model of markets in disequilibrium, *Econometrica* 45-5 : 1205-1220.
- Maddala, G.S. and Nelson, F.D., 1974, Maximum likelihood methods for models of markets in disequilibrium, *Econometrica* 42-6 : 1013-1030.
- Maddala, G.S., 1983, Limited-dependent and qualitative variables in econometrics, Cambridge University Press (Chapter 10 : Disequilibrium models).
- Muellbauer, J. 1978, Macrotheory vs. macroeconometrics : the treatment of "Disequilibrium" in macromodels, Discussion paper n°50 : 42.
- Quandt, R.E, 1982, Econometric disequilibrium models, *Econometric Reviews* 1 : 1-63.
- Richard, J.F. 1980, C-type distributions and disequilibrium models, working paper, Core, Université Catholique D.E. louvain-la-Neuve, Belgium.
- Richard, J.F., 1982, Comment, *Econometric Reviews*, 1 : 81-87.
- Schmidt, P. 1976, Note on the Statistical Estimation of Parametric Frontier Production Functions, *Review of Economics and Statistics*, 58 : 238-239.
- Sneessens, H.R., 1985, Two Alternative Stochastic Specification and Estimation Methods for Quantity Rationing Models - A Monte-carlo Comparison, *European Economic Review*, 29 : 111-136.
- Tischler, A. and I.Zang 1979, A Switching Regression Method Using Inequality Conditions, *Journal of Econometrics*, 11 : 259-274.
- Weinrich, G. 1984, On the Theory of Effective Demand under Stochastic Rationing, *Journal of Economic Theory*, 34 : 95-115.
- Weinstein, M.A., 1964, The sum of values from a normal and a truncated normal distribution, *Technometrics* 6 : 104-105.