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Free vibration Behavior of Functionally Graded Nanocomposite Plates Reinforced with Linear and Nonlinear Distributed Carbon Nanotube Nanofillers in Thermal Environment

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ABSTRACT

The present study investigates the influence of temperature and carbon nanotube (CNT) distribution patterns on the free vibration behaviour of nanocomposite plates. The plate is composed of a polymethyl methacrylate (PMMA) matrix reinforced with CNTs that are either uniformly distributed or functionally graded through the plate thickness. Both linear and nonlinear distribution patterns are considered. The analysis is performed using the First-Order Shear Deformation Theory (FSDT) combined with a layer-wise formulation. Temperature-dependent material properties are incorporated, while the effective properties of the nanocomposite are evaluated using the extended rule of mixtures. The accuracy of the developed model is verified through comparison with results available in the literature. A detailed parametric study is carried out to investigate the effects of temperature variation, CNT weight fraction, and distribution patterns on the dimensionless natural frequencies of functionally graded CNT-reinforced composite plates. The results indicate that increasing temperature leads to a reduction in natural frequencies due to the degradation of structural stiffness. Furthermore, the distribution pattern of CNTs significantly influences the vibrational response, with certain graded configurations providing improved stiffness and higher natural frequencies compared to the uniform distribution.

1 Introduction

Functionally Graded Materials (FGMs) allow tailoring properties by combining different materials, often ceramics and metals. Following Shen's work [1], CNTs became popular as reinforcements in FG polymer composites due to their remarkable properties [2]. These composites are widely used in sensitive engineer sectors [3–5]. However, beyond improving material properties, understanding environmental effects (especially temperature) is crucial, as elevated temperatures degrade strength, stiffness, and stability. Several studies have explored how this factor affect the vibration. A general solution was presented by Zeighami et al. [6] to calculate thermal stresses around elliptical hole in unsymmetric

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FG-CNTRC plates. Kang et al. [7] developed an analytical method grounded in 3D elasticity theory to assess the free vibration of FG plates with arbitrary boundary conditions in thermal environments. Uymaz et al. [8] conducted a three-dimensional thermal vibration analysis of FG-CNT reinforced plates under uniform, linear, and sinusoidal temperature distributions. Their findings demonstrated that both the amount and distribution type of temperature significantly influence the frequencies. F. Ebrahimi et al. [9] studied the thermal vibration analysis of the graphene-oxide powder-reinforced nanocomposite plates, once the plate is embedded on the viscoelastic substrate. F. Mohd and M. Talha [10] studied the vibrational behavior of functionally graded porous arches reinforced with graphene platelets under thermal environment. S. Qaderi et al. [11] investigated the free vibration analysis of graphene platelets-reinforced composites plates in thermal environment based on higher-order shear deformation plate theory. Haghighi and Alibeigloo [12] conducted thermal buckling and free vibration analysis of a rectangular plate reinforced with CNT within the framework of third-order shear deformation theory.

In the present work, a numerical investigation is undertaken to evaluate the natural frequencies of FG-CNTRC laminated plates under various thermal conditions. Four different CNT distribution patterns are considered in this study. The FG patterns are using the piecewise approach, and the effective mechanical properties of the nanocomposites are derived through the extended rule of mixture. The governing equations are formulated using the FSDT and subsequently solved numerically via an internally developed finite element method (FEM) code. Following rigorous validation of the code's accuracy, extensive results are presented to survey the effects of temperature and nanofillers distribution patterns (both linear and nonlinear) on the free vibrational behavior of nanocomposite plates using a layer-wise formulation approach.

2 Theoretical formulation

2.1 Geometry of FG-CNTRC plate

Considering a CNTRC plate composed of a polymer matrix and CNT fillers with dimensions of length a , width b , and thickness h within a Cartesian coordinate system (x, y, z) , as depicted in Figure 1. The CNT volume fraction can be distributed across the nanocomposite plate's thickness uniformly (UD) or in a functionally graded (FG) patterns.

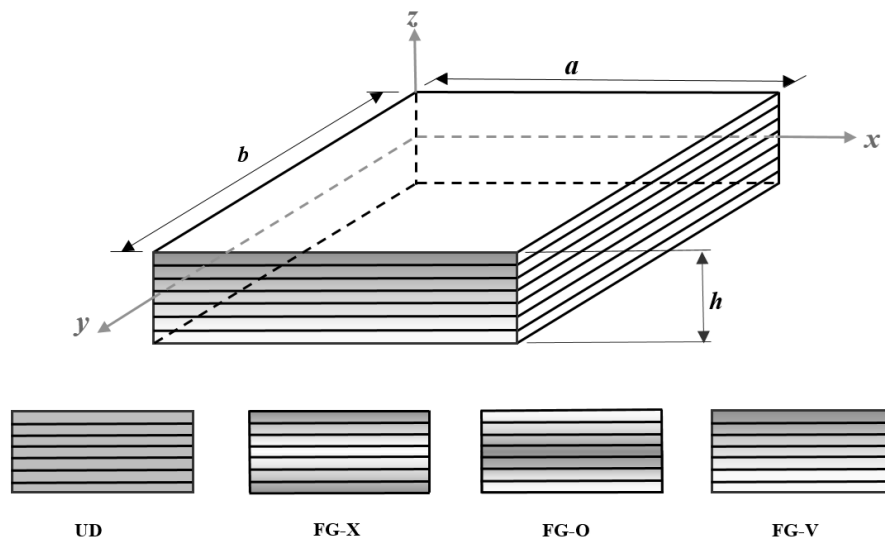


Fig. 1 – Geometry and distribution patterns of CNTs in composite plate

The mathematical expressions of the CNT volume fraction ($V_{CNT}^{(k)}$) of the k^{th} layer ($k = 1, 2, 3, \dots, N_L$) for UD and FG patterns are given as [13]:

$$\text{UD: } V_{CNT}^{(k)} = V_{CNT}^* \quad (1a)$$

$$\text{FG-X: } V_{CNT}^{(k)} = (1 + P_{in}) \left(\frac{|2k - N_L - 1|}{N_L} \right)^{P_{in}} V_{CNT}^* \quad (1b)$$

$$\text{FG-V: } V_{CNT}^{(k)} = (1 + P_{in}) \left(\frac{k - 0.5}{N_L} \right)^{P_{in}} V_{CNT}^* \quad (1c)$$

$$\text{FG-O: } V_{CNT}^{(k)} = (1 + P_{in}) \left(1 - \frac{|2k - N_L - 1|}{N_L} \right)^{P_{in}} V_{CNT}^* \quad (1d)$$

Where, N_L is the number of layers constituting the plate, and V_{CNT}^* is the CNT volume fraction given by:

$$V_{CNT}^* = \frac{w^{CNT}}{(1 - w^{CNT})(\rho^{CNT} / \rho^m) + w^{CNT}} \quad (2)$$

In which, w^{CNT} is the CNTs weight fraction, ρ^{CNT} and ρ^m are the mass densities of CNTs and matrix, respectively.

2.2 Material properties of CNTRC laminated plate

According to the extended rule of mixture, the effective Young's modulus in longitudinal E_{11}^{CNT} and transverse E_{22}^{CNT} directions and shear modulus $G_{12}^{(k)}$ of the k^{th} CNTRC layer are expressed as follows [14–16]:

$$E_{11}^{(k)} = \eta_1 V_{CNT}^{(k)} E_{11}^{CNT} + V_m^{(k)} E^m \quad (3a)$$

$$\frac{\eta_2}{E_{22}^{(k)}} = \frac{V_{CNT}^{(k)}}{E_{22}^{CNT}} + \frac{V_m^{(k)}}{E^m} \quad (3b)$$

$$\frac{\eta_3}{G_{12}^{(k)}} = \frac{V_{CNT}^{(k)}}{G_{12}^{CNT}} + \frac{V_m^{(k)}}{G^m} \quad (3c)$$

η_1 , η_2 and η_3 are the CNT efficiency parameters that are determined by the mixing law of CNTs material properties obtained from the molecular dynamics simulation, for $V_{CNT}^* = 0.12$, $\eta_1 = 0.137$, $\eta_2 = 1.022$ and $\eta_3 = 0.715$, for $V_{CNT}^* = 0.17$, $\eta_1 = 0.142$, $\eta_2 = 1.626$ and $\eta_3 = 0.715$, and for $V_{CNT}^* = 0.28$, $\eta_1 = 0.141$, $\eta_2 = 1.585$ and $\eta_3 = 1.109$ [17].

According to the same micromechanics model (i.e. extended rule of mixture) the Poisson's ratio (ν_{12}^{CNT}) and mass density (ρ_{12}^{CNT}) of the k^{th} CNTRC layer are given as follows [18, 19]:

$$\nu_{12}^{(k)} = V_{CNT}^* \nu_{12}^{CNT} + V_m^{(k)} \nu^m \quad (3d)$$

$$\rho_{12}^{(k)} = V_{CNT}^* \rho_{12}^{CNT} + V_m^{(k)} \rho^m \quad (3e)$$

Thermal ($\alpha^{(k)}$) expansion coefficients in longitudinal ($\alpha_{11}^{(k)}$) and transverse ($\alpha_{22}^{(k)}$) directions of the k^{th} CNTRC laminated plate layer are assumed to be graded in the thickness direction and are expressed as [20]:

$$\alpha_{11}^{(k)} = V_{CNT}^{(k)} \alpha_{11}^{CNT} + V_m^{(k)} \alpha^m \quad (4a)$$

$$\alpha_{22}^{(k)} = (1 + \nu_{12}^{CNT}) V_{CNT}^{(k)} \alpha_{22}^{CNT} + (1 + \nu^m) V_m^{(k)} \alpha^m - \nu_{12}^{CNT} \alpha_{11}^{(k)} \quad (4b)$$

2.3 Governing Equations

Displacement fields of the CNTRC plate are defined using the FSDT, as follows:

$$\begin{cases} u(x, y, z) = u_0(x, y) + z\theta_x(x, y) \\ v(x, y, z) = v_0(x, y) + z\theta_y(x, y) \\ w(x, y, z) = w_0(x, y) \end{cases} \quad (5)$$

Where, $u_0(x, y)$, $v_0(x, y)$ and $w_0(x, y)$ are displacements components of a point situated on the mid-plane of the plate. $\theta_x(x, y)$ and $\theta_y(x, y)$ refer respectively to the rotations of the transverse normal about the x and y axes.

The strain-displacement relationship can be expressed as follows:

$$\begin{Bmatrix} \varepsilon_x^{(k)} \\ \varepsilon_y^{(k)} \\ \gamma_{xy}^{(k)} \\ \gamma_{yz}^{(k)} \\ \gamma_{xz}^{(k)} \end{Bmatrix} = \begin{Bmatrix} \varepsilon_x^{(0)} \\ \varepsilon_y^{(0)} \\ \gamma_{xy}^{(0)} \\ \gamma_{yz}^{(0)} \\ \gamma_{xz}^{(0)} \end{Bmatrix} + \begin{Bmatrix} \varepsilon_x^{(1)} \\ \varepsilon_y^{(1)} \\ \gamma_{xy}^{(1)} \\ \gamma_{yz}^{(1)} \\ \gamma_{xz}^{(1)} \end{Bmatrix} = \begin{Bmatrix} u_{0,x} \\ v_{0,y} \\ u_{0,y} + v_{0,x} \\ w_{0,y} + \theta_y \\ w_{0,x} + \theta_x \end{Bmatrix} + \begin{Bmatrix} \theta_{x,x} \\ \theta_{y,y} \\ \theta_{x,y} + \theta_{y,x} \\ 0 \\ 0 \end{Bmatrix} \quad (6)$$

The linear constitutive equations of the nanocomposite plates are introduced as:

$$\begin{Bmatrix} \sigma_x^{(k)} \\ \sigma_y^{(k)} \\ \tau_{xy}^{(k)} \\ \tau_{yz}^{(k)} \\ \tau_{xz}^{(k)} \end{Bmatrix} = \begin{bmatrix} Q_{11}^{(k)} & Q_{12}^{(k)} & 0 & 0 & 0 \\ Q_{12}^{(k)} & Q_{22}^{(k)} & 0 & 0 & 0 \\ 0 & 0 & Q_{44}^{(k)} & 0 & 0 \\ 0 & 0 & 0 & Q_{55}^{(k)} & 0 \\ 0 & 0 & 0 & 0 & Q_{66}^{(k)} \end{bmatrix} \begin{Bmatrix} \varepsilon_x^{(k)} \\ \varepsilon_y^{(k)} \\ \gamma_{xy}^{(k)} \\ \gamma_{yz}^{(k)} \\ \gamma_{xz}^{(k)} \end{Bmatrix} - \Delta T \begin{Bmatrix} \alpha_{11}^{(k)} \\ \alpha_{22}^{(k)} \\ 0 \\ 0 \\ 0 \end{Bmatrix} \quad (7)$$

Where, ΔT is the temperature increment from the reference temperature T_0 ($\Delta T = T - T_0$), and $T_0 = 300K$.

$$Q_{11}^{(k)} = \frac{E_{11}^{(k)}}{1 - \nu_{12}^{(k)}\nu_{21}^{(k)}}, \quad Q_{22}^{(k)} = \frac{E_{22}^{(k)}}{1 - \nu_{12}^{(k)}\nu_{21}^{(k)}}, \quad Q_{12}^{(k)} = \frac{\nu_{21}^{(k)}E_{11}^{(k)}}{1 - \nu_{12}^{(k)}\nu_{21}^{(k)}}, \quad Q_{44}^{(k)} = G_{23}^{(k)}, \quad Q_{55}^{(k)} = G_{13}^{(k)}, \quad Q_{66}^{(k)} = G_{12}^{(k)}$$

The axial forces (N_x, N_y, N_{xy}), shear forces (Q_x, Q_y) and the moment resultants (M_x, M_y, M_{xy}) are:

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} = \begin{bmatrix} A_{ij} \end{bmatrix} \begin{Bmatrix} \varepsilon_x^{(0)} \\ \varepsilon_y^{(0)} \\ \varepsilon_{xy}^{(0)} \end{Bmatrix} + \begin{bmatrix} B_{ij} \end{bmatrix} \begin{Bmatrix} \varepsilon_x^{(1)} \\ \varepsilon_y^{(1)} \\ \varepsilon_{xy}^{(1)} \end{Bmatrix} - \begin{Bmatrix} N_x^{Th} \\ N_y^{Th} \\ N_{xy}^{Th} \end{Bmatrix} \quad (8a)$$

$$\begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} B_{ij} \end{bmatrix} \begin{Bmatrix} \varepsilon_x^{(0)} \\ \varepsilon_y^{(0)} \\ \varepsilon_{xy}^{(0)} \end{Bmatrix} + \begin{bmatrix} D_{ij} \end{bmatrix} \begin{Bmatrix} \varepsilon_x^{(1)} \\ \varepsilon_y^{(1)} \\ \varepsilon_{xy}^{(1)} \end{Bmatrix} - \begin{Bmatrix} M_x^{Th} \\ M_y^{Th} \\ M_{xy}^{Th} \end{Bmatrix} \quad (8b)$$

$$\begin{Bmatrix} Q_x \\ Q_y \end{Bmatrix} = \frac{5}{6} \begin{bmatrix} F_{ij} \end{bmatrix} \begin{Bmatrix} \gamma_{yz}^{(0)} \\ \gamma_{xz}^{(0)} \end{Bmatrix}, \quad (i, j = 1, 2, 6) \quad (8c)$$

Where, $(N_x^{Th}, N_y^{Th}, N_{xy}^{Th})$ and $(M_x^{Th}, M_y^{Th}, M_{xy}^{Th})$ are in-plane thermal force resultants and thermal moment resultants, respectively.

The stiffness components of the plate are:

$$(A_{ij}, B_{ij}, D_{ij}) = \sum_{k=1}^{N_L} \int_{Z_k}^{Z_{k+1}} Q_{ij}^{(k)}(1, z, z^2) dz, (i, j = 1, 2, 6) \quad (9a)$$

$$F_{ij} = \sum_{k=1}^{N_L} \int_{Z_k}^{Z_{k+1}} Q_{ij}^{(k)} dz, (i, j = 1, 2, 6) \quad (9b)$$

2.4 Governing Equations

The free vibration problem of the CNTRC plate is solved using a nine node isoparametric quadrilateral element with five degrees of freedom (three displacements u, v, w and two rotations θ_x, θ_y).

The elementary displacements are defined through the following expression:

$$\begin{Bmatrix} u_0 \\ v_0 \\ w_0 \\ \theta_x \\ \theta_y \end{Bmatrix} = \sum_{i=1}^9 \begin{bmatrix} N_i & 0 & 0 & 0 & 0 \\ 0 & N_i & 0 & 0 & 0 \\ 0 & 0 & N_i & 0 & 0 \\ 0 & 0 & 0 & N_i & 0 \\ 0 & 0 & 0 & 0 & N_i \end{bmatrix} \begin{Bmatrix} u_{0i} \\ v_{0i} \\ w_{0i} \\ \theta_{xi} \\ \theta_{yi} \end{Bmatrix} \quad (10)$$

Written Equation (10) in a matrix form:

$$\{\delta\} = [N]\{\delta_e\} \quad (11)$$

The elementary strain field is written as:

$$\begin{Bmatrix} \varepsilon_{0x} \\ \varepsilon_{0y} \\ \gamma_{0xy} \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \\ \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix} = \sum_{i=1}^9 \begin{bmatrix} \partial N_i / \partial x & 0 & 0 & 0 & 0 \\ 0 & \partial N_i / \partial y & 0 & 0 & 0 \\ \partial N_i / \partial y & \partial N_i / \partial x & 0 & 0 & 0 \\ 0 & 0 & 0 & \partial N_i / \partial x & 0 \\ 0 & 0 & 0 & 0 & \partial N_i / \partial y \\ 0 & 0 & 0 & \partial N_i / \partial y & \partial N_i / \partial x \\ 0 & 0 & \partial N_i / \partial x & N_i & 0 \\ 0 & 0 & \partial N_i / \partial y & 0 & N_i \end{bmatrix} \begin{Bmatrix} u_{0i} \\ v_{0i} \\ w_{0i} \\ \theta_{xi} \\ \theta_{yi} \end{Bmatrix} \quad (12)$$

Rewriting Equation (12) in compact matrix form as follows:

$$\{\varepsilon\} = [B]\{\delta_e\} \quad (13)$$

The governing equation of motion for free vibration analysis of a nanocomposite plates under different thermal environments is written as:

$$[M]\{\ddot{\delta}\} + ([K] - [K_{Th}])\{\delta\} = 0 \quad (14)$$

$[K_e]$, $[M_e]$ and $[K_{Th}]$ are elementary stiffness matrices, elementary mass, and elementary initial stress stiffness matrices, respectively, given as follows:

$$[M_e] = \iint [N]^T [\bar{m}] [N] dx dy \quad (16)$$

$$\begin{bmatrix} K_{Hy_e} \end{bmatrix} = \iint [G]^T [S] [G] dx dy \quad (17)$$

Where, $[C]$ is the elasticity matrix and $[\bar{m}]$ is the inertia matrix, expressed as:

$$[C] = \begin{bmatrix} A & B & 0 \\ B & D & 0 \\ 0 & 0 & F \end{bmatrix}, [\bar{m}] = \begin{bmatrix} I_0 & 0 & 0 & I_1 & 0 \\ 0 & I_0 & 0 & 0 & I_1 \\ 0 & 0 & I_0 & 0 & 0 \\ I_1 & 0 & 0 & I_2 & 0 \\ 0 & I_1 & 0 & 0 & I_3 \end{bmatrix} \quad (18)$$

Note that, $I_i = \sum_{k=1}^{N_L} \int_{Z_{k-1}}^{Z_k} Z^i \rho^k dz$, ($i=0,1,2$) and ρ^k is the material density of the k^{th} layer.

[G] represents the shape function operating matrix.

$$[G] = \sum_{i=1}^9 \begin{bmatrix} N_{i,x} & 0 & 0 & 0 & 0 \\ N_{i,y} & 0 & 0 & 0 & 0 \\ 0 & N_{i,x} & 0 & 0 & 0 \\ 0 & N_{i,y} & 0 & 0 & 0 \\ 0 & 0 & N_{i,x} & 0 & 0 \\ 0 & 0 & N_{i,y} & 0 & 0 \\ 0 & 0 & 0 & N_{i,x} & 0 \\ 0 & 0 & 0 & N_{i,y} & 0 \\ 0 & 0 & 0 & 0 & N_{i,x} \\ 0 & 0 & 0 & 0 & N_{i,y} \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (19)$$

[S] matrix contains in-plane initial internal force, moment resultants, and transverse shear resultant terms.

[illegible]

Where,

$$\begin{aligned} S_{11} = S_{33} = S_{55} = N_x^i, \quad S_{22} = S_{44} = S_{66} = N_y^i, \quad S_{12} = S_{43} = S_{65} = N_{xy}^i, \quad S_{77} = S_{99} = N_x^i h^2 / 12, \quad S_{88} = S_{1010} = N_y^i h^2 / 12, \\ S_{87} = S_{109} = N_{xy}^i h^2 / 12, \quad -S_{73} = S_{91} = M_x^i, \quad -S_{74} = -S_{83} = S_{92} = S_{101} = M_{xy}^i, \quad -S_{84} = S_{102} = M_y^i, \quad -S_{113} = S_{121} = Q_x^i, \\ -S_{114} = S_{122} = Q_y^i \end{aligned}$$

3 Numerical results and discussions

3.1 Comparative study

Table 1 compares the non-dimensional natural frequencies of CNT-reinforced plates with UD, FG-V, and FG-O distributions at 300 K, 500 K, and 700 K, between the present results and Refs. [21] and [17]. A very good agreement is observed at all temperatures, confirming the validity of the model.

Table 1 - Comparison of non-dimensional frequencies of CNTRC plates at different temperatures

Temperature	UD			FG-V			FG-O		
	Present	Ref. [21]	Ref. [17]	Present	Ref. [21]	Ref. [17]	Present	Ref. [21]	Ref. [17]
300 K	12.2159	12.1261	12.2696	11.3277	11.3095	11.3074	10.5669	10.4535	---
	16.7212	16.5545	16.8071	16.2493	16.2611	16.1790	15.5529	15.3530	---
500 K	10.9703	10.9644	11.0402	9.5801	10.2442	10.2068	9.6358	9.5378	---
	14.5695	14.4941	14.7052	13.7348	14.2264	14.1873	13.6289	13.4627	---
700 K	9.2597	9.2518	9.3611	6.9236	8.7751	8.8190	8.3535	8.2728	---
	11.5310	11.5159	12.0533	9.9871	8.7751	11.6928	11.2314	11.1033	---

3.2 Parametric study

Table 2 summarizes the effect of the aspect ratio a/b on the non-dimensional frequency parameter $\bar{\omega}$ of CCCC CNTRC plates under different temperature conditions. Four CNT distribution patterns (UD, FG-V, FG-O, and FG-X), three aspect ratios ($a/b=1, 1.5, 2$), and three temperature levels are examined. The power-law index and CNT volume fraction are fixed at 1 and 0.12, respectively. The results clearly indicate that the natural frequency decreases with increasing temperature for all distribution types due to thermal softening. Among the configurations, the FG-X pattern consistently yields the highest frequency values, reflecting its superior stiffness enhancement. Furthermore, square plates ($a/b = 1$) exhibit greater resistance to temperature-induced frequency reduction compared to rectangular plates ($a/b=1.5$ and $a/b=2$), highlighting the significant role of geometric proportions in the dynamic stiffness behaviour of the plates.

Table 2 - Effect of aspect ratio, distribution patterns and temperature environments on the first non-dimensional frequency

Distribution	a/ b = 1			a/b = 1.5			a/b = 2		
	300K	400K	500K	300K	400K	500K	300K	400K	500K
UD	15.8763	14.8527	13.7297	25.5465	24.2646	22.8221	36.1066	35.2298	34.2757
FG-V	15.5641	14.3663	13.0086	23.8247	22.117	20.1268	31.2603	27.2250	22.0165
FG-O	15.1409	14.2250	13.2138	22.2923	21.3418	20.2756	27.8736	27.2617	26.6360
FG-X	16.2766	15.1990	14.0210	27.2555	25.7656	24.0979	41.7429	40.5849	39.2883

Table 3 presents the influence of the CNT volume fraction on the non-dimensional frequency parameter $\bar{\omega}$ of square CNTRC plates with CCCC boundary conditions under thermal environments. Three values of the CNT volume fraction are considered while maintaining a width-to-thickness ratio of 20. The results clearly show that increasing the CNT volume fraction leads to a significant rise in $\bar{\omega}$, reflecting the enhancement of the effective stiffness. In contrast, the frequency

parameter decreases as temperature increases due to thermal-induced material softening. Among the considered distribution patterns, the FG-X configuration provides the highest stiffness, followed by the uniform distribution (UD), whereas the FG-O distribution yields the lowest stiffness response.

Table 3 - Effect of volume fraction, distribution patterns and temperature environments on the first non-dimensional frequency

Distribution	$V_{CNT}^* = 0.12$			$V_{CNT}^* = 0.17$			$V_{CNT}^* = 0.28$		
	300K	400K	500K	300K	400K	500K	300K	400K	500K
UD	25,5465	24,2646	22,8221	15,1756	14,5136	13,7761	10,8764	10,3826	9,8469
FG-V	23,8247	22,1173	20,1268	13,9422	12,8705	11,6251	10,1227	9,2550	8,2484
FG-O	22,2923	21,3418	20,2756	12,8908	12,3667	11,7990	9,4291	8,9794	8,5022
FG-X	27,2555	25,7656	24,0979	16,6062	15,8206	14,9398	11,9351	11,3799	10,7692

This final study aims to examine the impact of power law index ' P_{in} ' on the $\bar{\omega}$ parameter of a fully clamped square CNTRC plates subjected to various temperature environments. Different values of P_{in} (0, 0.4, 1, and 1.8) are considered. The data provided in Table 4 clearly indicate that as the P_{in} increases, the $\bar{\omega}$ parameter experiences an increase for the plates with FG-X configuration and decreases for the plates with FG-V and FG-O patterns. Consequently, higher P_{in} values result in an increased concentration of CNT nanofillers in the middle surface for FG-O pattern, in the upper surface for FG-V pattern and in both the top and bottom surfaces for the FG-X pattern. Additionally, it is noteworthy that across all P_{in} values, the dimensionless frequencies decrease with rising temperatures.

Table 4 - Effect of power law index, distribution patterns and temperature environments on the first non-dimensional frequency

P_{in}	300K			400K			500K		
	FG-V	FG-O	FG-X	FG-V	FG-O	FG-X	FG-V	FG-O	FG-X
0	20,6675	20,6675	20,6675	19,8101	19,8101	19,8101	18,8424	18,8424	18,8424
0.4	19,9638	19,21	21,7063	18,7184	18,4733	20,753	17,2785	17,6461	19,6762
1	18,777	17,2626	22,6456	17,2906	16,6557	21,5998	15,5319	15,9849	20,4197
1.8	17,3827	15,2998	23,3935	15,6782	14,7854	22,2728	13,599	14,2307	21,0101

4 Conclusion

This study examined the free vibration behaviour of functionally graded carbon nanotube-reinforced composite (FG-CNTRC) plates subjected to thermal environments. The analysis was performed using the First-Order Shear Deformation Theory combined with a layer-wise formulation, while the effective material properties were evaluated through the extended rule of mixtures considering temperature-dependent behaviour. Various distribution patterns of carbon nanotubes across the plate thickness were investigated in order to assess their influence on the vibrational response.

The numerical results demonstrated that temperature plays a significant role in the dynamic behaviour of FG-CNTRC plates. An increase in temperature leads to a reduction in the dimensionless natural frequencies due to the degradation of the overall stiffness of the nanocomposite structure. In addition, the aspect ratio was found to strongly influence the vibrational response, where larger aspect ratios result in lower stiffness and consequently lower natural frequencies. The distribution pattern of CNTs across the plate thickness also has a considerable impact on the dynamic characteristics. Increasing the power-law index P_{in} leads to higher natural frequencies for the FG-X configuration, while the opposite trend

is observed for the FG-V and FG-O patterns. These results highlight the importance of the CNT distribution strategy in tailoring the vibrational performance of nanocomposite plates.

Future work may extend the present study by considering additional factors such as hygrothermal environments, nonlinear vibration behaviour, different boundary conditions, and more advanced material models for CNT-reinforced composites. Furthermore, the proposed formulation could be applied to the analysis of other nanocomposite structural components such as shells or sandwich structures.

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