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Research Paper

Evaluation of dynamic responses of concrete gravity dam considering dam-reservoir-foundation interaction with inclined absorptive reservoir bottom

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ABSTRACT

The present paper shows the dynamic responses of concrete gravity dam considering inclined reservoir bed. Responses of gravity dam influenced by the adjacent reservoir and soil foundation. Interaction between dam, reservoir and foundation can give the authentic idea about the responses of the gravity dam subjected to dynamic excitation. Hydrodynamic pressure originated at the face of the gravity dam due to dynamic force. Bottom absorption and geometrical profile of the reservoir influence the behavior of the reservoir along with the adjacent gravity dam. In the present study, dam-reservoir-foundation system has been analyzed by direct coupling approach due to harmonic excitation including the fluid-structure and soil-structure interaction simultaneously. Reservoir bottom is considered as inclined and absorptive. Geometry of the dam, reservoir and foundation are discretized using finite element technique. Suitable non-reflective boundary conditions are applied along the truncated surface of the reservoir and soil foundation. Dynamic analysis of the dam, reservoir and foundation coupled system has been performed by applying Newmarks' technic. Responses of the reservoir, dam and foundation have been observed for different inclination of reservoir bottom.

1 Introduction

Gravity dam is a massive structure that holds huge amount of water for civilization. Hydrodynamic pressure incidence on the dam face influences dynamic responses of the structure. Effect of soil foundation below the dam is also important for the analysis and design of gravity dam. Thus, interaction between dam, reservoir and foundation should be included for dynamic analysis of concrete gravity dam. Bed slope and bottom absorption of the reservoir influence the responses of the dam and reservoir. So, responses of the gravity dam are observed in present study for inclined absorptive reservoir bottom.

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Analysis of gravity dam and reservoir have been conducted by several researchers. Among them, Saini et al. [1] analysed the fluid-structure system using the finite element technique. Bathe and Hahn [2] performed the analysis of fluid-structure system assuming the fluid as non-viscous. The fluid domain was discretized by finite element with lumped or consistent mass idealization. Chwang [3] presented the effect of vertical and horizontal ground excitation on the development of hydrodynamic pressure in finite reservoir. An approximate method was suggested by Muller [4] in case of fluid-structure interaction problem. Eigenvalue solution for the coupled system for compressible and incompressible fluid was performed to compare the results. Humar and Roufaei [5] used finite element technique to estimate the hydrodynamic pressure adjacent to gravity dam subjected to horizontal exciting force and proposed a new radiation boundary condition that gives better results over the Sommerfeld boundary condition. Das and Aki [6] proposed a mathematical model for vibration analysis of gravity dam considering the dam-reservoir interaction. Fluid was assumed as compressible and the dam was assumed as elastic. Fenves and Loli [7] suggested a procedure that involved cavitation of fluid and nonlinear model of structure for analysis of fluid-structure coupled system. Chen and Taylor [8] suggested a solution to solve the fluid-structure systems using finite element. Bougacha and Tassoulas [9] evaluated responses of dam subjected to dynamic excitations and also determined the effect of sedimentary material at the reservoir bottom. Li et al. [10] suggested an exact far boundary condition for analysis of the fluid-structure coupled problem. A closed-form solution was suggested by Bouaanani et al. [11] for the determination of hydrodynamic pressure on the gravity dam subjected to earthquake excitations. The dam, water and foundation were assumed as linear and two-dimensional. Maity and Bhattacharya [12] presented a novel procedure to solve fluid-structure problem with finite element technique. A new truncation boundary was suggested by Gogoi and Maity [13] for evaluation of hydrodynamic pressure. They included the effect of the absorptive bottom in the analysis. Papazafeiropoulos et al. [14] presented a numerical simulation for the determination of responses of dam-water-foundation systems using finite element. Neya and Ardeshtir [15] solved dam-reservoir problem by performing the dynamic analysis in frequency domain. Mandal and Maity [16] evaluated the responses of gravity dam considering the ageing effect of concrete. Yang et al. [17] presented the formulation of different boundary conditions to solve the Soil-Structure problem using ABAQUAS software. A mesh-free numerical method was proposed by Behrooz and Vaghefi [18] for the determination of earthquake responses of dam-reservoir-foundation systems. Khiavi and Sari [19] suggested an analytical technique for the determination of hydrodynamic pressure. The reservoir was assumed as rectangular and harmonic excitation was applied vertically at the base of the foundation. Dastgerdi et al. [20] developed a two-dimensional modelling for dam-reservoir-foundation arrangement using ABAQUS software. Effect of change of cross-section of the dam and face angle at the upstream side was examined. Rasa et al. [21] evaluates dynamic behaviour of dam-reservoir-foundation interaction problem using efficient finite element model using Fortran 90 programming language. Warin [22] used machine learning methods for optimization of storage of reservoir.

Sharma et al. [23] evaluated the seismic responses of concrete gravity dam with dam-reservoir-foundation interaction using finite element technic.

Very few literatures have been found for solving the dam-reservoir problem with the effect of the inclined reservoir bottom using direct coupling approach. In this study hydrodynamic pressure and responses of gravity dam and foundation has been observed considering inclined reservoir bed by applying harmonic excitation. Here dam, reservoir and foundation are considered as a single system and the whole system is analysed by direct coupling approach. Dam-reservoir and dam-foundation interaction has been included simultaneously to obtain the responses of structure. Fluid has been considered compressible and non-viscous. Effect of surface wave is neglected and reservoir bed is considered absorptive. Length of the reservoir is truncated to make it finite for saving the computational time. Appropriate non reflective boundary conditions are implemented along truncated surface of fluid and soil domain. Two-dimensional geometry of dam-reservoir-foundation system has been discretised using finite element. Pressure is assumed as unknown parameter in fluid domain and displacement is considered as nodal unknown for dam and foundation. Harmonic excitation has been applied to observe the responses of dam, reservoir and foundation. A MATLAB code is developed for the analysis purpose. The dynamic equilibrium equation is solved using newmarks' integration technique.

2 Theoretical formulation

The dynamic equilibrium equation for the concrete gravity dam subjected to external force is govern by the following equation:

$$[M_d]\{\ddot{u}_d\} + [C_d]\{\dot{u}_d\} + [K_d]\{u_d\} = \{F_d\} \quad (1)$$

Here $[M_d]$, $[C_d]$ and $[K_d]$ present mass matrix, damping matrix and stiffness matrix of dam. $\{\ddot{u}_d\}$, $\{\dot{u}_d\}$, $\{u\}$ are nodal acceleration, velocity and displacement. $\{F_d\}$ is the nodal force vector. The stiffness matrix can be determined as below.

$$[K_d] = \iint [B]^T [D] [B] |J| d\xi d\eta \quad (2)$$

$[B]$ presents the strain displacement matrix, $[D]$ is elasticity matrix, J present Jacobian matrix. Mass matrix of the structure can be calculated as follows.

$$[M_d] = \int [N]^T \rho [N] d\xi d\eta \quad (3)$$

Here ρ denotes the density of material of the structure. N denotes the shape function of structure. Rayleigh damping model is used for the structure and can be shown as below.

$$[C_d] = a'[M_d] + b'[K_d] \quad (4)$$

a' and b' are damping constants and their relationship can be expressed by the following equation.

$$\xi' = \frac{1}{2} \left(a' \omega + \frac{b'}{\omega} \right) \quad (5)$$

Where

$$a' = \frac{2(\xi_2' \omega_2 - \xi_1' \omega_1)}{(\omega_2^2 - \omega_1^2)} \quad (6a)$$

$$b' = \frac{2\omega_1 \omega_2 (\xi_2' \omega_1 - \xi_1' \omega_2)}{(\omega_2^2 - \omega_1^2)} \quad (6b)$$

ξ_1' and ξ_2' are damping ratios for two different frequencies ω_1 and ω_2 .

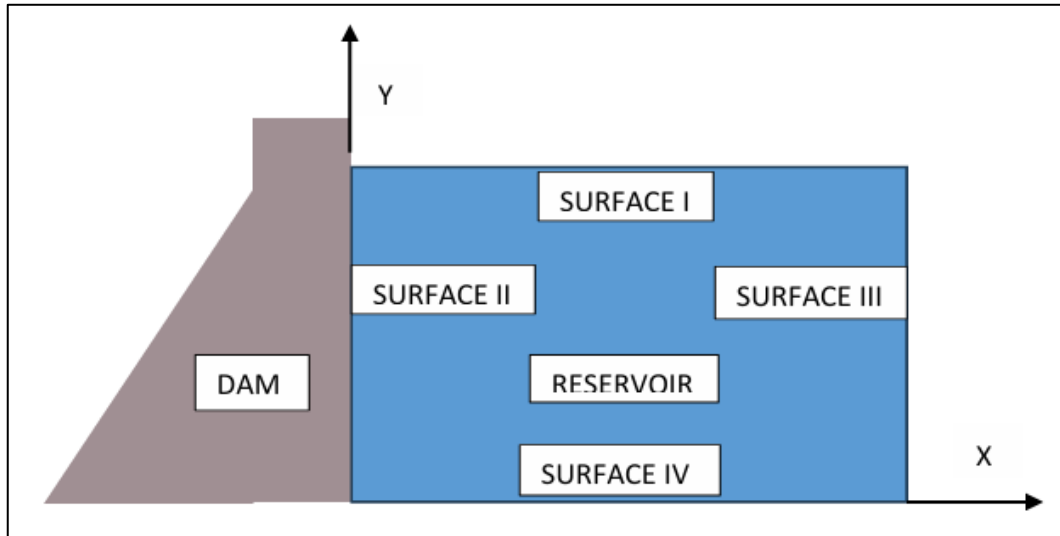


Fig. 1 – Schematic diagram of dam-reservoir system

The Equation for compressible fluid is given below.

$$\nabla^2 p(x, y, t) = \frac{1}{c^2} \ddot{p}(x, y, t) \quad (7)$$

Here ∇_2 is a two-dimensional Laplacian operator. Fluid is assumed as compressible and non-viscous. The reservoir domain has been discretized by two-dimensional eight-node finite element. p is hydrodynamic pressure considered to be nodal variable of fluid region. c denotes acoustic velocity in water. Following boundary conditions should be maintained at different reservoir surfaces.

Surface I:

At the top surface of reservoir, the following equation to be considered with effect of surface wave.

$$\frac{1}{g} \ddot{p} + \frac{\partial p}{\partial y} = 0 \quad (8)$$

surface II:

At fluid-structure interface, the following expression has to satisfy.

$$\frac{\partial p}{\partial n}(0, y, t) = \rho_f a e^{i\omega t} \quad (9)$$

Here $a e^{i\omega t}$ denotes the horizontal ground acceleration. ρ_f is density of the water.

Surface III:

Along the fluid bed, considering bottom absorption effect, the numerical expression is given below.

$$\frac{\partial p}{\partial n}(x, 0, t) = i\omega q \dot{p}(x, 0, t) \quad (10)$$

$$q = \frac{1}{c} \left(\frac{1-\alpha}{1+\alpha} \right) \quad (11)$$

α is the reflection coefficient of the reservoir bottom

Surface IV:

Along the truncated face of the fluid domain, following equation is to be satisfied.

$$\frac{\partial p}{\partial n} = \left(\xi_m - \frac{1}{c} \right) \dot{p} \quad (12)$$

Here ξ_m is calculated as suggested by I. Gogoi and D. Maity [13].

$$\zeta_m = - \frac{i \sum_{m=1}^{\infty} \frac{\lambda_m^2 I_m}{\beta_m} e^{(-k_m x)} (\psi_m)}{\Omega c \sum_{m=1}^{\infty} \frac{\lambda_m^2 I_m}{\beta_m k_m} e^{(-k_m x)} (\psi_m)} \quad (13)$$

Here,

$$I_m = \frac{1}{H_f} \int_0^{H_f} \psi_m dy \quad (14)$$

$$\psi_m = \frac{1}{2\lambda_m} [(\lambda_m + \omega q) e^{i\lambda_m y} + (\lambda_m - \omega q) e^{-i\lambda_m y}] \quad (15)$$

$$\lambda_m^2 = \left(\frac{(2m-1)\pi}{2H_f} \right)^2 + i2\omega q/H_f \quad (16)$$

$$\beta_m = (\lambda_m^2 - \omega^2 q^2) \left(H_f - \frac{\chi}{\lambda_m^2 - \chi^2} \right) + i\omega q \quad (17)$$

$$K_m = \sqrt{\lambda_m^2 - \Omega^2} \quad (18)$$

$$\Omega = \frac{\omega}{c} \quad \text{and} \quad \chi = \frac{\omega^2}{g} \quad (19)$$

Pressure is considered as the nodal variable in the reservoir medium. Now implementing the Galerkin approach, Eq. (7) can be discretized as given below.

$$\int_{\Omega} N_{rj} \left[\nabla^2 \sum N_{ri} p_i - \frac{1}{c^2} \sum N_{ri} \ddot{p}_i \right] d\Omega = 0 \quad (20)$$

N_{rj} are interpolation functions of reservoir domain and Ω denotes fluid region. Eq. (20) may be written below after the application of Green's theorem.

$$- \int_{\Omega} \left[\frac{\partial N_{rj}}{\partial x} \sum \frac{\partial N_{ri}}{\partial x} p_i + \frac{\partial N_{rj}}{\partial y} \sum \frac{\partial N_{ri}}{\partial y} p_i \right] d\Omega - \frac{1}{c^2} \int_{\Omega} N_{rj} \sum N_{ri} d\Omega \ddot{p}_i + \int_{\Gamma} N_{rj} \sum \frac{\partial N_{ri}}{\partial n} d\Gamma p_i = 0 \quad (21)$$

Here Γ is the boundary of the reservoir domain. The final equation become as follows.

$$[J]\{\ddot{p}\} + [A]\dot{p} + [H]\{p\} = \{F_r\} \quad (22)$$

Here,

$$[J] = [\bar{J}] + \frac{1}{g} [R_f] \quad (23)$$

$$[A] = \frac{1}{c} [R_t] \quad (24)$$

$$[H] = [\bar{H}] + \zeta_m [R_t] - i\omega q [R_{fb}] \quad (25)$$

$$\{F_r\} = -\rho_f [R_{fs}] \{a\} \quad (26)$$

$$[R_f] = \sum \int_{\Gamma_f} [N_r]^T [N_r] d\Gamma \quad (27)$$

$$[R_t] = \sum \int_{\Gamma_t} [N_r]^T [N_r] d\Gamma \quad (28)$$

$$[R_{fb}] = \sum \int_{\Gamma_{fb}} [N_r]^T [N_r] d\Gamma \quad (29)$$

$$[R_{fs}] = \sum \int_{\Gamma_{fs}} [N_r]^T [T] [N_d] d\Gamma \quad (30)$$

f, f_s, f_b and t denote the top of the fluid domain, dam-reservoir boundary, reservoir bed and truncated face of the reservoir.

The soil foundation region has been discretized using eight-node isoparametric element. Mass, stiffness and damping matrix of the soil medium has been calculated in the same manner as in the structure domain. Infinite soil medium is truncated to make it finite. The well-known viscous boundary condition (Kontoe [24]) has been used in the present work.

In two-dimensional modelling (Fig.2), the absorbing boundary condition can be formulated from the following equations.

$$\sigma(s) + \rho c_p \dot{u}_{sl}(s) = 0 \quad (31)$$

$$\tau(s) + \rho c_s \dot{v}_{sl}(s) = 0 \quad (32)$$

Here, σ and τ are stresses at the truncation surface of the soil medium. u_{sl} is the normal displacement and v_{sl} is the shear displacement of soil and s denote the coordinate of artificial boundary. c_p and c_s are velocity of compression and shear wave. The normal and tangential damping coefficients c_n and c_t can be expressed as follows.

$$c_n = A_1 \rho c_p \quad (33)$$

$$c_t = A_2 \rho c_s \quad (34)$$

The coefficients c_s and c_p in two mutually orthogonal directions can be expressed below.

$$c_s = \sqrt{\frac{G}{\rho}} \quad (35)$$

$$c_p = \sqrt{\frac{E(1-\nu)}{(1+\nu)(1-2\nu)\rho}} \quad (36)$$

G is the shear modulus and can be expressed as below

$$G = \frac{1}{2(1+\nu)} \quad (37)$$

For an isotropic medium, A_1 and A_2 become as follows. Here, s can be expressed as below.

$$A_1 = \frac{8}{15\pi} (5 + 2s - 2s^2) \quad (38)$$

$$A_2 = \frac{8}{15\pi} (3 + 2s) \quad (39)$$

$$s = \sqrt{\frac{(1-2\nu)}{2(1+\nu)}} \quad (40)$$

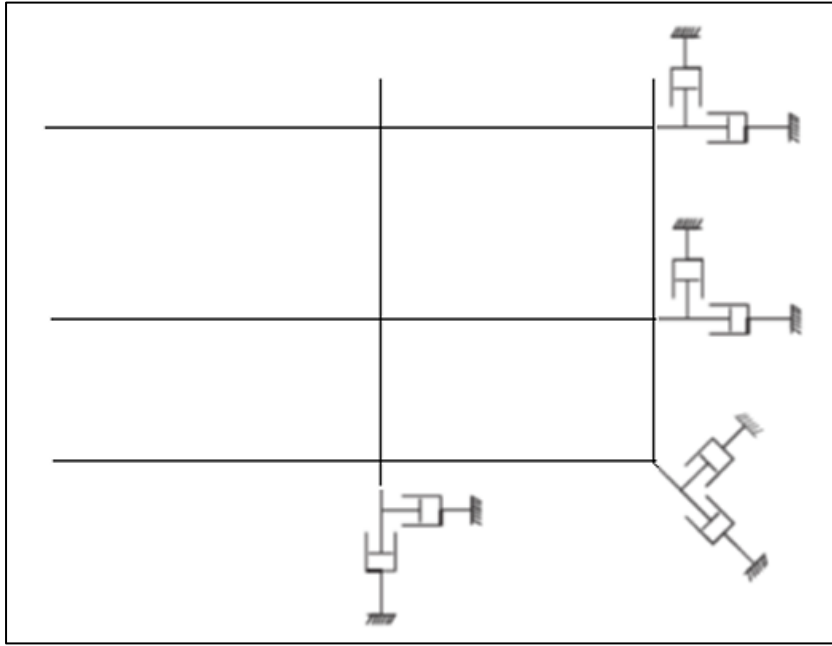


Fig. 2 - Schematic diagram of the standard viscous boundary

The dam, reservoir and soil foundation systems are coupled using direct coupling approach. The coupling equations are given below. The discrete equation of dam considering damping may be written as follows.

$$M_d \ddot{u}_d + C_d \dot{u}_d + K_d u_d - Q_p - M_d \ddot{u}_g = 0 \quad (41)$$

[Q] is the coupling term that originated at dam-reservoir interface as follows:

$$\int_{\Gamma_{dr}} N_{dr}^T n p d\Gamma = \left(\int_{\Gamma_{dr}} N_{dr}^T n N_{dr} d\Gamma \right) p = Qp \quad (42)$$

Here, N_{dr} is the interpolation function of line element along the dam-reservoir boundary. The equation of the fluid domain can be expressed below.

$$J \ddot{p} + A \dot{p} + H p - Q^T \ddot{u}_d - F_r = 0 \quad (43)$$

The pair of Eq. (42) and (43) are coupled in matrix form, which expressed the coupled equation of dam and reservoir is given below.

$$\begin{bmatrix} J & Q^T \\ 0 & M_d \end{bmatrix} \begin{Bmatrix} \ddot{p} \\ \ddot{u}_d \end{Bmatrix} + \begin{bmatrix} A & 0 \\ 0 & C_d \end{bmatrix} \begin{Bmatrix} \dot{p} \\ \dot{u}_d \end{Bmatrix} + \begin{bmatrix} H & 0 \\ -Q & K_d \end{bmatrix} \begin{Bmatrix} p \\ u_d \end{Bmatrix} = \begin{Bmatrix} F_r \\ M_d \ddot{u}_g \end{Bmatrix} \quad (44)$$

The coupled equation for dam-reservoir-foundation system can be written as follows.

$$\begin{bmatrix} J & Q_d^T & Q_c^T & 0 \\ 0 & M_{dd} & M_{dc} & 0 \\ 0 & M_{cd} & M_{cc} & M_{cf} \\ 0 & 0 & M_{fc} & M_{ff} \end{bmatrix} \begin{Bmatrix} \ddot{P} \\ \ddot{U}_d \\ \ddot{U}_c \\ \ddot{U}_f \end{Bmatrix} + \begin{bmatrix} A & 0 & 0 & 0 \\ 0 & C_{dd} & C_{dc} & 0 \\ 0 & C_{cd} & C_{cc} & C_{cf} \\ 0 & 0 & C_{fc} & C_{ff} \end{bmatrix} \begin{Bmatrix} \dot{P} \\ \dot{U}_d \\ \dot{U}_c \\ \dot{U}_f \end{Bmatrix} + \begin{bmatrix} H & 0 & 0 & 0 \\ -Q_d & K_{dd} & K_{dc} & 0 \\ 0 & K_{cd} & K_{cc} & K_{cf} \\ 0 & 0 & K_{fc} & K_{ff} \end{bmatrix} \begin{Bmatrix} P \\ U_d \\ U_c \\ U_f \end{Bmatrix} = \begin{Bmatrix} F_r \\ 0 \\ 0 \\ 0 \end{Bmatrix} - \begin{bmatrix} J & Q_d^T & Q_c^T & 0 \\ 0 & M_{dd} & M_{dc} & 0 \\ 0 & M_{cd} & M_{cc} & M_{cf} \\ 0 & 0 & M_{fc} & M_{ff} \end{bmatrix} \begin{Bmatrix} \ddot{U}_d^g \\ \ddot{U}_c^g \\ \ddot{U}_f^g \end{Bmatrix} \quad (45)$$

Here subscript ‘d’, ‘f’ and ‘c’ indicates the nodes of the dam, foundation and dam-foundation interface. $\ddot{U}_g$ present ground acceleration. Added motion approach has been used in Eq. (47). The coupling matrix $Q = [[Q_d] \quad [Q_c]]^T$, matrix $[Q_d]$ related to the body of the dam and matrix $[Q_c]$ is related to dam-reservoir interface. Absolute responses of the coupled system are combination of free field responses and added part responses.

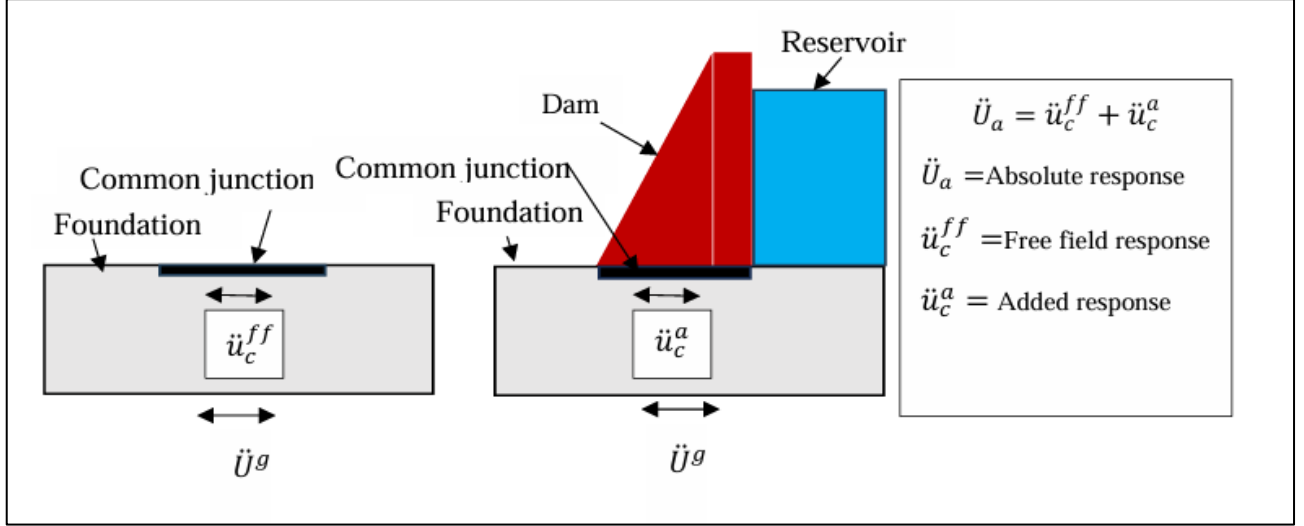


Fig. 3 - Schematic diagram of dam-reservoir-foundation for application of ground motion

$$M_{cc} = M_c^d + M_c^f, C_{cc} = C_c^d + C_c^f \text{ and } K_{cc} = K_c^d + K_c^f \quad (46)$$

$$\begin{Bmatrix} \ddot{U}_d \\ \ddot{U}_c \\ \ddot{U}_f \end{Bmatrix} = \begin{Bmatrix} \ddot{P}^{ff} \\ \ddot{u}_d^{ff} \\ \ddot{u}_c^{ff} \\ \ddot{u}_f^{ff} \end{Bmatrix} + \begin{Bmatrix} \ddot{P}^a \\ \ddot{u}_d^a \\ \ddot{u}_c^a \\ \ddot{u}_f^a \end{Bmatrix}, \quad \begin{Bmatrix} \dot{U}_d \\ \dot{U}_c \\ \dot{U}_f \end{Bmatrix} = \begin{Bmatrix} \dot{P}^{ff} \\ \dot{u}_d^{ff} \\ \dot{u}_c^{ff} \\ \dot{u}_f^{ff} \end{Bmatrix} + \begin{Bmatrix} \dot{P}^a \\ \dot{u}_d^a \\ \dot{u}_c^a \\ \dot{u}_f^a \end{Bmatrix}, \text{ and } \begin{Bmatrix} P \\ U_d \\ U_c \\ U_f \end{Bmatrix} = \begin{Bmatrix} P^{ff} \\ u_d^{ff} \\ u_c^{ff} \\ u_f^{ff} \end{Bmatrix} + \begin{Bmatrix} P^a \\ u_d^a \\ u_c^a \\ u_f^a \end{Bmatrix} \quad (47)$$

Here superscripts ‘ff’ stands for free field response and ‘a’ denotes added part response. After putting Eq. (47) in Eq. (45) we can get the following equations.

$$\begin{bmatrix} J & Q_d^T & Q_d^T & 0 \\ 0 & M_{dd} & M_{dc} & 0 \\ 0 & M_{cd} & M_{cc} & M_{cf} \\ 0 & 0 & M_{fc} & M_{ff} \end{bmatrix} \begin{Bmatrix} \ddot{P}^a \\ \ddot{u}_d^a \\ \ddot{u}_c^a \\ \ddot{u}_f^a \end{Bmatrix} + \begin{bmatrix} A & 0 & 0 & 0 \\ 0 & C_{dd} & C_{dc} & 0 \\ 0 & C_{cd} & C_{cc} & C_{cf} \\ 0 & 0 & C_{fc} & C_{ff} \end{bmatrix} \begin{Bmatrix} \dot{P}^a \\ \dot{u}_d^a \\ \dot{u}_c^a \\ \dot{u}_f^a \end{Bmatrix} + \begin{bmatrix} H \\ -[Q_d] \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} K_{dd} & K_{dc} & 0 \\ K_{cd} & K_{cc} & K_{cf} \\ 0 & K_{fc} & K_{ff} \end{bmatrix} \begin{Bmatrix} P^a \\ u_d^a \\ u_c^a \\ u_f^a \end{Bmatrix} = R + F \quad (48)$$

Here

$$R = - \begin{bmatrix} J & Q_d^T & Q_d^T & 0 \\ 0 & M_{dd} & M_{dc} & 0 \\ 0 & M_{cd} & M_{cc} & M_{cf} \\ 0 & 0 & M_{fc} & M_{ff} \end{bmatrix} \begin{Bmatrix} \ddot{P}^{ff} \\ \ddot{u}_d^{ff} \\ \ddot{u}_c^{ff} \\ \ddot{u}_f^{ff} \end{Bmatrix} - \begin{bmatrix} A & 0 & 0 & 0 \\ 0 & C_{dd} & C_{dc} & 0 \\ 0 & C_{cd} & C_{cc} & C_{cf} \\ 0 & 0 & C_{fc} & C_{ff} \end{bmatrix} \begin{Bmatrix} \dot{P}^{ff} \\ \dot{u}_d^{ff} \\ \dot{u}_c^{ff} \\ \dot{u}_f^{ff} \end{Bmatrix} - \begin{bmatrix} H \\ -[Q_d] \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} K_{dd} & K_{dc} & 0 \\ K_{cd} & K_{cc} & K_{cf} \\ 0 & K_{fc} & K_{ff} \end{bmatrix} \begin{Bmatrix} P^{ff} \\ u_d^{ff} \\ u_c^{ff} \\ u_f^{ff} \end{Bmatrix} \quad (49)$$

and

$$F = - \begin{bmatrix} J & Q_d^T & Q_d^T & 0 \\ 0 & M_{dd} & M_{dc} & 0 \\ 0 & M_{cd} & M_{cc} & M_{cf} \\ 0 & 0 & M_{fc} & M_{ff} \end{bmatrix} \begin{Bmatrix} F_r \\ \ddot{U}_d^g \\ \ddot{U}_c^g \\ \ddot{U}_f^g \end{Bmatrix} \quad (50)$$

Free field responses are determined by analysing the soil foundation separately. Here the foundation has been analysed separately. Therefore, the responses of structure and fluid are assumed zero. Hence, equations can be written as below.

$$\begin{Bmatrix} \ddot{P} \\ \ddot{U}_d \\ \ddot{U}_c \\ \ddot{U}_f \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ \ddot{u}_c^{ff} \\ \ddot{u}_f^{ff} \end{Bmatrix} + \begin{Bmatrix} \ddot{P}^a \\ \ddot{U}_d^a \\ \ddot{U}_c^a \\ \ddot{U}_f^a \end{Bmatrix}, \quad \begin{Bmatrix} \dot{P} \\ \dot{U}_d \\ \dot{U}_c \\ \dot{U}_f \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ \dot{u}_c^{ff} \\ \dot{u}_f^{ff} \end{Bmatrix} + \begin{Bmatrix} \dot{P}^a \\ \dot{U}_d^a \\ \dot{U}_c^a \\ \dot{U}_f^a \end{Bmatrix}, \quad \text{and} \quad \begin{Bmatrix} P \\ U_d \\ U_c \\ U_f \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ u_c^{ff} \\ u_f^{ff} \end{Bmatrix} + \begin{Bmatrix} P^a \\ U_d^a \\ U_c^a \\ U_f^a \end{Bmatrix} \quad (51)$$

Free-field responses are determined by applying dynamic excitation to the foundation.

$$\begin{bmatrix} M_{cc} & M_{cf} \\ M_{fc} & M_{ff} \end{bmatrix} \begin{Bmatrix} \ddot{u}_c^{ff} \\ \ddot{u}_f^{ff} \end{Bmatrix} + \begin{bmatrix} C_{cc} & C_{cf} \\ C_{fc} & C_{ff} \end{bmatrix} \begin{Bmatrix} \dot{u}_c^{ff} \\ \dot{u}_f^{ff} \end{Bmatrix} + \begin{bmatrix} K_{cc} & K_{cf} \\ K_{fc} & K_{ff} \end{bmatrix} \begin{Bmatrix} u_c^{ff} \\ u_f^{ff} \end{Bmatrix} = - \begin{bmatrix} M_{cc} & M_{cf} \\ M_{fc} & M_{ff} \end{bmatrix} \begin{Bmatrix} \ddot{U}_c^g \\ \ddot{U}_f^g \end{Bmatrix} \quad (52)$$

After getting the free field responses, the interaction force R can be determined by using the following expression.

$$R = - \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & M_{dd} & M_{dc} & 0 \\ 0 & M_{cd} & M_{cc}^d & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ \ddot{u}_c^{ff} \\ 0 \end{Bmatrix} - \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & C_{dd} & C_{dc} & 0 \\ 0 & C_{cd} & C_{cc}^d & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ \dot{u}_c^{ff} \\ 0 \end{Bmatrix} - \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & K_{dd} & K_{dc} & 0 \\ 0 & K_{cd} & K_{cc}^d & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ u_c^{ff} \\ 0 \end{Bmatrix} \quad (53)$$

For determination of absolute responses of dam-reservoir-foundation coupled systems the interaction force R in Eq. (53) is determined. Then the added responses are determined as per Eq. (48). After that free field response and added responses are summing up for calculating the absolute responses of dam-reservoir-foundation coupled systems.

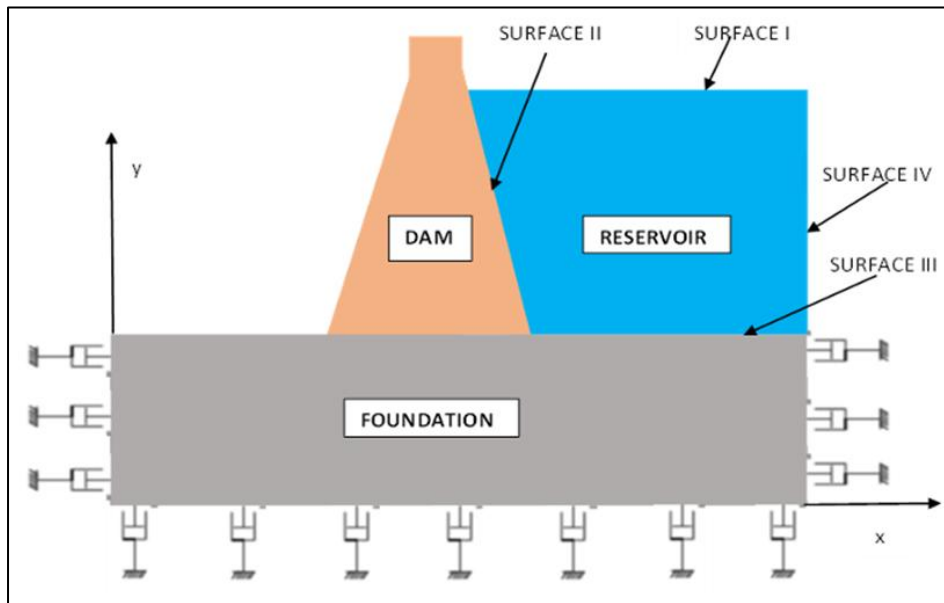


Fig. 4 - Geometry of the coupled systems

3 Validation of the Program

3.1 Validation of Dam, reservoir and dam-reservoir system

MATLAB code has been developed for analysis purpose. The algorithm is validated with the problem considered by Samii and Lotfi [25]. The properties and boundary conditions are taken as considered by them. The frequencies of first three modes of dam, reservoir and dam-reservoir system have been compared with the frequencies obtained from the literature of Samii and Lotfi [25] are shown in Table 1 to 3.

Table 1 - Comparison of frequencies of first three modes for reservoir only

Mode no.	Frequency (Hz) from present study	Frequency (Hz) from Samii and Lotfi [25]
1	3.1247	3.1151
2	4.7522	4.7491
3	7.8037	7.7955

Table 2 - Comparison of frequencies of first three modes for dam only

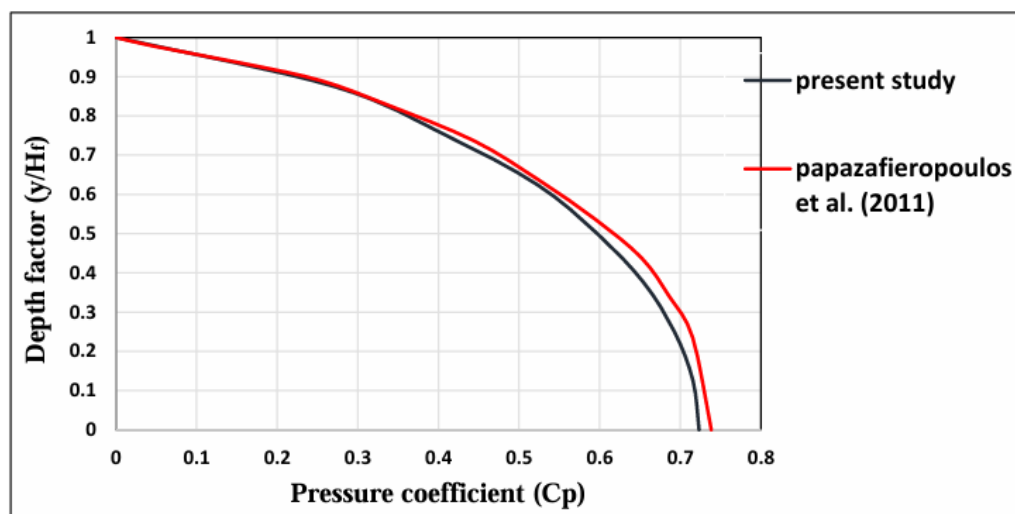
Mode no.	Frequency (Hz) from present study	Frequency (Hz) from Samii and Lotfi [25]
1	3.1372	3.1464
2	6.4883	6.4751
3	8.7394	8.7386

Table 3 – Comparison of frequencies of first three modes for dam-reservoir system

Mode no.	Frequency (Hz) from present study	Frequency (Hz) from Samii and Lotfi [25]
1	2.5788	2.5267
2	3.2796	3.2681
3	4.6882	4.6665

3.2 Validation of Dam-reservoir-foundation system

A program has been developed for analysis of dam-reservoir-foundation coupled system. The Developed program is compared with the example solved by Papazafeiropoulos et al. [14]. The dimensions of the problem are taken as taken by Papazafeiropoulos et al. The density of fluid (ρ_f) is assumed to be 1000kg/m³. Acoustic speed in water is considered 1440 m/s. Modulus of elasticity of dam is assumed as 3.15x 1010 N/m² and density of the gravity dam is taken as 2400kg/m³. Modulus of elasticity of the foundation is taken as 3.15x 1010 N/m² and density is assumed to be 2400kg/ m³. Boundary conditions are applied as suggested by Papazafeiropoulos et al. [14]. Distribution of pressure at dam face for dynamic excitation is shown in Fig. 5. Our results are in conformity with Papazafeiropoulos et al. (2011).

**Fig. 5 - Hydrodynamic pressure at face of the dam**

4 Numerical results

Variation of hydrodynamic pressure and stresses of the gravity dam are determined by applying harmonic excitation to the dam-reservoir-foundation system. Schematic diagram of coupled system has been given in Fig. 6. Size of the soil foundation (length=350 m., height =100 m.) is taken as per the reference of Mandal and Maity [26]. Height of fluid (H_f) has been assumed 103 m. Density of water (ρ_f) is taken as 1000kg/m^3 and acoustic speed in water is considered 1438.7 m/s . Elastic modulus of structure is considered as $3.15 \times 10^{10}\text{ N/m}^2$ and density of the structure is assumed as 2415.816 kg/m^3 . Poisson ratio of the structure is considered as 0.235. Modulus of elasticity of the foundation is assumed as $1.75 \times 10^{10}\text{ N/m}^2$ and density is taken as 1800kg/m^3 . Poisson ratio for soil foundation is taken as 0.2. Hydrodynamic pressure and stresses of the dam and foundation are observed for different bed slopes and reflection coefficients at base of the reservoir.

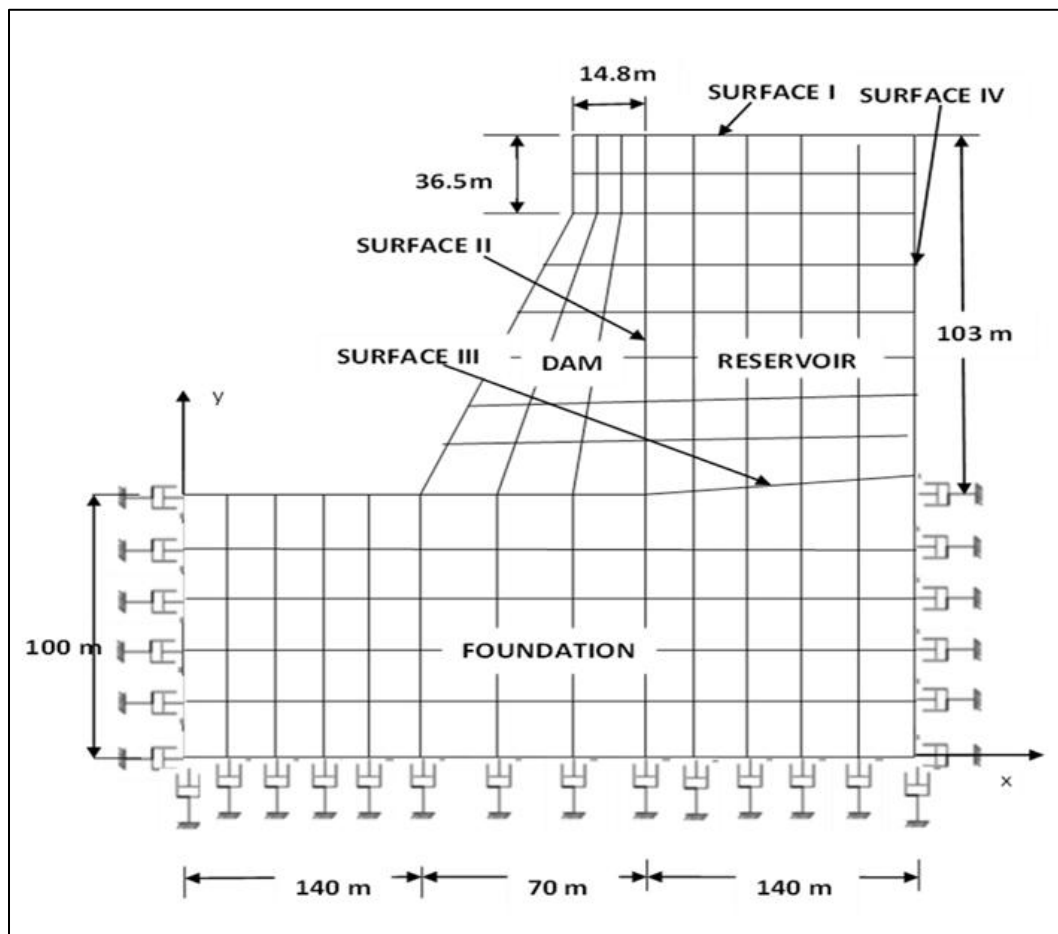


Fig. 6 - Typical Finite element discretization of dam-reservoir-foundation system

4.1 Part I:

Changes of pressure distribution in reservoir and stresses in the soil and structure have been observed for different bottom slope angles (θ_b) of the reservoir bed. Anticlockwise slope is assumed as positive slope of reservoir bed and clockwise slope is assumed as negative slope of reservoir bottom. The external excitation is assumed to be harmonic of Tc/H_f equal to 100 and reflection coefficient (α) of reservoir bed is taken 0.95. Fig 7.a presents variation of pressure coefficient ($C_p = p/\rho_f a H_f$) at dam face due to anticlockwise bed slope ($+5^\circ$, $+10^\circ$ and $+15^\circ$). Fig 7.b shows the variation of pressure coefficient (C_p) at dam face due to clockwise bed slope (-5° , -10° and -15°). Fig. 8.a displays distribution of pressure with time at heel of the structure due to positive angles at the reservoir base ($+5^\circ$, $+10^\circ$ and $+15^\circ$) and Fig. 8.b displays distribution of pressure with time at heel of the structure due to negative angles at the reservoir base (-5° , -10° and -15°). From these figures, increasing value of pressure coefficient has been observed at heel of the structure due to anticlockwise bed slope and decreasing value of pressure coefficient has been observed at heel of the structure due to clockwise bed slope.

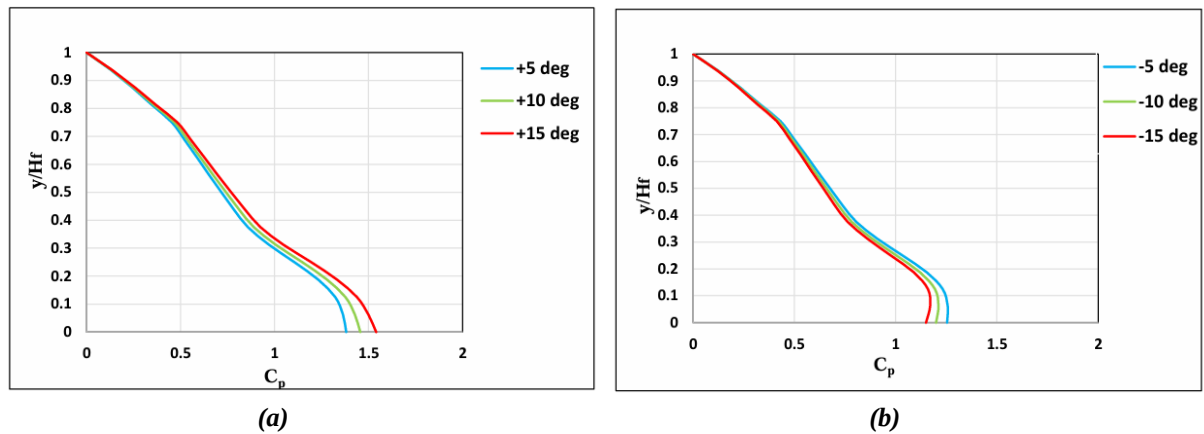


Fig. 7 - Variation of pressure coefficient (C_p) at dam face with different bed slopes θ_b a) positive and b) negative due to harmonic loading

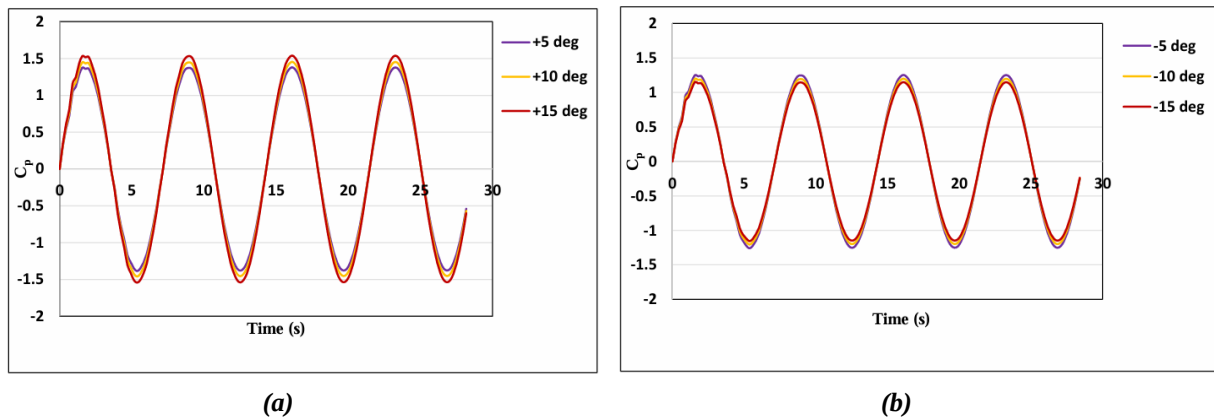


Fig. 8 - Distribution of pressure coefficient (C_p) with time at heel of structure for a) positive and b) negative bed slopes

Fig 9.a and Fig. 9.b present stresses at heel of the structure for anticlockwise bed slope. Fig. 10.a and Fig. 10.b present stresses at heel of the structure for clockwise bed slope of reservoir. The maximum stress (major and minor) at the heel of the dam is increased for an increase in positive slope angle ($+5^\circ$, $+10^\circ$ and $+15^\circ$) and the maximum stress (major and minor) at the heel of the dam is decreased due to the increase in negative slope angle (-5° , -10° and -15°).

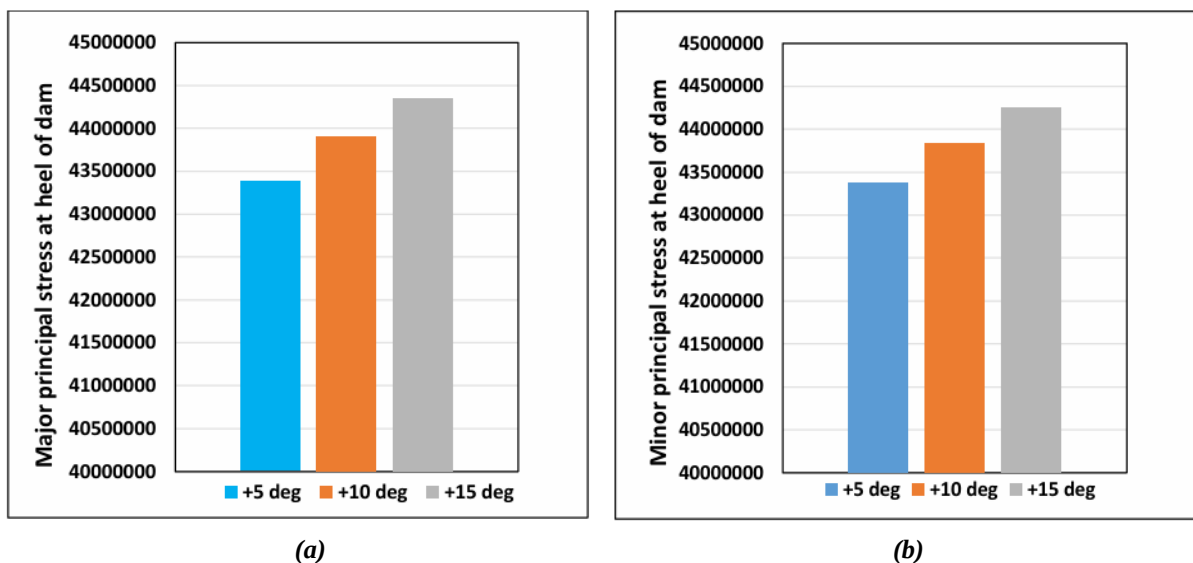


Fig. 9 - a) Major and b) Minor principal stress at heel of structure due to anticlockwise bed slopes

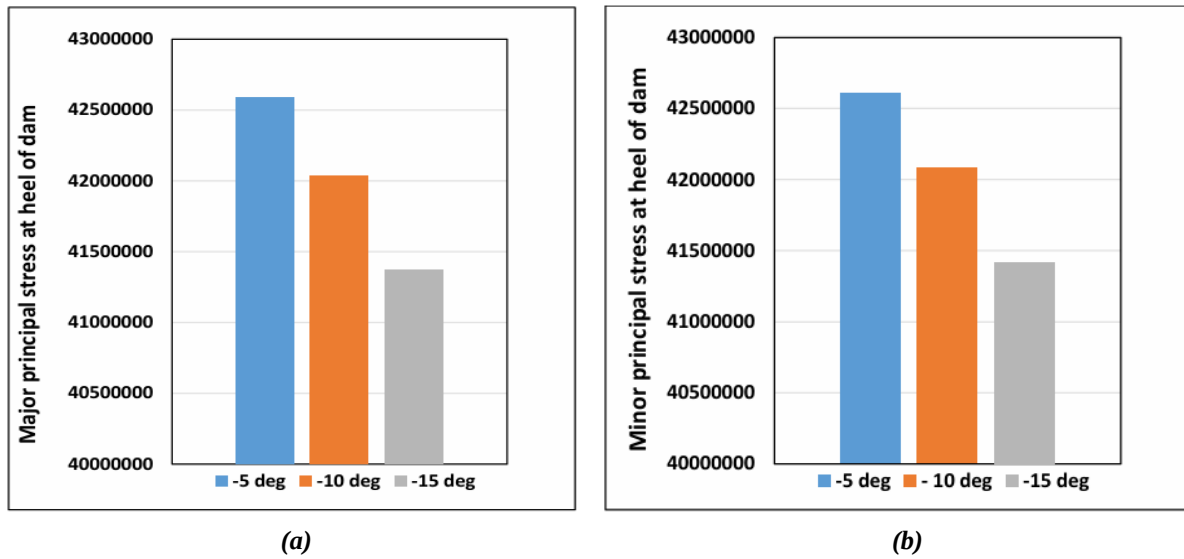


Fig. 10 - a) Major and b) Minor principal stress at heel of structure due to clockwise bed slopes.

Fig. 11.a shows distribution of major stress with respect to time and Fig. 11.b shows distribution of minor stress with respect to time for foundation near the heel of the dam for anticlockwise slope angles ($+5^\circ$, $+10^\circ$ and $+15^\circ$). Fig. 12.a shows distribution of major stress with respect to time and Fig. 12.b shows distribution of minor stress with respect to time for foundation near the heel of the dam for clockwise slope angles (-5° , -10° and -15°). From these figures, it is seen that the maximum stress i.e. both major and minor principal stress in foundation near heel of structure is decreased due to increase of anticlockwise slope angle. This has also been observed that the maximum stress in foundation near heel of structure is increased due to increase of clockwise slope angle.

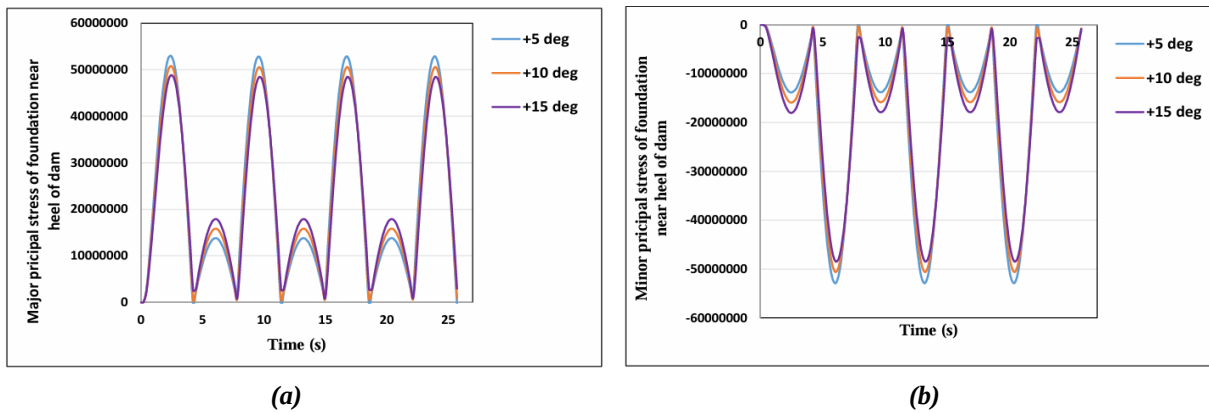


Fig. 11 - Distribution of a) major and b) minor principal stress of foundation near heel of structure for anticlockwise bed slopes

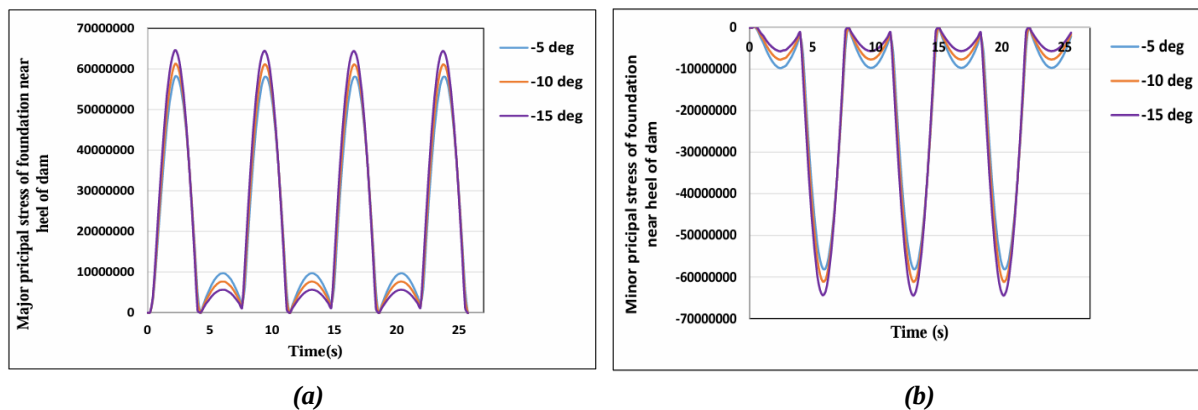


Fig. 12 - Distribution of a) major and b) minor principal stress of foundation near heel of structure for clockwise bed slopes

4.2 Part II:

Coefficient of hydrodynamic pressure at heel of the dam is observed for variation of reflection coefficient (α) for inclined reservoir bed considering fluid-structure and soil-structure interaction simultaneously. Geometry and properties are considered same as in part I. In this part of work Tc/H_f is taken as 100. Table 4 presents the pressure coefficient (C_p) at the heel of the dam for different reflection coefficients with different slopes (θ_b) of reservoir bottom. It has been seen that pressure is increased with the increase of reflection coefficient for any value of bottom slope.

Table 4 - Pressure coefficient (C_p) at the heel of the dam for different reflection coefficient with inclined base

Slope angle (θ_b)	Reflection coefficient (α)	C_p
$+5^\circ$	0.0	1.37868
	0.5	1.38052
	1.0	1.38075
$+10^\circ$	0.0	1.45312
	0.5	1.45525
	1.0	1.45552
$+15^\circ$	0.0	1.53721
	0.5	1.53969
	1.0	1.54000
-5°	0.0	1.25264
	0.5	1.25392
	1.0	1.25408
-10°	0.0	1.19853
	0.5	1.19952
	1.0	1.19965
-15°	0.0	1.14899
	0.5	1.14986
	1.0	1.14996

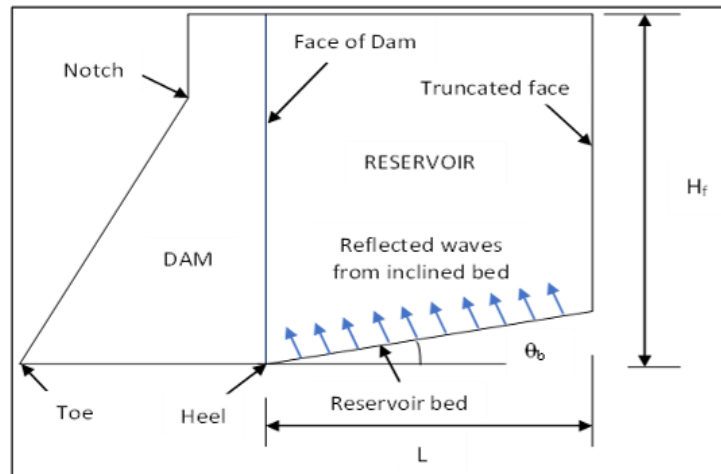
5 Discussion

Here, responses of dam, reservoir and foundation have been studied with considering dam-reservoir and dam-foundation interactions simultaneously. Maximum hydrodynamic pressure and maximum stress in the structure obtained at heel of the dam. Hydrodynamic pressure and stresses of structure increase due to increment of anticlockwise bed slope of reservoir. However, Hydrodynamic pressure and stresses of structure decrease due to increment of clockwise bed slope of reservoir. When the slope of the reservoir bed is in the anticlockwise direction, the inclined reservoir bed is towards the concrete gravity dam (Fig. 13.a). Due to this inclination, the reservoir bed reflects the wave towards the gravity dam and this is the main reason for obtaining the higher value of pressure and stresses of structure due to increment of anticlockwise bed slope of the reservoir. When the slope of the reservoir bed is in the clockwise direction, the inclined reservoir bed is away from the concrete gravity dam (Fig. 13.b). Due to this inclination, the reservoir bed reflects the wave that is going away from the gravity dam. This is the main reason for obtaining the lower pressure and stresses of structure due to the increase of clockwise slope of reservoir bed.

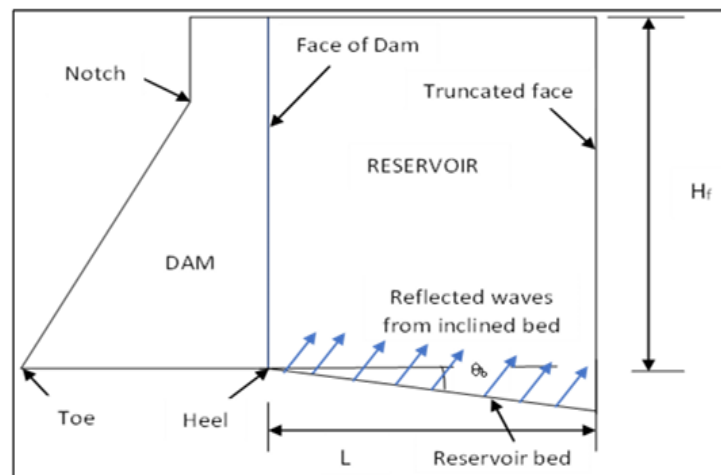
The stress in soil near heel of the structure decreases with increase in anticlockwise bed slope and the stress in foundation near heel of the structure rises with increment of clockwise bed slope. When the bottom slope is anticlockwise, the stiffness contribution of the foundation increases due to the increase in the volume of foundation. As a result, the stress in the foundation near the heel of the structure is reduced with anticlockwise increment of reservoir bed slope. Similarly, when the reservoir bed slope is in the clockwise direction, the stiffness contribution of the foundation decreases due to the decrease in

the volume of foundation. As a result, the stresses in the foundation near heel of the structure is increased with anticlockwise increment of reservoir bed slope. It has been also seen that pressure increases due to increment of reflection coefficient for any value of inclination of reservoir bed.

Very little work has been found on dynamic analysis of dam-reservoir-foundation system till now. Most of the authors have modelled the fluid domain considering Lagrange approach. In their analysis displacement is considered as nodal unknown for fluid domain. But in the present method pressure is consider as unknown for fluid domain. This reduces the computational time and avoid zero frequency mode. Most of the author have considered the reservoir base horizontal. In the present study the effect of base angle has been studied for dynamic responses of dam and reservoir. It has been found that that the inclined reservoir bottom influences the dynamic responses of dam and reservoir.



(a)



(b)

Fig. 13 - Inclined reservoir bed a) anticlockwise slope and b) clockwise slope with reflected waves

6 Conclusion

Based on the numerical results and the observations from the present study following conclusions can be made.

- Maximum stress and maximum hydrodynamic pressure occurred at heel of the dam due to the dynamic excitation. Hydrodynamic pressure increases for increment of reflection coefficient of the reservoir bottom for any value of inclination of the reservoir bed.
- Hydrodynamic pressure at heel of the dam increases for increment of anticlockwise bed slope of reservoir. However, pressure coefficient decreases at the same place due to increment of clockwise bed slope.

- Stresses at heel of the structure increased due to increment of anticlockwise bed slope of reservoir. However, reverse condition occurred for increase of clockwise bed slope of the reservoir.
- The stress of foundation under heel of the structure decreases for increment of anticlockwise bed slope. However, stress at this location increases due to increment of clockwise bed slope.
- Bed slope of the reservoir may change due to the scouring or siltation at the river bed in front of the dam. This may vary the slope in clockwise or anticlockwise direction. The stresses at the bottom of dam and the hydrodynamic pressure at heel of dam changes accordingly. So, the present study is very much important for design and stability of the gravity dam.
- Proper planning of dredging is required accordingly to maintain the bed slope of the reservoir that may affect the dynamic responses of gravity dam and reservoir.

Declarations

Conflict of interest: The authors declare no conflict of interest

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