



Journal of Materials and Engineering Structures

Research Paper

Performance comparison of hyperbolic paraboloidal shell footing with its flat counterpart

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ARTICLE INFO

Article history :

Received : 10 December 2024

Revised : 18 January 2025

Accepted : 1st February 2025

Keywords:

Shell footing

Hyperbolic paraboloidal shell

Finite element method

Mohr-Coulomb yield criteria

ABSTRACT

While traditional flat footings are commonly used in construction, hyperbolic paraboloidal shell footings present potential benefits in load distribution and settlement reduction. The study encompasses two primary aspects: the design of both footing types in accordance with Indian standard practices, followed by a comparative analysis of their performance utilizing finite element methodology. Performance comparison is carried out concentrating on vertical settlement, stress distribution, and the amount of concrete required under centric gravity loads. The hyperbolic paraboloidal shell footing was modeled with curved surfaces, and the underlying soil was treated as nonlinear using the Mohr-Coulomb yield criteria. The amount of concrete required for a hyperbolic paraboloidal shell footing is significantly less, calculated to be 0.61 times that of a flat footing.

1 Introduction

The curvilinear shape of a shell enables it to withstand membrane stresses including tensile, compressive, and shear forces. It distributes loads to the support via its membrane surface. Its effectiveness depends on its curvature, which facilitates various alternative stress paths. Shells offer advantages for use in foundations as cost-effective alternatives to traditional flat foundations, particularly in scenarios where heavy column loads must be transferred to soils with lower bearing capacities.

Shell foundations provide a cost-effective option compared to flat foundations, improving both the bearing capacity and settlement behaviour [1]. They have been used worldwide since the 1950s and present notable benefits over conventional foundations [2]. Research has shown that shell foundations are superior to flat foundations in terms of ultimate load capacity and settlement characteristics. Additionally, shell foundations are more effective in resisting lateral forces caused by wind and earthquakes [2, 3]. Shell foundations are particularly effective in weak soils because they can transfer large loads more

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economically than shallow foundations [4]. In loose sandy soils, pyramidal shell foundations, particularly when reinforced with geogrids, demonstrate greater load capacity and better settlement curves compared to flat foundations [5]. Furthermore, they have the potential to reduce embodied carbon emissions in building foundations by as much as two-thirds under high applied loads [6]. Research combining theoretical and practical studies by [7] examined the load-bearing capacity of conical shell foundations on sand with 10% Kaolinite. The theoretical approach extended the traditional solution for flat circular foundation bearing capacity to conical shell foundations using an upper bound solution from limit analysis. Results showed conical shell foundations had better load-bearing capacity and resistance to settlement than flat foundations. A conical angle of 120° increased maximum load capacity by 15% and reduced settlement by 51%. The study found an inverse relationship between the foundation's angle and both bearing capacity and settlement. Researchers proposed preliminary design charts for modified bearing capacity coefficients for conical shell foundations, accounting for the soil's internal friction angle and the shell's angle. Laboratory tests evaluated the geotechnical performance of shell foundations on treated and untreated soil, comparing them with flat counterparts [8]. These experiments used pyramidal frustum and semi-cylindrical shell footing models. Results show shell foundations have higher load-bearing capacities than flat foundations. Foundations on treated soil perform better, with cement treatment altering the load settlement curves. Shell foundations present a viable alternative to flat foundations, offering increased load-bearing capacity for similar cross-sectional areas. They also result in reduced settlements, potentially leading to cost savings in substructure construction and overall project expenses.

Unlike traditional shallow foundations, which tend to settle significantly [9], shell foundations provide enhanced performance. Research conducted by [3] on dry sand indicated that shell foundations have superior load-bearing capabilities compared with flat footings of the same plan dimensions under similar loading conditions. [10] conducted a study examining the interaction between conical shell footings and underlying soil, focusing on the frictional behavior of sloped contact surfaces. The finite element model included the shell-soil interface, covering separation, finite-amplitude kinetic frictional slip, unrestricted surface rotation, and pressure-overclosure relationships. It accounted for geometric nonlinearity to address large deformations and displacements from slip. Compared to the tie constraint and Winkler model, the tangential friction model provided more accurate predictions of shell response. The study revealed that the cone's half-angle and the ratio of soil to foundation moduli significantly influence the interaction. [11] utilized PLAXIS, a nonlinear finite element analysis software, to model two triangular strip footings. They assessed the load-bearing capacities of these footings compared to flat strip footings by varying the rise-to-half-width ratio. Their results showed that the shell models had over 40 percent higher load capacities and experienced less settlement than the flat models. [12] investigated the load-bearing capacities of conical and pyramidal shell foundations on dry sand and compared them with those of circular and square flat foundations. They utilized four different foundation models to examine how the shell shape affects the load capacity and settlement. The results showed that shell foundations possess greater ultimate capacities than their flat-foundation counterparts. Research by [13] used PLAXIS, a finite element code, for non-linear analysis of shell footings' geotechnical performance. Results showed shell footings had superior load-bearing capacity compared to flat footings of the same area. The findings matched laboratory data with a variance of 11 to 25%. Numerical studies on edge beams at the shell footing base revealed enhanced load-bearing capacity. The analysis also showed fully embedded shell footings had greater load-carrying capabilities than surface ones. [14] conducted a study on triangular strip shell foundations using the FELA method and examined the soil's failure surface beneath these foundations. The research revealed that the failure pattern under shell foundations is not influenced by their dimensions. Instead, key factors determining the failure surface are the friction between foundation and soil, and depth of embedment. The study also found that increasing foundation roughness leads to an increase in bearing capacity.

Shell foundations have various shapes, including arched, circular, triangular, conical, cylindrical, spherical, hyperbolic, parabolic, and pyramidal, each of which is designed for optimal performance. Among these, the hyperbolic paraboloid (hyper) shell is particularly notable, as it features a doubly curved anticlastic shape formed by two sets of parabolas that curve in opposite directions. [15] examined the behaviour of pyramid, equivalent hyperbolic paraboloidal, and standard square shell footings under vertical loads at varying embedment depths. These were compared with a square footing with the same cross-sectional area. The findings revealed that the load-bearing capacity of shell footings improved with increased height-radius ratios (0.3, 0.4, and 0.5) and deeper embedment depths (0.3 to 0.5), showing superior load-carrying capacity compared to flat footings. Numerous studies have aimed at reducing the amount of steel reinforcement and concrete used in construction [16]. [17] performed both experimental modelling and numerical analysis to evaluate the maximum load-bearing capacity of three different conical and pyramidal shell foundations placed on sand with and without reinforcement. [18] outlines the unique aspects of calculations based on soil load-bearing capacity when using strip-shell foundations, which are highly effective for constructing medium and tall buildings on compressible soils. The study presents a novel static solution to the Coulomb's

wedge theory for soil load-bearing capacity with Shell Strip Footing. This approach considers the curved soil surface beneath the shell and the counterweight from strip foundations, allowing for a justifiable increase in the soil's load-bearing capacity. [19] experimentally determined the ultimate load capacities for inverted and upright conical, inverted and upright pyramidal, and hemispherical shell foundations on silty soil, comparing them to conventional flat squares and circles. The "upright conical" shell footing demonstrated the highest load capacity at 12.7 kN, with an efficiency of 51%. The "upright pyramidal" shell footing exhibited superior settling characteristics, with a settlement factor of 0.017. Shell foundations generally provide greater efficiency compared to traditional systems. The inverted pyramid shell footing, with a side slope angle of 15° to 30° , shows remarkable performance, achieving efficiencies of 48.3% and 46.5%, respectively, surpassing other footing designs.

[20] explored hyperbolic paraboloidal shell structures through finite element modelling to analyze the interaction between shell foundations and the surrounding soil. They performed a parametric study that considered factors such as shell thickness, warp, ridge, and the dimensions of the edge beam cross-sections. Similarly, [21] utilized the PLAXIS software to evaluate the settlement characteristics of hyper shell foundations with different edge beam sizes, emphasizing the effects of embedding ratios on settlement, load-bearing capacity, and stresses at the soil-shell interface. Their results demonstrated that hyper footings can significantly minimize settlement. [22] investigated 11 configurations of vertical hyperbolic paraboloidal shell footings, emphasizing both geometric and geotechnical factors. Their findings indicated that shell footings with an f/a ratio of 0.75 yielded the best performance. This study evaluated the distribution of axial force and stress in beam and shell membranes, highlighting lower settlement characteristics.

2 Problem for investigation

This study examines the performance of hyperbolic paraboloidal shell footings in comparison with flat footings using the finite element program ANSYS Ver.20 (2020). Initially, both types of footings were designed to support a central gravity load of 1650 kN, considering a soil-bearing capacity of 100 kN/m^2 , in accordance with the Indian standards. Following the design phase, finite element analysis was conducted on both the shell and flat footings. The performances of the two footing types were compared based on the vertical settlement and stress levels. The hyperbolic paraboloidal shell footing model features curved surfaces, as illustrated in Fig. 1. The underlying soil is modeled nonlinearly by applying the Mohr-Coulomb yield criterion.

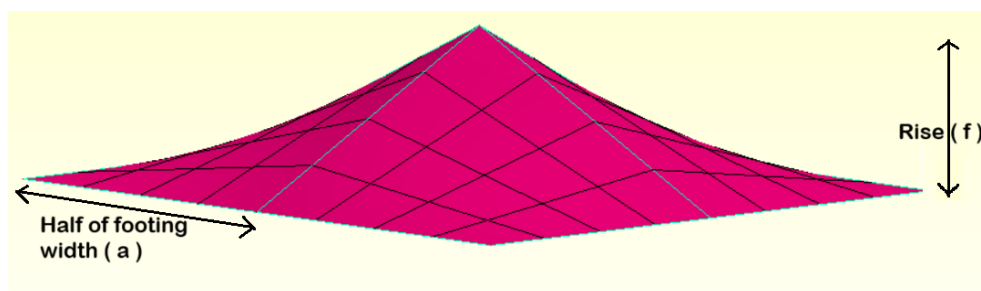


Fig. 1 - Hyperbolic paraboloidal shell having curved surfaces

2.1 Design of footings

The design of the hyperbolic paraboloidal shell footing was performed according to [23], as described in Table 1. The thickness of the shell was 150 mm with an edge and ridge beam with cross-sectional dimensions of $400 \times 400 \text{ mm}$. The design of the flat footing was carried out as per [24] for identical loads and soil bearing capacity, and the thickness of the footing required was derived to be 650 mm.

Table 1 - Design of hyperbolic paraboloidal shell footing

Description	Unit	Values	Remarks
Column load	kN	1650	Vertical point load
Safe soil-bearing capacity	kN/m ²	100	
Vertical Pressure, P_v	kN/m ²	150	
Shell central rise ratio, (f/a)		0.5	f/a should be between 0.5 to 1, as per IS 9456:1980
Grade of Concrete		25	
Permissible Shear stress in concrete = C_{xy}	N/mm ²	3.1	
Grade of Steel, f_y	N/mm ²	415	
Area of footing required, A	m ²	16.5	
Edge Dimension of footing, $2a = 2b$	m	4.06	Square Plan-footing is considered
Half of the edge dimension, $a = b$	m	2.125	
Shell central rise, f	m	1.0625	$f/a = 0.5$
Shell warp, $k = ((f/(a \times b)))$	m ⁻¹	0.235	
Membrane shear, $N_{xy} = P_v/2k$	N/mm	318.75	
Minimum thickness of shell required, $t = N_{xy}/C_{xy}$	mm	102.82	
Area of steel in shell, $A_{st \text{ shell}} = N_{xy} \times 1000 / (0.87 \times f_y)$	mm ²	882.84	
Percentage steel in shell, $P_{st \text{ shell}} = A_{st \text{ shell}} \times 100 / (1000 \times t_{\text{provided}})$	%	0.59	Minimum reinforcement in the shell as per IS code = 0.5 %
Max. tension in edge beam, $T = N_{xy} \times a$	kN	677.34	
Max. compression in ridge beam, $C = 2N_{xy} (a^2 + f^2)^{1/2}$	kN	1514.59	
Ridge beam cross-section required	mm	389.18	
Total A_{st}	mm ²	2758.88	
The thickness of the shell provided	mm	150	The minimum thickness of the shell as per IS code is 150 mm satisfying the cover requirements
Ridge beam cross-section (Square)	mm	400	
Edge beam cross-section (Square)	mm	400	
Provided Edge dimension, $2a = 2b$	m	4.25	

2.2 Modelling of hyperbolic paraboloidal shell footing

To investigate the performance of hyperbolic paraboloidal shell footing concerning a flat footing, finite element methodology is used because the hyperbolic paraboloidal shell footing designed before is modeled using ANSYS Ver.20 (2020) with curved surfaces. A previously designed flat footing was also modeled and analyzed. The material properties utilized in the FEM modeling are summarized in Tables 2 and 3.

Table 4. describes the specifications of all footing models under investigation.

Table 2 - Material Properties of Footing Models

Parameters	Value	Units
Young's modulus	25000	MPa
Poisson's ratio	0.2	-
Bulk Modulus	13889	MPa
Shear Modulus	10417	MPa

Table 3 - Soil Properties- Mohr-Coulomb Model (C _ Ø soil)

Parameters	Value	Units
Density	1500	kg/m ³
Young's modulus	7	MPa
Poisson's ratio	0.2	-
Bulk Modulus	3.88	MPa
Shear Modulus	2.92	MPa
Friction angle	15	Degree
Dilatancy angle	0	Degree
Cohesion	20	Kpa

Table 4 - Specifications of all footing models

Type	Plan (m×m)	f/a	Shell rise (m)	Shell Thickness (m)	Beam dimensions (m×m)	Volume (m ³)
Shell	4.25 x 4.25	0.5	1.0625	0.15	0.4 x 0.4	7.164
Flat	4.25 x 4.25	0	0	0.65	-	11.74

2.3 Modelling of soil domain

The safe bearing capacity of the soil considered before, while designing footings, equal to 100 kN/m² is taken into account during finite element analysis. According to Terzaghi's ultimate bearing capacity equation for shallow foundations:

$$q(\text{ult}) = c * N_c + Y * D * N_q + 0.5 * B * Y * N_y \quad (1)$$

$$q(\text{safe}) = q(\text{ult})/FS \quad (2)$$

where $q(\text{ult})$ denotes the ultimate bearing capacity, $q(\text{safe})$ is the safe bearing capacity of soil-derived soil, which is approximately 100 kN/m² with the soil parameters listed in Table 3. c is Cohesion of soil, Y is unit weight of soil, D is depth of footing, B denotes width of footing and N_c , N_q , N_r represents Terzaghi's bearing capacity factors which depend on soil friction angle ϕ . Soil yielding occurs when the shear stress reaches the following criterion:

$$\tau = c + \sigma \tan \phi \quad (3)$$

where τ is the shear stress, c is the cohesion, σ is the normal stress, and ϕ is the inner friction angle.

The soil domain is represented as a uniform and isotropic cuboid measuring 42.5 m in each dimension. These specific measurements were determined through iterative testing during the analysis of linear interaction. The selected dimensions proved effective in minimizing horizontal strains caused by footing loads to negligible levels. The conceptualized soil mass retained its uniform and isotropic properties, validating the appropriateness of the chosen cuboid size.

2.4 Finite element analysis

The present work employs the fem software ANSYS (2020) to analyse and assess the mechanism of shell footing on soil. In the analysis, we can employ the SOLID187 element toward the soil representing elastoplastic materials since it has been formulated to follow the Mohr-Coulomb yield criterion interface. Three translational degrees of freedom (x, y, and z) are allowed in each of the nodes of this higher-order 10-noded element, which is suitable for irregular meshes due to its quadratic displacement behaviour. To portray the profiled soil underlying the footing, the SOLID186 element, a higher order, 3 D 20 noded element with x, y, z degree of liberty at each node, is used. The shell footing was modelled using the SHELL181 element, which is ideal for applications involving large rotations and strains. This 4-noded element provides six degrees of freedom at each node: three for rotation (x, y, z), and three for translation (x, y, z). Through the use of the BEAM188 element, which incorporates six degrees of freedom in terms of translations (x,y,z) and rotations (x,y,z) at each of the nodes, both edge and ridge beams were modelled successfully. In terms of boundary conditions, normal fixities were applied where full fixity was provided at the bottom and partial fixity at the sides of the soil domain. The interaction of footing and soil interface was modelled using CONTA174 and TARGE170 interface elements. For the contact elements different 3-D ‘target’ surfaces are represented by TARGE170. In Figure 2, the discretization of soil mass included finer meshes around the footing where stresses were high, so as to enhance computational accuracy. The various elements used in analysis are listed in Table 5.

Table 5 - FEM elements used in modeling

Component	Element
Beam	BEAM188
Shell footing	SHELL181
Soil	SOLID186 and SOLID187
Interface: footing surface	CONTA174
Interface: soil surface	TARGET170

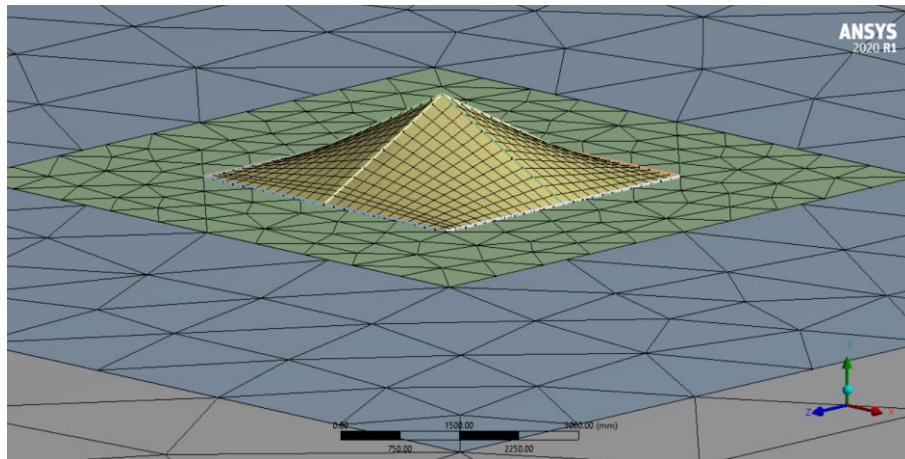


Fig. 2 - Finite element discretization of shell-soil interaction model

3 Results and discussions

In the present work, the performance of a hyperbolic paraboloidal shell footing is studied with respect to a flat footing using design aids and finite element analysis.

3.1 Design of footings

Hyperbolic paraboloidal shell footing and flat footing were designed and subjected to an incremental centric gravity load, leading to soil failure for a soil-bearing capacity of 100 kN/m². Both footings have a plan of 4.25 x 4.25 m², and the shell

footing has been provided with edge and ridge beams of cross-section $400 \times 400 \text{ mm}^2$. It is observed that the thickness required in the shell membrane for a f/a ratio of 0.5 is 150 mm, which is approximately 0.23 times the thickness required by flat footing. The volume of the hyperbolic paraboloidal shell footing along with beams is 7.16 m^3 and is approximately 0.61 times required for flat footing. The ridge and edge beams are subjected to axial compression and tension, respectively. These beams must be designed primarily to resist axial forces.

3.2 Finite element analysis

The projected results for the hyperbolic paraboloidal shell footing reveal that the shell footings with a f/a ratio of 0.5 performed significantly better than the traditional flat footing. The load versus deflection curve was plotted between the footings, as shown in Figure 3. The load leading to soil failure for a flat footing with a thickness of 650 mm is 1780 kN, and that for a shell footing with a thickness of 150 mm is 1650 kN (0.93 times). The vertical settlement in the shell footing for a centric load of 1650 kN was found to be 1.04 times that of the flat footing at the centre. The variation in settlement along the projected diagonal of the footings for a centric load of 1650 kN is illustrated in Figure 4.

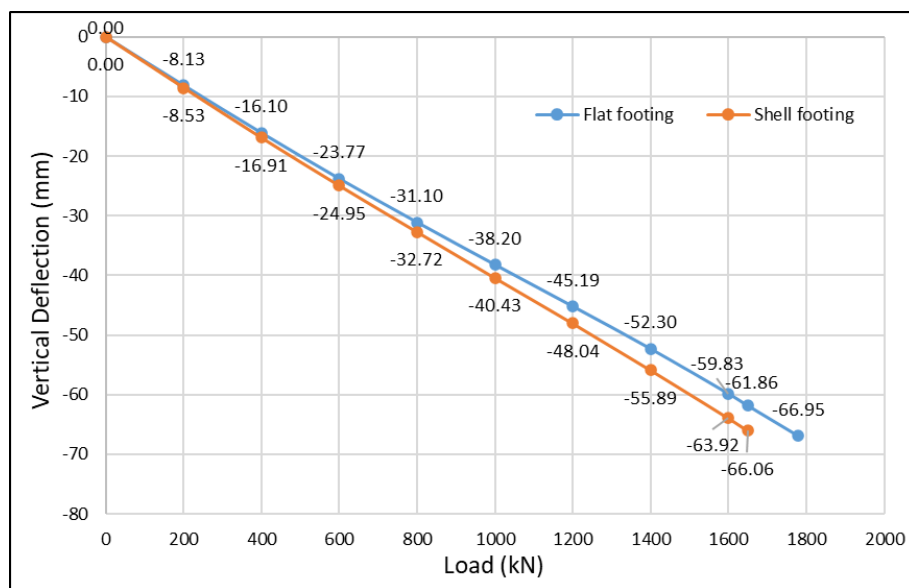


Fig. 3 - Load-settlement curves for various footing models

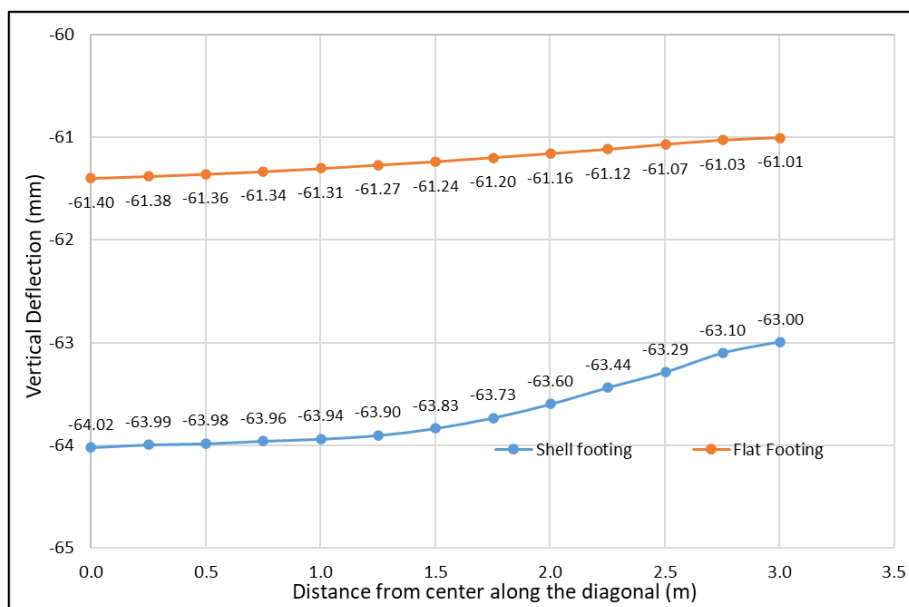


Fig. 4 - Vertical settlement in various shell footing models with variations in beam dimensions

Figures 5, 6, and 7 describe the variation in the von Mises stresses in the shell membrane along the diagonal in the top and bottom layers. Through the analysis of stress distribution in the shell footing, it was visible that the complete concave and convex curvature of the shell membrane part of the foundation is utilized in transferring forces from column to soil. The variation of Von-Mises stresses in the bottom layer of the shell membrane is indicating that the stress is well-distributed under the footing, the soil is less likely to experience local shear failure, enhancing the effective bearing capacity. Confinement of soil due to curvature of shell footing raises the soil's shear strength, improving its ability to support higher loads without failure and mobilizing the soil's full strength. Maximum Von-Mises stress occurs at the footing and column junction due to punching shear by column. Increased Von Mises stress at the top surface of the footing indicates localized stress concentration near the column, which is a precursor to punching shear failure.

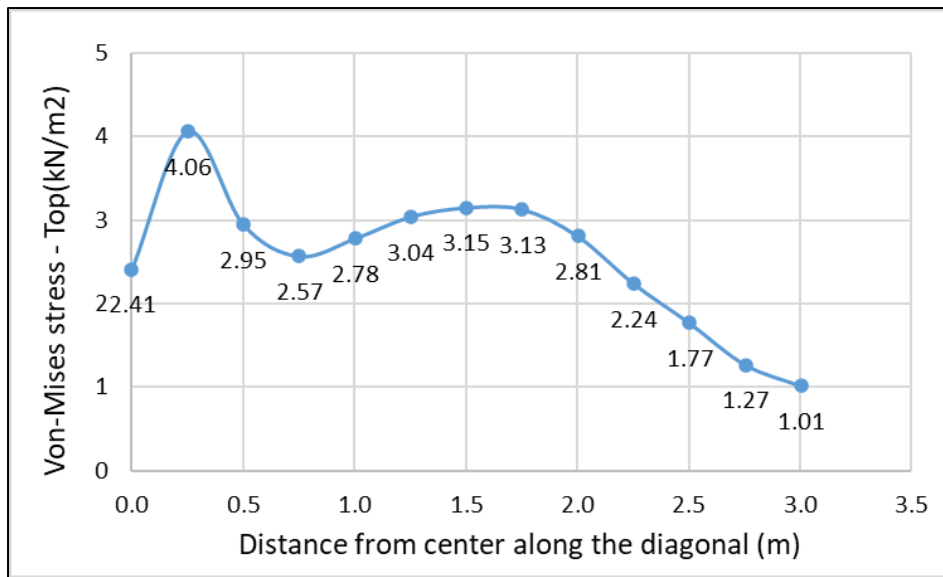


Fig. 5 - Von-Mises stress in the top layer of Shell footing ($f/a = 0.5$)

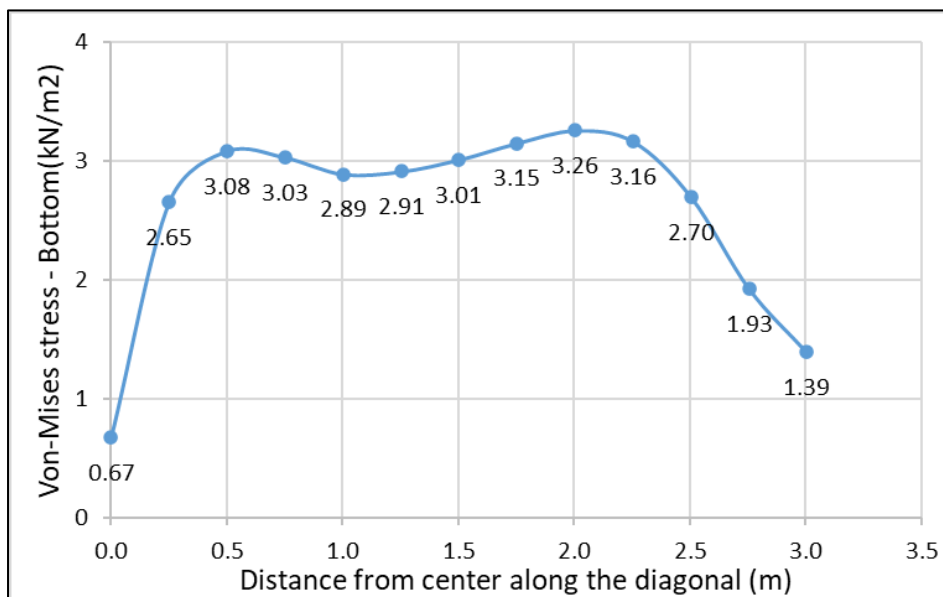


Fig. 6 - Von-Mises stress in the bottom layer of Shell footing ($f/a = 0.5$)

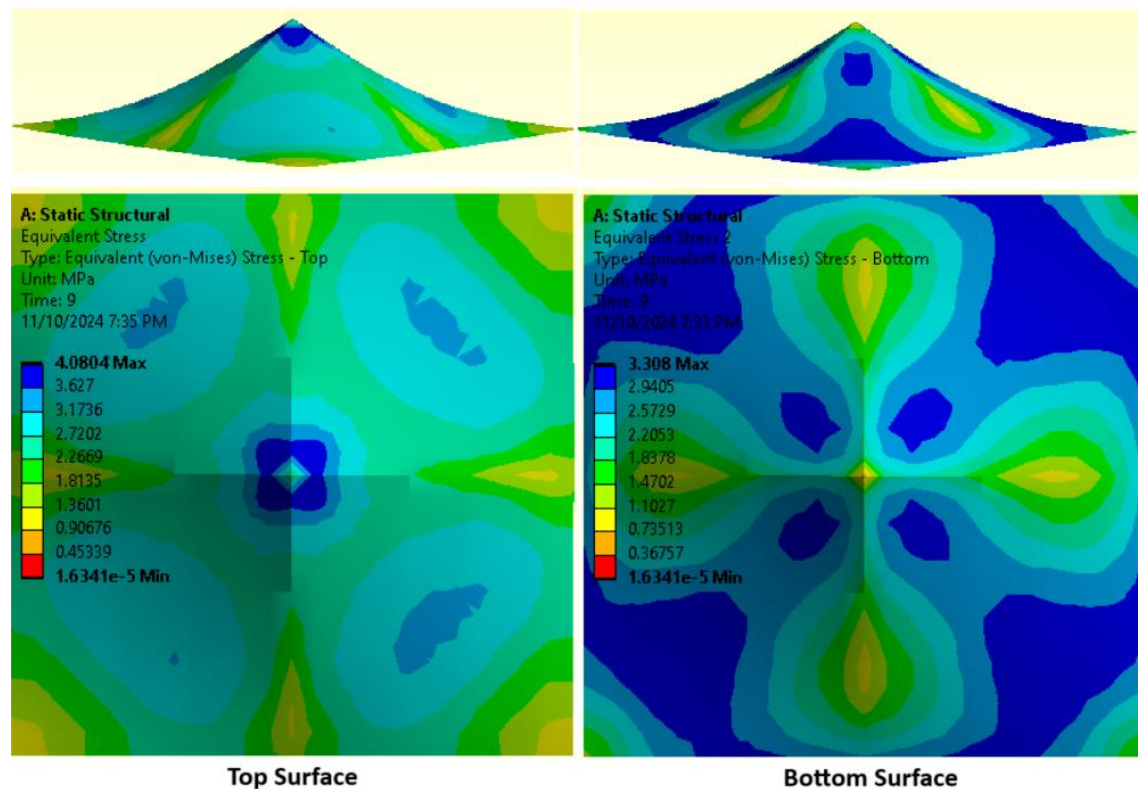


Fig. 7 - Variation in Von-Mises stress in the Shell footing ($f/a = 0.5$)

4 Conclusions

This research examines the comparative performance of hyperbolic paraboloidal shell footing and flat footing when subjected to gravitational loads. The shell footing was designed in accordance with Indian standards, and its settlement characteristics and force developments were analyzed using finite element analysis conducted with ANSYS (2020) fem software. The following conclusions can be drawn based on these findings.

- A hyperbolic paraboloidal shell footing requires substantially less material than a traditional flat footing. For a centric gravity load of 1650 kN, compared to flat footing, hyperbolic paraboloidal shell footing with a f/a ratio of 0.5 requires only 0.61 times of volume.
- The required thickness of a hyperbolic paraboloidal shell footing is considerably less than that of a traditional flat footing in accordance to Indian standard design practices. When subjected to a centric gravity load of 1650 kN with a f/a ratio of 0.5, the necessary design thickness for a hyperbolic paraboloidal shell footing is found to be 0.23 times that of a flat footing.
- The bearing capacity of the shell footing is significantly higher than that of its flat counterpart with the same volume and plan. The ultimate bearing capacity of the inverted hyperbolic paraboloidal shell footing with a f/a ratio of 0.5 and a thickness of 150 mm was determined to be 0.93 times that of the flat footing with a thickness of 650 mm.
- The maximum vertical settlement in the shell footing with a f/a ratio of 0.5 and thickness of 150 mm for a centric load of 1650 kN was found to be 1.04 times that of the flat footing of a thickness of 650 mm.
- The footing–column junction in shell footing exhibits the peak Von Mises stress, predominantly attributed to the punching shear force imposed by the column structure.

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