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Research Paper

Study on the Stiffness of Rail Support for Some Lines of Urban Railway in Vietnam

Tran Anh Dung ^{a,*}, Le Trung Hieu ^b, Le Hai Ha ^a, Pham Van Ky ^a

^a University of Transport and Communications, Hanoi, Vietnam

^b Hanoi Authority for Planning and Investment, Hanoi, Vietnam.

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ABSTRACT

Track stiffness significantly affects how forces are transferred from the train to the substructure. Vietnam is currently in the process of constructing and operating three urban railway lines (URL). This research focuses on analyzing the stiffness of the rail support system and examining the forces exerted on these supports in relation to their stiffness properties. To achieve this, the study employs analytical methods, finite element method (FEM), and simulation techniques for structural analysis. The results reveal that the forces transmitted to the rail bearings are 26,117 N, 24,332 N, and 24,570 N, corresponding to axle loads of 16 tons/axle, 14 tons/axle, and 14.5 tons/axle, respectively. Furthermore, the reduction ratios of train loads transferred to the substructure are 67.35%, 65.24%, and 66.11%. These values are associated with bearing stiffness values of 23.1×10^6 N/mm, 30.8×10^6 N/mm, and 27.5×10^6 N/mm, respectively. Proper determination of rail support stiffness is crucial for efficient railway operations and maintenance. Accurate stiffness values ensure better load distribution across the infrastructure. This prevents excessive stress on specific structural components. In addition, optimizing stiffness helps minimize maintenance costs and prolongs infrastructure lifespan.

1 Introduction

The rail support' stiffness is a crucial specification for track maintenance and design, as it influences load-bearing capacity, rail track quality, the longevity of track components, and vehicle dynamics [1-5]. High rail stiffness increases the load transmitted to the substructure, destroying the substructure and unsmooth track. Reduced rail stiffness leads to greater lateral displacement of the rail, and it increases the risk of derailment due to gauge widening.

Many studies in the world have been studying the stiffness of rail supports. In 2023, Bai et al. studied the stability of the rail pad structural, and how it influences the dynamic behaviour of the vehicle-track matched system [6]. In 2005, Morison

* Corresponding author. Tel.: +(84)983841175.

E-mail address: trananhdung@utc.edu.vn

et al. researched methods to measure the dynamic stiffness of elastic fastenings [7]. Cox and Wang presented their findings on how track stiffness affects vibration levels in railway tunnels in 2003 [8]. In 2022, Othman et al. employed the FEM to evaluate the non-linear static stiffness of the rail pad [9]. Ge et al. investigated the influence of the distributed support of rail pads on vertical interactions of the vehicle-track [10]. Ferreño et al. examined the use of machine learning algorithms to anticipate the mechanical properties of rail pads under in service conditions [11]. The stiffness of track in turnout implemented by Dung et. al in 2003 [12]. Nowadays, the ballast less track is mainly used as the superstructure of urban railways instead of the ballast track structure. The elasticity of this track structure is mainly determined by the rail pad. At present, Vietnam is constructing and operating URL in both Hanoi capital and Ho Chi Minh city. These lines have different structures and train types. This study examines how support stiffness affects the impact of trains on the track structure. The stiffness values were determined for three urban railway lines currently operating in Vietnam. Additionally, the load reduction rate from trains on various bearing structures was analyzed to assess the performance and efficiency of each structural type.

2 Train Load and Support Stiffness of Urban Railways in Vietnam

2.1 Load model of train

This section provides the train load information for three lines: Ben Thanh - Suoi Tien (BT-ST), Cat Linh - Ha Dong (CL-HD), and Nhon - Hanoi Station (N-HS).

The train of BT - ST line comprises three cars, with 16 tons of an axle load.



Fig. 1 – The train of BT – ST line

The train of CL – HD line consists of four cars, with 14 tons of an axle load.



Fig. 2 – The train of CL – HD line

The train on the N – HS line consists of four cars, with 14.5 tons of an axle load.

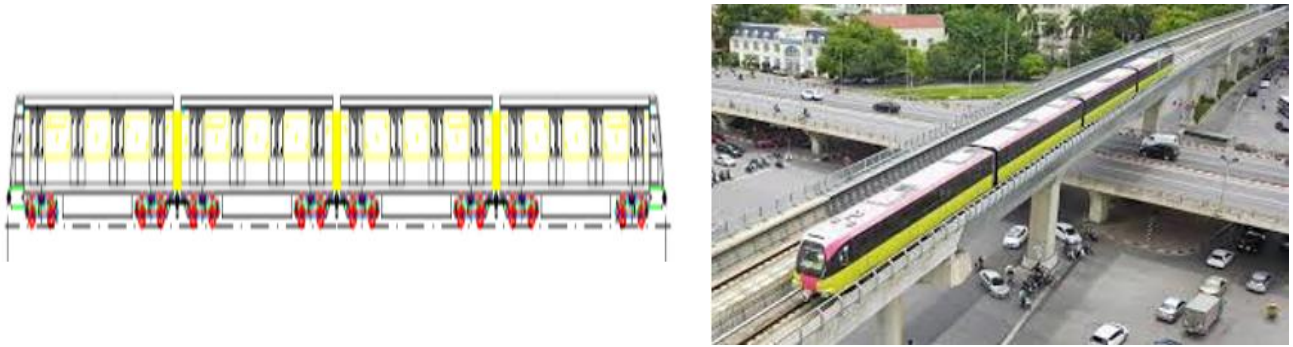


Fig. 3 – The train of N-HS line

2.2 Structure of the Urban Railway Track

The ballastless track structure of the BT - ST line consists of rails placed on sleepers through rail pads. The sleepers were placed on reinforced concrete slabs cast in place through anti-vibration boxes (AVT Box).

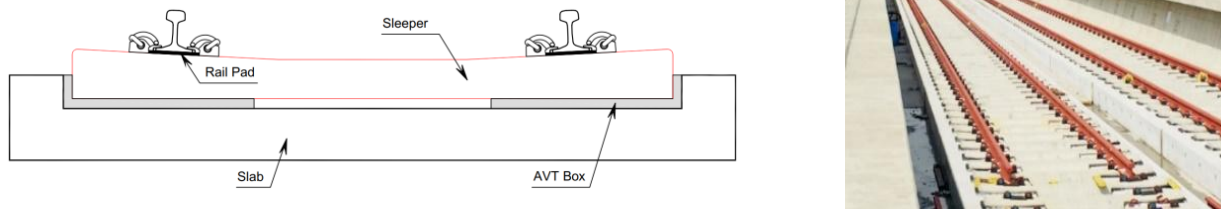


Fig. 4 – Superstructure of the BT – ST line

The ballastless track structure of the CL – HD line consists of rails positioned on short sleepers via rail pads. The sleepers are mounted on reinforced concrete slabs that are cast in place.

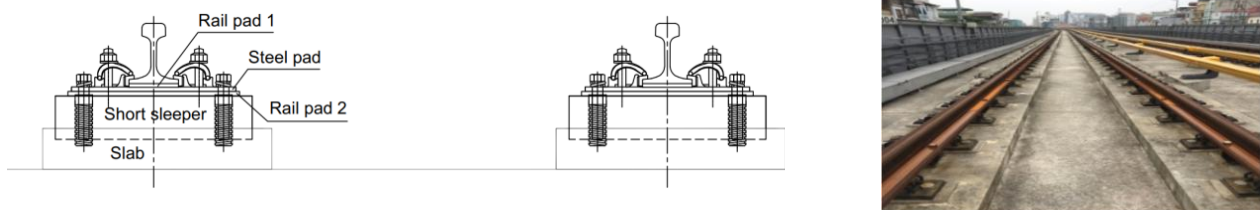


Fig. 5 – Superstructure of CL – HD line

The ballastless track structure of the N – HS line consists of rails mounted on longitudinal reinforced concrete slabs that are cast in place, using rail pads for support.

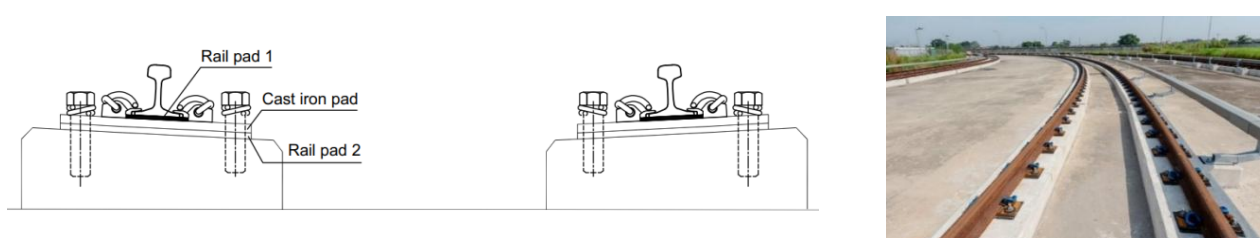


Fig. 6 – Superstructure of N – HS line

2.3 Support Stiffness of Urban Railways

The support stiffness is the total stiffness including the rail pad to the sleeper, anti-vibration box (AVT Box), slab, and substructure.

Fig.7-Fig.9 shows one of the simplest models of the corresponding urban railway systems of the BT - ST, CL - HD, and N - HS lines.

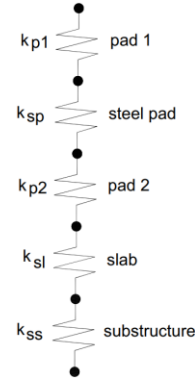
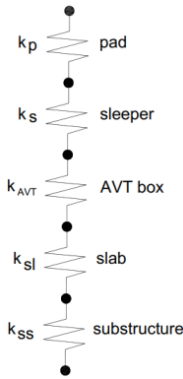


Fig. 7 – Total stiffness of rail support of BT – ST line

Fig. 8 – Total stiffness of the rail support for the CL – HD line

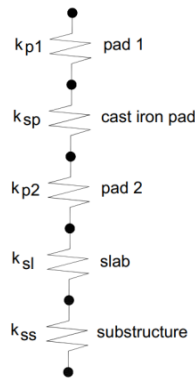


Fig. 9 – Total stiffness of the rail support for the N-HS station line

The equation for determining the rail support' stiffness of the BT - ST line is as follows

$$\frac{1}{k_1} = \frac{1}{k_p} + \frac{1}{k_s} + \frac{1}{k_{AVT}} + \frac{1}{k_{sl}} + \frac{1}{k_{ss}} \quad (1)$$

The equation for determining the rail support stiffness of the CL – HD line is as follows

$$\frac{1}{k_2} = \frac{1}{k_{p1}} + \frac{1}{k_{sp}} + \frac{1}{k_{p2}} + \frac{1}{k_{sl}} + \frac{1}{k_{ss}} \quad (2)$$

The equation for determining the rail support stiffness of the N – HS line is as follows

$$\frac{1}{k_3} = \frac{1}{k_{p1}} + \frac{1}{k_{sp}} + \frac{1}{k_{p2}} + \frac{1}{k_{sl}} + \frac{1}{k_{ss}} \quad (3)$$

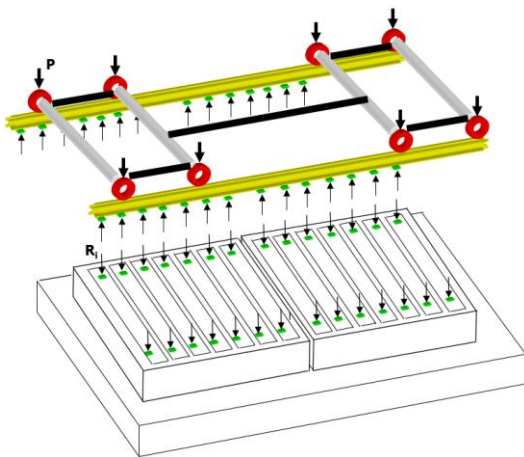


Fig. 10 – Model of rail support of BT-ST line

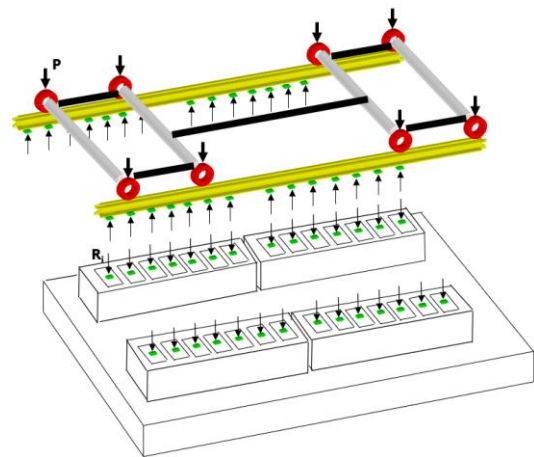


Fig. 11 – Model of rail support of CL-HD line

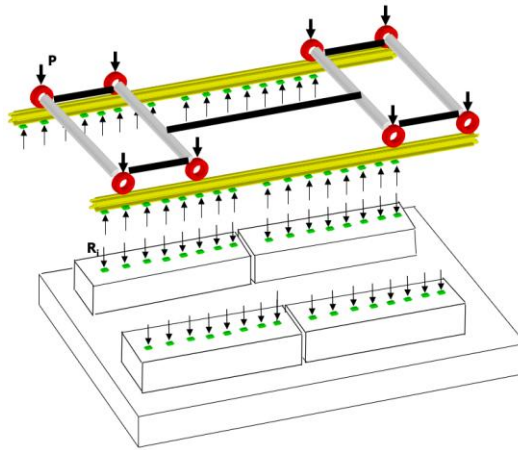


Fig. 12 – Model of rail support of N-HS line

3 Calculation of Support Stiffness and Forces of Urban Railway Acting on the Substructure by Finite Element Method

According to formulas (1), (2) and (3) we have the support stiffness of URL as shown in the table below

Table 1 – Support stiffness

Stiffness (N/m)	BT – ST	CL – HD	N – HS
k_p	10^8		
k_s	∞		
k_{AVT}	3×10^7		
k_{sl}	∞	∞	∞
k_{ss}	∞	∞	∞
k_{p1}		5×10^7	55×10^6
k_{p2}		8×10^7	55×10^6
k_{sp}		∞	∞
k	23.1×10^6	30.8×10^6	27.5×10^6

The rail is treated as a continuous long beam positioned on resilient supports. The calculation is conducted on an 80meter long, rail segment divided into 80 elements, with each element measuring 0.625 meters in length. The P60 rail has elastic modulus of 210 Gpa, and inertia moment of 3,217 cm⁴. The axle load of the train is 16 tons per axle on the BT - ST line, 14 tons per axle on the CL - HD line, and 14.5 tons per axle on the N - HS line.

The process of solving problems using the FEM is as follows:

In Figure 13, we divide the rail structure into elements

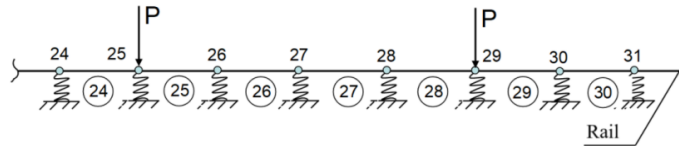


Fig. 13 – Discrete rail structure

A rail element is illustrated in Figure 14.

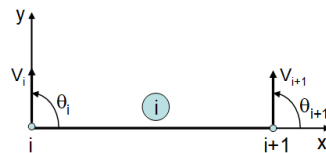


Fig. 14 – Rail element

Stiffness matrix of the element

$$[k] = \begin{bmatrix} \frac{12EJ}{a^3} + c & \frac{6EJ}{a^2} & -\frac{12EJ}{a^3} & \frac{6EJ}{a^2} \\ \frac{6EJ}{a^2} & \frac{4EJ}{a} & -\frac{6EJ}{a^2} & \frac{2EJ}{a} \\ -\frac{12EJ}{a^3} & -\frac{6EJ}{a^2} & \frac{12EJ}{a^3} + c & -\frac{6EJ}{a^2} \\ \frac{6EJ}{a^2} & \frac{2EJ}{a} & -\frac{6EJ}{a^2} & \frac{4EJ}{a} \end{bmatrix} \quad (4)$$

Node displacement vector of element

$$\{\delta\}^e = \begin{Bmatrix} v_i \\ \theta_i \\ v_{i+1} \\ \theta_{i+1} \end{Bmatrix} \quad (5)$$

Element node force

$$\{F\}^e = \begin{Bmatrix} V_i \\ M_i \\ V_{i+1} \\ M_{i+1} \end{Bmatrix} \quad (6)$$

Solve the equations

$$[K^*]\{\Delta^*\} = \{F^*\} \quad (7)$$

We find the displacements at the nodes

$$\{\Delta^*\} = [K^*]^{-1} \{F^*\} \quad (8)$$

Substitute into the equation

$$[k]\{\delta\}^e = \{F\}^e \quad (9)$$

We have

$$\{F\}^e = \begin{Bmatrix} V_i \\ M_i \\ V_{i+1} \\ M_{i+1} \end{Bmatrix} \quad (10)$$

The forces R_i acting on the rail supports are determined based on the following formula:

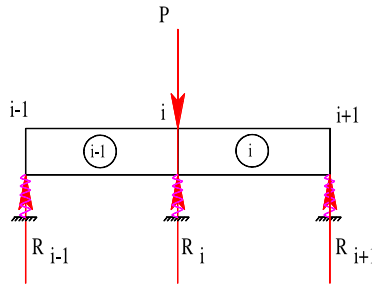


Fig.15 – Reaction at the rail supports

We have

$$\Delta Q_i = Q_i^i - Q_i^{i-1} = V_i^i - (-V_i^{i-1}) = R_i - P \rightarrow R_i = V_i^i + V_i^{i-1} + P \quad (11)$$

At the rail supports, we can determine the force values such as in Figure 16.

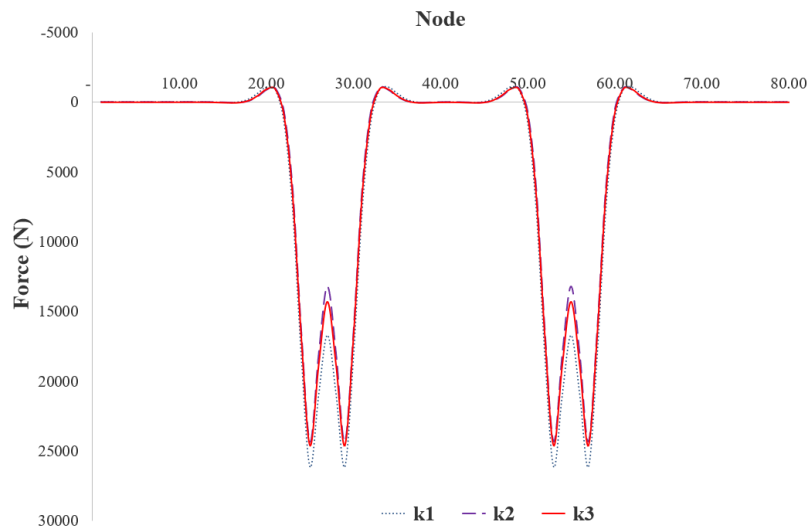


Fig. 16 – Diagram of the forces at the rail support

The max force values acting on the rail support corresponding to stiffnesses k_1 , k_2 , and k_3 are 26,117 N, 24,332 N, and 24,570 N. Based on the results, we recognize that the force values affecting the rail supports depend on the train load and the stiffness of the rail supports.

In addition, the authors use a simulation method with MIDAS software [12] with input data as above, the results are shown as follows:

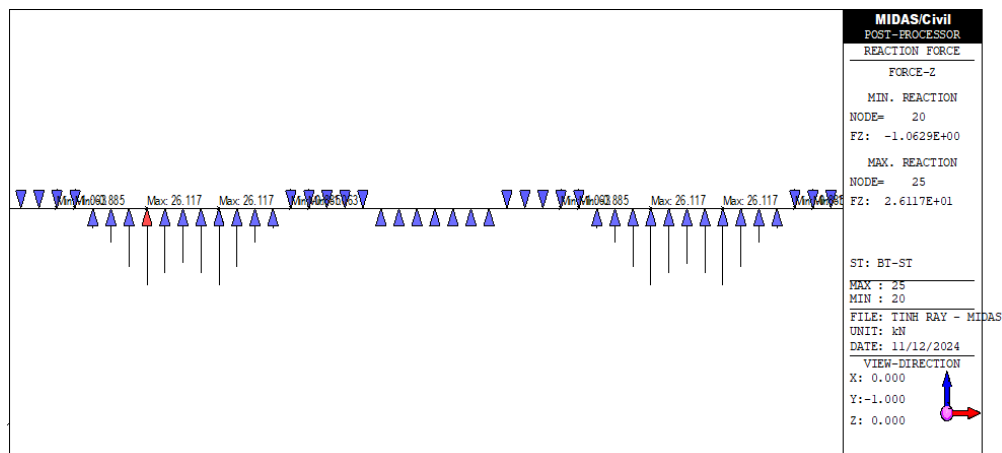


Fig. 17 - Simulation results of BT-ST line

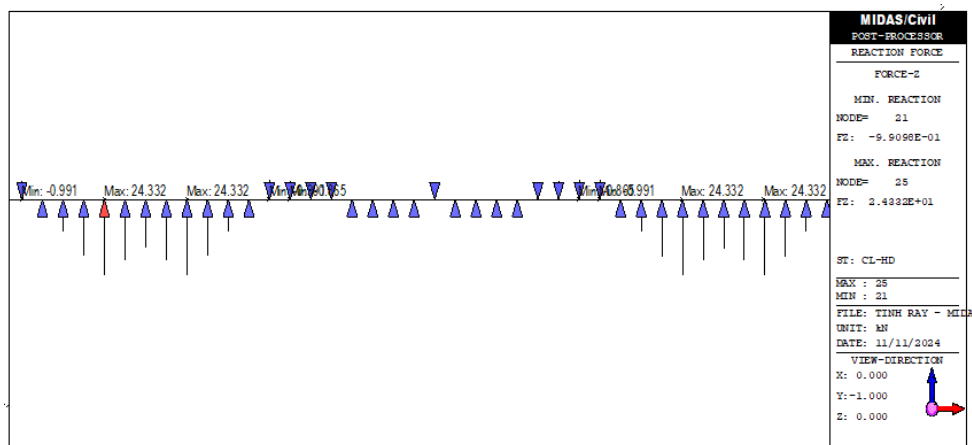


Fig. 18 - Simulation results of CL-HD line

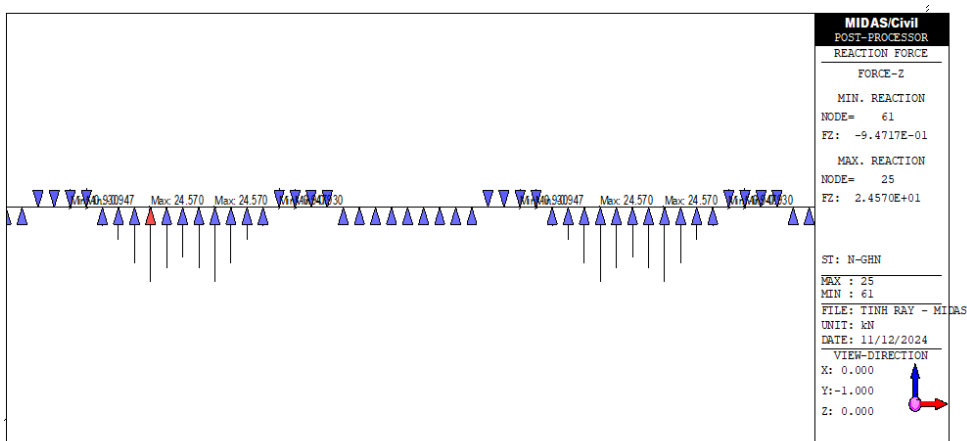


Fig. 19 – Simulation results of N-HS line

The maximum force values acting on the bearing corresponding to the 3 lines BT-ST, CL-HD, and N-HS according to the simulation method are 26,117 N, 24,332 N, and 24,570 N.

4 Discussion

The calculation results for the force value transmitted to the bearing between the finite element method and the simulation method are consistent.

Table 2 – Compare calculation results

Line	BT-ST	CL-HD	N-HS
Finite Element Method	26,117	24,332	24,570
Simulation Method	26,117	24,332	24,570

The better the elasticity stiffness of the bearing, the greater the load reduction value transmitted to the substructure.

Table 3 – Rate of load reduction transmitted to the bearing

Line	BT-ST	CL-HD	N-HS
Maximum force value transmitted to the bearing (N)	26,117	24,332	24,570
Wheel load (N)	80,000	70,000	72,500
Rate of load reduction transmitted to the bearing (%)	67.35	65.24	66.11

Based on the above results, we see that the bearing's stiffness of the BT-ST line is the best, so the ratio of load reduction transmitted to the substructure is the largest.

5 Conclusions

The stiffness of rail support affects the ability to transmit the force of the train from the superstructure to the substructure. The greater the elasticity of the bearing, the more effectively it reduces the train load transferred to the substructure. This specification impacts the smoothness of the track railway. Support stiffness is highly valuable for identifying the causes of certain substructure issues. The results can be used to give track conditions to guide maintenance planning and implementation. Accurate measurement of bearing stiffness is a tool for engineers to perform maintenance activities correctly, optimizing the maintenance budget. The study's findings provide valuable insights into structural performance under different loading conditions. Engineers can use these results to refine track design specifications. Enhanced stiffness management contributes to improved safety and reliability of urban railway systems. Moreover, the analysis highlights the importance of balancing stiffness values to reduce dynamic effects. A systematic approach to stiffness evaluation can address operational challenges effectively. In conclusion, understanding rail support stiffness is essential for sustainable urban railway development in Vietnam.

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