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Determine cable dynamic parameters from vibration measurement results

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ABSTRACT

Structural health monitoring is a particularly important issue for cable-stayed bridges. The main components of a cable-stayed bridge include the main girder, tower, and stay cables. Stay cables act as a support for the girder, helping it have a large span length. Therefore, stay cables are essential parts of cable-stayed bridges and extrados bridges. Determining the parameters of the stay cable is necessary work. Among the parameters of cable-stayed cables, dynamic parameters such as frequency and damping ratio play an important role. This paper presents a method combining the Hilbert transform and Bandpass filter to determine the frequency, damping ratio and logarithmic decrement of the stay cables. The analysis results show the logarithmic decay changes over time during the vibration. Therefore, to estimate the details, it is necessary to divide the data segment before calculating the logarithmic decay. The results of calculating the vibration frequency using the Hilbert transform method are similar to those of calculating the vibration frequency using the FFT method. The calculation results from the proposed method are compared with those of the traditional method and are highly accurate.

1 Introduction

The operation of stay cables depends heavily on cable parameters, so determining the main parameters of stay cables attracts much attention from researchers worldwide. Determining the parameters of stay cables through vibration measurement results is a highly effective method. Initially, the study of cable-stayed bridge vibrations was closely related to the theory of tensioned wire vibrations. Over many years, with the contributions of many scientists worldwide, vibration theories have been increasingly improved. In the early stages, most studies did not consider the simultaneous effects of cable sag extension and stiffness. In addition, the cable boundary conditions are also considered simple boundary condition cases to facilitate the solution of partial differential equations. But in reality, the operation of the cable is affected not only

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by the sag and stiffness of the cable but also by the cable damping. These parameters significantly influence the actual performance of the stay cables.

Recently, in a review article on cable structures in the Journal of Bridge Engineering, Dongzhou Huang and Wei-zhen Chen [1], the content of estimating tension in cables still attracts the attention of scientists worldwide. The first of the three typical articles mentioned by Huang and Chen is the article by Morgenthal et al. [2]; in this article, the research team also used the vibration method to estimate the cable tension; the new point in this article is that the research team used a mobile vibration measurement device connected to a smartphone. The nonlinear finite element model supports estimating the cable's tension force to give accurate results. In the second article, Yong-Hui Huang et al. [3] used the Form-Finding Theory to determine the tension in the cable. With this method, only by measuring the coordinates of the main cable, the information about the main cable and the hangers can be accurately determined. Furthermore, the vibration method is almost impossible when hangers with shock absorbers are used, while the proposed method does not affect accuracy. In the third article, Zarbaf et al. [4] presented a method using an optimization algorithm to estimate the tension force in the cable. This is a problem when determining the structural system, so in addition to the tension force in the cable, the stiffness parameters, cable sag, and cable area can also be determined.

The above studies show the importance of cable parameter estimation research. Knowing the cable parameters will help us see the structure's condition, which is an essential job in bridge health monitoring. Cable parameter determination can be obtained by various methods such as analytical methods, methods supported by optimization algorithms or artificial intelligence... The work of determining parameters in cables is only one step in assessing the actual condition of the main structure (such as a cable-stayed bridge, suspension bridge or arch bridge). With the current trend, determining parameters in cables determines the stationary value and the value changing in real-time. To do this, modern methods are needed to overcome the limitations of methods that only apply to stationary signals. Besides, with the traditional method of evaluating damping from the displacement (acceleration) diagram and determining frequency through Fast Fourier transform (FFT) of the measured signal, it is impossible to decide on the damping of the corresponding vibration modes. In this paper, the author presents a method based on Hilbert transform and signal filtering to determine the main dynamic parameters of the cable, such as vibration frequency and damping ratio with corresponding vibration mode. Once the damping ratio is obtained, the logarithmic decrement can be determined. The results of the proposed method are compared with traditional methods and give highly accurate results.

In practice, the results of determining the logarithmic decrement of the cable are also used to determine whether or not a damper is required.

2 Theoretical bases

2.1 Hilbert transform

Hilbert transform of the real value function $x(t)$ is the real value function, defined by

$$y(t) = H[x(t)] = \frac{1}{\pi} P \int_{-\infty}^{+\infty} \frac{x(u)}{t-u} du \quad (1)$$

where P is the Cauchy principal value

we can define the analytic signal

$$z(t) = x(t) + iy(t) \quad (2)$$

With the instantaneous parameters have magnitude

$$a(t) = \sqrt{x(t)^2 + y(t)^2} \quad (3)$$

and

$$\theta(t) = \arctan\left(\frac{y(t)}{x(t)}\right) \quad (4)$$

The above equation can be rewritten as:

$$z(t) = a(t)e^{i\theta(t)} \quad (5)$$

Then, the instantaneous frequency is defined as follows

$$\omega(t) = \frac{d\theta(t)}{dt} \text{ and } f(t) = \frac{1}{2\pi} \frac{d\theta(t)}{dt} \quad (6)$$

2.2 Analytic signal

According to Armin B. Mehrabi and Habib Tabatabai [5], the theoretical basis of cable vibration is as follows

$$\frac{\partial^2}{\partial x^2} \left(EI \frac{\partial^2 v}{\partial x^2} \right) - H \frac{\partial^2 v}{\partial x^2} - h \frac{d^2 y}{dx^2} + k'v + c' \frac{\partial v}{\partial t} + m \frac{\partial^2 v}{\partial t^2} = 0 \quad (7)$$

where: EI is the bending stiffness; v is the displacement in the plane due to vibration; y is the displacement due to the self-weight of the cable; H is the horizontal force component of the cable tension; h is the horizontal force component added to the cable tension due to the vibration process; k' is the spring constant; c' is coefficient of viscosity; and m is mass.

Transform into an eigenvalue problem, we have

$$v(x, t) = w(x)Y(t) \quad (8)$$

Here we have $w(x)$ and $Y(t)$ are the parts of $v(x, t)$

For the case of free vibration with cable damping, Y of the n th vibration mode is represented as follows

$$Y_n(t) = e^{p_n t} \quad (9)$$

Here p_n is a complex number represented in the form

$$p_n = -\xi_n \omega_n \pm i\omega_{nd} \quad (10)$$

In there, ξ_n is the damping ratio; ω_n is the undamped natural angular frequency; ω_{nd} is the angular frequency of the damped vibration

$$\omega_{nd} = \omega_n \sqrt{1 - \xi_n^2} \quad (11)$$

we can write the form following

$$Y_n(t) = [A_n \cos(\omega_{nd}t + \varphi_n)] \exp(-\xi_n \omega_n t) \quad (12)$$

Hilbert transform, we have

$$\tilde{Y}_n(t) = [A_n \sin(\omega_{nd}t + \varphi_n)] \exp(-\xi_n \omega_n t) \quad (13)$$

Thus, we have an analytic signal as follows

$$Z_n(t) = Y_n(t) + j\tilde{Y}_n(t) = A_n e^{-\xi_n \omega_n t} \cdot e^{j(\omega_{nd} t + \varphi_n)} = B_n(t) e^{j\theta_n(t)} \quad (14)$$

$$B_n(t) = A_n e^{-\xi_n \omega_n t} \Rightarrow \ln(B_n(t)) = -\xi_n \omega_n t + C \quad (15)$$

$$\theta_n(t) = \omega_{nd} \cdot t + \varphi_n \Rightarrow \frac{d\theta}{dt} = \omega_{nd} \quad (16)$$

Thus, we will use linear curve fitting of plot of $\theta_n(t)$ and $\ln(B_n(t))$ to determine ω_{nd} and ξ_n .

Once ξ_n is obtained, the logarithmic decrement can be determined using the following approximate formula [6].

$$\delta_n = \frac{2\pi\xi_n}{\sqrt{1-\xi_n^2}} \text{ so } \delta_n \approx 2\pi\xi_n \text{ if } \xi_n \ll 1 \quad (17)$$

2.3 Bandpass Filter

Using BPF for filtering frequency transforms the signal from the time domain to the frequency domain, and then the window function cuts unwanted frequencies in the frequency domain. Finally, the obtained results will be inverted to the time domain. This approach will eliminate unwanted frequencies, but it will contain the limitations of the process of transformation from the time domain to the frequency domain and vice versa. The BPF is a combination of the Low Pass Filter and High Pass Filter, which can be illustrated as shown in the figure below:

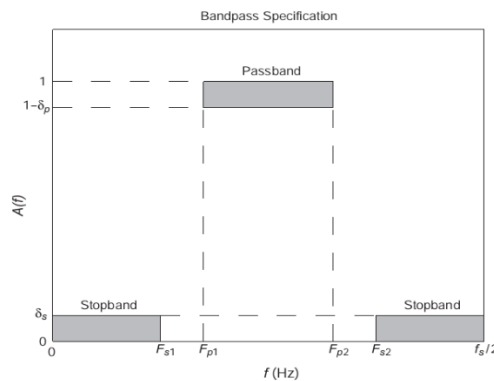


Fig. 1 – Linear Magnitude Response Specifications for a Bandpass Filter

The design specification for the magnitude response of a band pass filter is more involved because two transition bands are bracketing the pass band, as seen in Figure 1. Thus, there are four cutoff frequencies plus a pass band ripple δ_p and a stop band attenuation δ_s .

Thus, if we use a BPF, the obtained signal, then the obtained result will contain a frequency band corresponding to the bandwidth that we have just designed. By adjusting bandwidth reasonably, we can restrict errors in filtering.

In the case of a signal with multiple frequencies, then we need to perform BPF before performing the Hilbert transform.

3 Structure parameters identification

3.1 Theoretical verification

To verify the theory proposed above, the paper analyzes a simple example to clearly see the effectiveness of the proposed method. The signal input to the analysis is a mathematical function defined as follows

$$\text{Function} = A * \sin(2 * \pi() * f * t) * \exp(-\text{damp} * 2 * \pi() * f * t) \quad (18)$$

with $f=7.88\text{Hz}$; $damp=0.005$; Since the amplitude A does not affect the result of the method, it can be chosen arbitrarily.

The input signal is shown as follows:

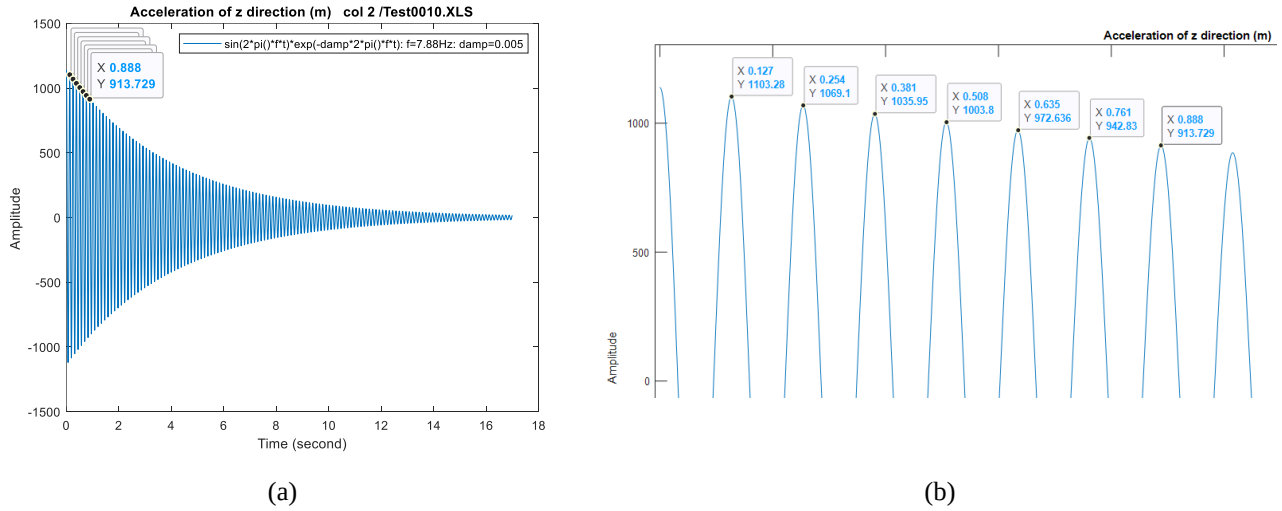


Fig. 2 – Shows the signal (a) and details of the signal amplitude peaks (b).

From the above signal, analysis according to the proposed theory gives the following results:

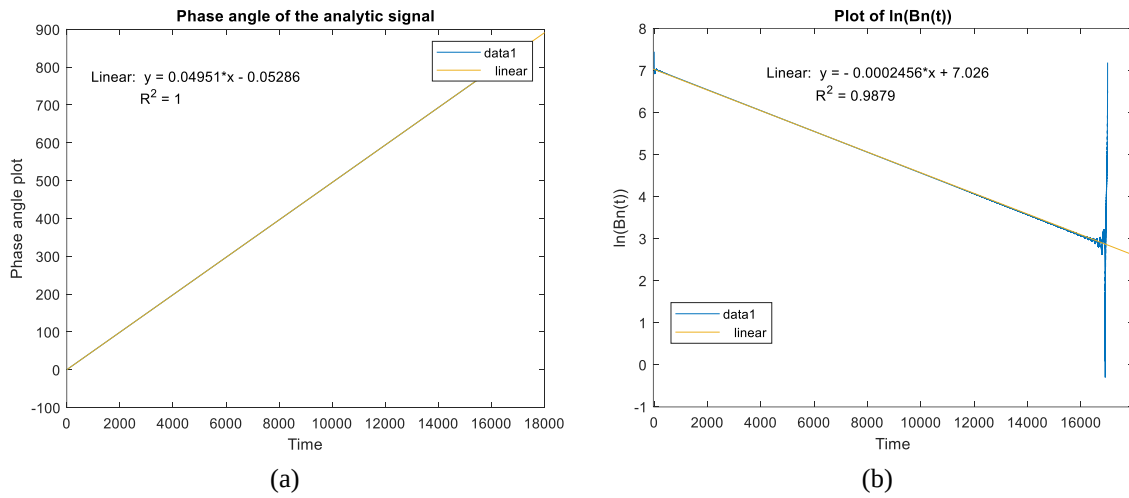


Fig. 3 – The phase angle diagram (a) and $\ln(Bn(t))$ diagram of the signal (b)

Applying the curve fit linear line to the phase angle diagram and $\ln(Bn(t))$ yields equations of the form $y=a*x+b$ as shown in the figures. These linear lines completely agree with the above theory, so we can base the calculation on the coefficients a of the equation below to determine ω_{nd} and ξ_n .

From equation 15, we can determine $\omega_n \xi_n = a / \text{time_step} = a * 1000$ (because time step = 1/1000s).

From equation 16, we can determine $\omega_{nd} = a / \text{time_step} = a * 1000$ (because time step = 1/1000s).

From the above figures, we can be identified $\omega_{nd} = 49.51 \text{ rad/s}$, $f_{nd} = 7.88\text{Hz}$ and $\xi_n = 0.005$

From equation 17, we can determine $\delta_n = 0.0314$ (3.14%).

As we know, the logarithmic decrement can also be calculated by the formula.

$$\delta_n = \ln \left(\frac{A_n}{A_{n+1}} \right) \quad (19)$$

To increase the accuracy we can use the regression line of the function e^x from the peaks of the signal in figure 2 to determine logarithmic decrement δ_n .

From the peaks obtained in figure 2, draw a regression line to determine the logarithmic decrement as follows:

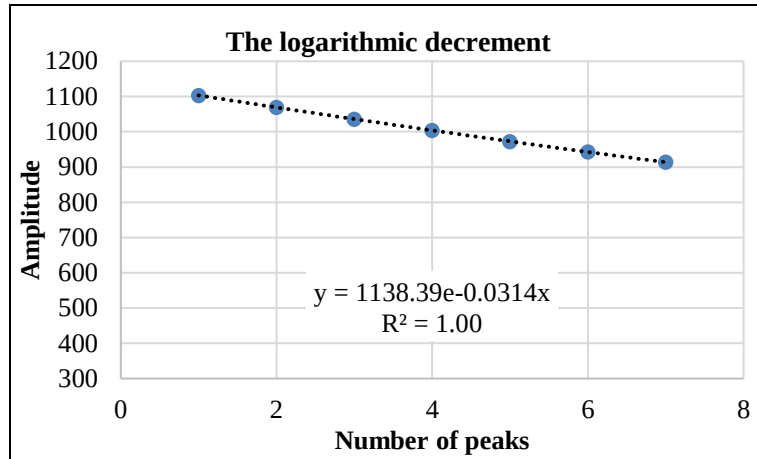


Fig. 4 – Shows the result of the logarithmic decrement

From the above picture shows $\delta_n = 0.0314$ (3.14%). This result is completely consistent with the analysis result by the proposed method.

Through a simple example, it is shown that the proposed method above can determine the vibration frequency, damping ratio and logarithmic decrement of any single-frequency vibration signal.

3.2 Application examples

Experimental cable length $L=4.68\text{m}$; modulus of elasticity $E=2 \times 10^{11}\text{N/m}^2$; cable cross-sectional area $A=9.871 \times 10^{-5}\text{m}^2$; material density $\rho=7.846\text{kN/m}^3\cdot\text{g}$; On the cable, an accelerometer is attached to collect vibration results with a sampling frequency of 1kHz. Cable tension $P=151.5\text{kN}$, controlled through gauges located at the cable end.



(a)



(b)

Fig. 5 – Experimental equipment layout. (a) Install cable and accelerometer (b) cable tension, vibration stimulation and data acquisition.

After the cable is stimulated to vibrate, we have an acceleration result during 22.5s, as shown below.

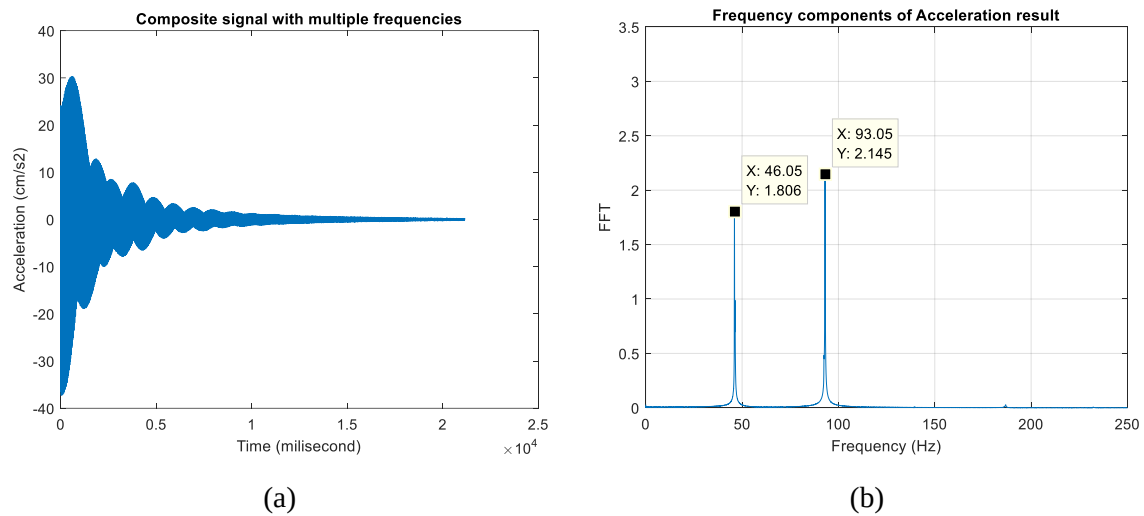


Fig. 6 – Acceleration result during 22.5s (a) and Its FFT (b)

The figure above shows the vibration measurement and FFT results, showing that this acceleration signal synthesizes two signals with corresponding vibration frequencies of 46.05Hz and 93.05Hz. With the results as shown in the figure, it is possible to obtain the frequency of the cable and the cable damping from the acceleration graph with only FFT, but it is not possible to separate the damping corresponding to a certain vibration mode (in other words, the damping coefficient is not accurate because it cannot be identified corresponding to the vibration mode).

Therefore, to get more accurate results (vibration frequency and damping of the corresponding vibration mode), it is necessary to decompose the above signal into two signals with vibration frequencies of 46.05Hz and 93.05Hz. Then, the above formulas are applied to calculate the frequency and damping ratio corresponding to the vibration mode of the cable.

In this article, the author uses the BPF method to filter and separate the signal into signals with only one frequency: the signal with a frequency of 46.05Hz and the signal with a frequency of 93.05Hz. The FFT result after being BPF is shown below.

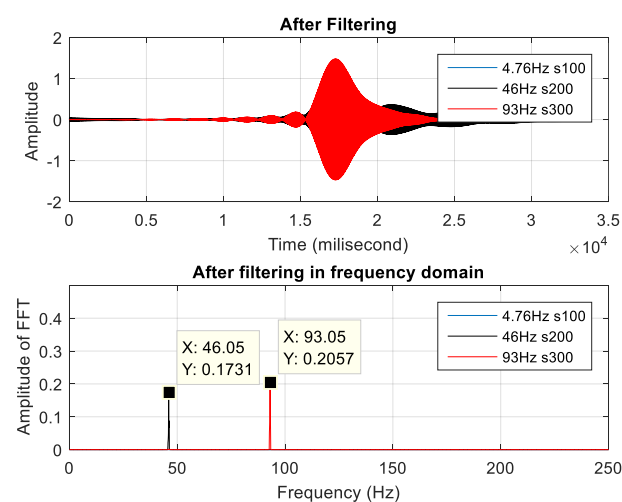


Fig. 7 – Acceleration result after filtering and Its FFT

Perform Hilbert transform on signal with frequency 93.05Hz and 46.05Hz, draw phase angle diagram and $\ln(Bn(t))$ diagram against time as shown below. Applying the curve fit linear line to the phase angle diagram and $\ln(Bn(t))$ yields equations of the form $y=a*x+b$ as shown in the figures. These linear lines completely agree with the above-mentioned

theory, so we can base the calculation on the coefficients a of the equation below to determine ω_{nd} and ξ_n . From equation 15, we can determine $\omega_h \xi_n = a / \text{time_step} = a * 1000$ (because time step = 1/1000s). From equation 16, we can determine $\omega_{nd} = a / \text{time_step} = a * 1000$ (because time step = 1/1000s).

- **For the case of signals with a frequency 93.05Hz**

From Figure 7, it shows that the damping ratio tends to change over time. Therefore, in order to accurately evaluate the damping ratio, it is necessary to divide the signal into small segments for analysis.

Select one second signal less affected by white noise for signal analysis. The analysis results are shown in the figures below.

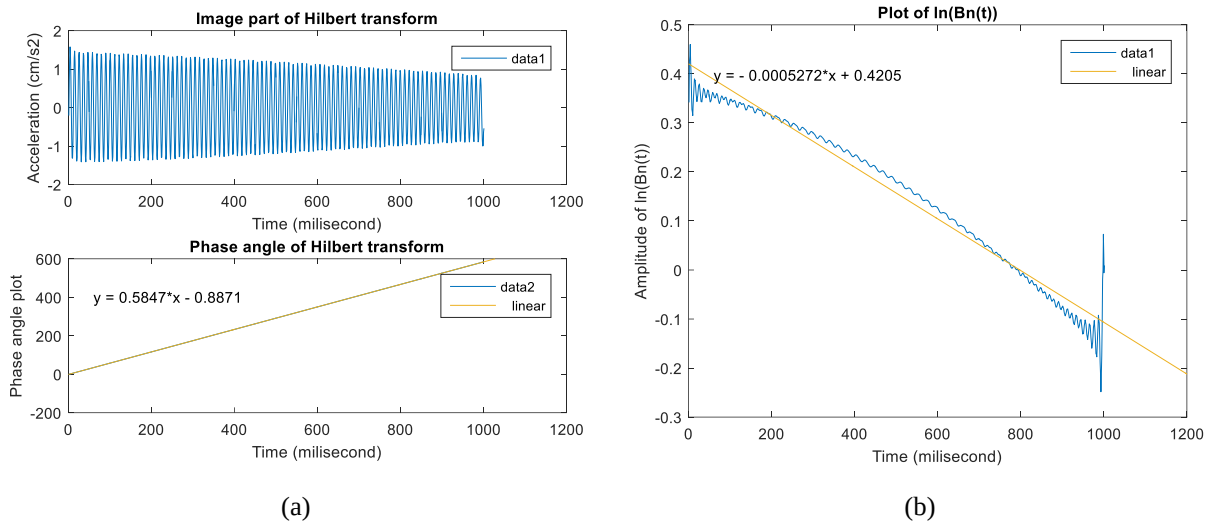


Fig. 8 – Its Hilbert transform and phase angle plot of signals with frequency 93.11Hz (a) Plot of $\ln(Bn(t))$ (b)

From the above figures, we can be identified $\omega_{nd} = 584.7 \text{ rad/s}$, $f_{nd} = 93.11 \text{ Hz}$ and $\xi_n = 0.09\%$.

From equation 17, we can determine logarithmic decrement $\delta_n = 0.57\%$.

From figure 8, it shows that the frequency of the vibration signal does not show any change over time, but the damping ratio will change over time.

To determine the logarithmic decrement, it is necessary to capture the peaks at the beginning of the signal and the peaks at the end of the signal.

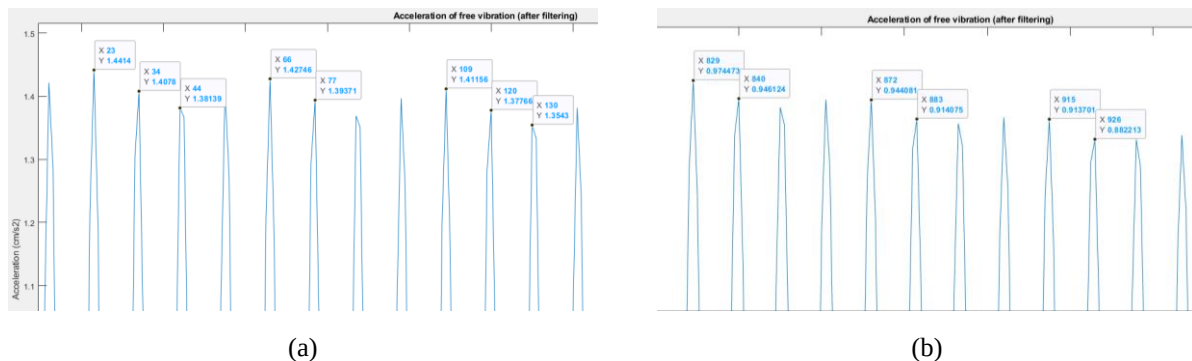


Fig. 9 – Details the signal amplitude peaks of beginning part (a) and ending part (b).

From the signal beginning part, the logarithmic decrement is calculated to be 0.38%; from the ending part, the logarithmic decrement is calculated to be 0.71%, the average is 0.55%, and the deviation compared to the above calculation is 3%. From the above results, it can be seen that it is necessary to divide the measured signal into smaller segments to obtain more accurate results.

Select 0.5 seconds of the signal beginning part for signal analysis. The analysis results are shown in the figures below.

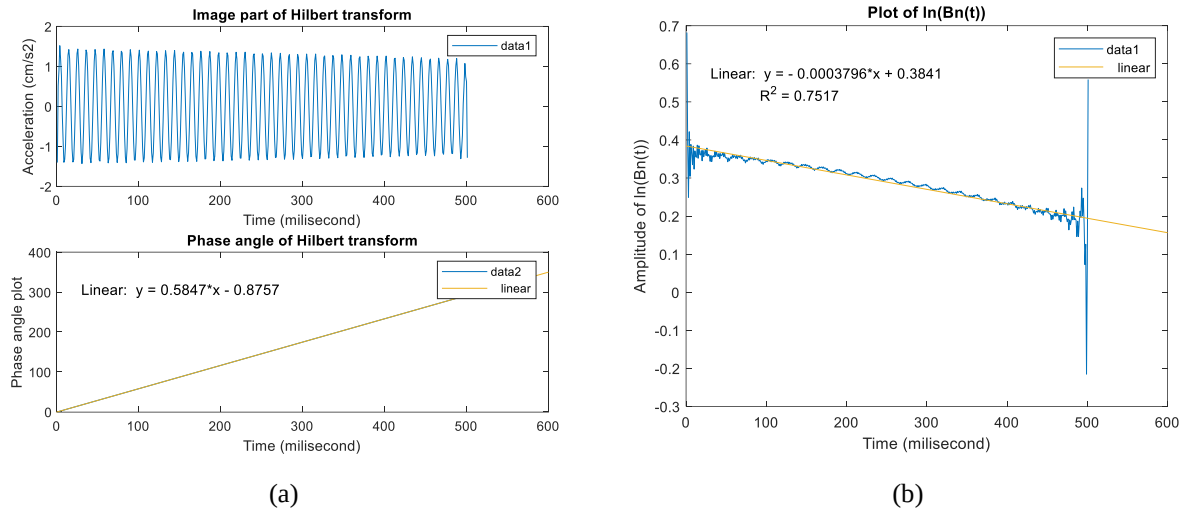


Fig. 10 – Its Hilbert transform and phase angle plot (a) and plot of $\ln(Bn(t))$ (b) of signal beginning part with 0.5 second

From the above figures, we can be identified $\omega_{nd}=584.7\text{rad/s}$, $f_{nd}=93.11\text{Hz}$ and $\xi_n=0.065\%$.

From equation 17, we can determine logarithmic decrement $\delta_n=0.4\%$. This result is not significantly different from the traditional amplitude peak capture method.

Select a 0.5-second signal at the beginning part for signal analysis. The analysis results are shown in the figures below.

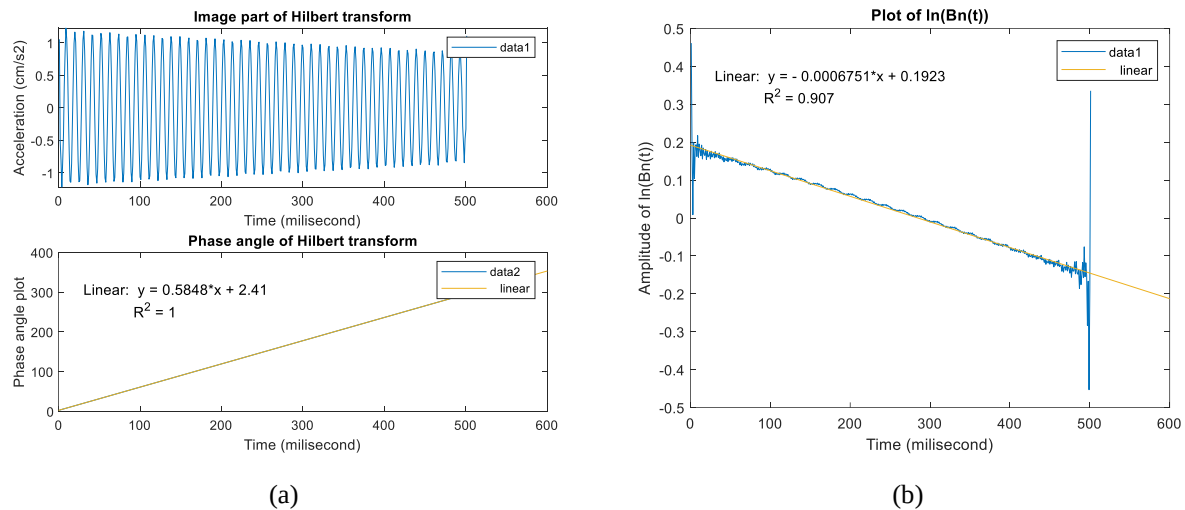


Fig. 11 – Its Hilbert transform and phase angle plot (a) and plot of $\ln(Bn(t))$ (b) of signal ending part with 0.5 second

From the above figures, we can be identified $\omega_{nd}=584.8\text{rad/s}$, $f_{nd}=93.12\text{Hz}$ and $\xi_n=0.115\%$.

From equation 17, we can determine logarithmic decrement $\delta_n=0.72\%$. So if we take the average result, we get the logarithmic decrement $\delta_n=0.56\%$.

This result shows that the logarithmic decrement changes over time during the vibration. To estimate the details, it is necessary to divide the data segment before calculating the logarithmic decrement.

- For the case of signals with frequency 46.05Hz

Select a 1 second signal that is less affected by white noise for signal analysis. The analysis results are shown in the figures below.

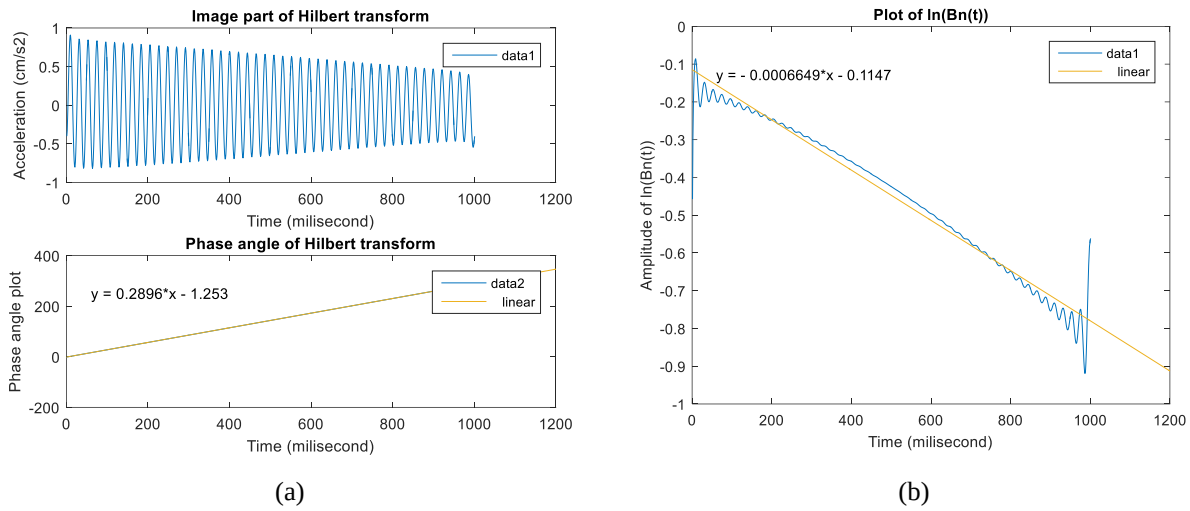


Fig. 12 – Its Hilbert transform and phase angle plot of signals with frequency 46.05Hz (a) Plot of $\ln(Bn(t))$ (b)

Figure 12 shows that the vibration frequency does not change in the signal segment being analyzed, but the damping ratio will change in the signal segment being analyzed.

From the above figures, we can be identified $\omega_{nd}=289.6\text{rad/s}$, $f_{nd}=46.11\text{Hz}$ and $\xi_n=0.23\%$.

From equation 17, we can determine $\delta_n=1.44\%$.

The above results show that the filtered signal clearly shows the nature of only one frequency through the phase angle diagram (the linear curve fitting almost coincides with the phase angle diagram over time). The frequency result calculated from the phase angle diagram shows a negligible deviation compared to the result obtained from the familiar FFT method.

As analyzed in the above case, here we also split into two data segments in 0.5 seconds for analysis

For the first 0.5-second data segment, the results are as follows:

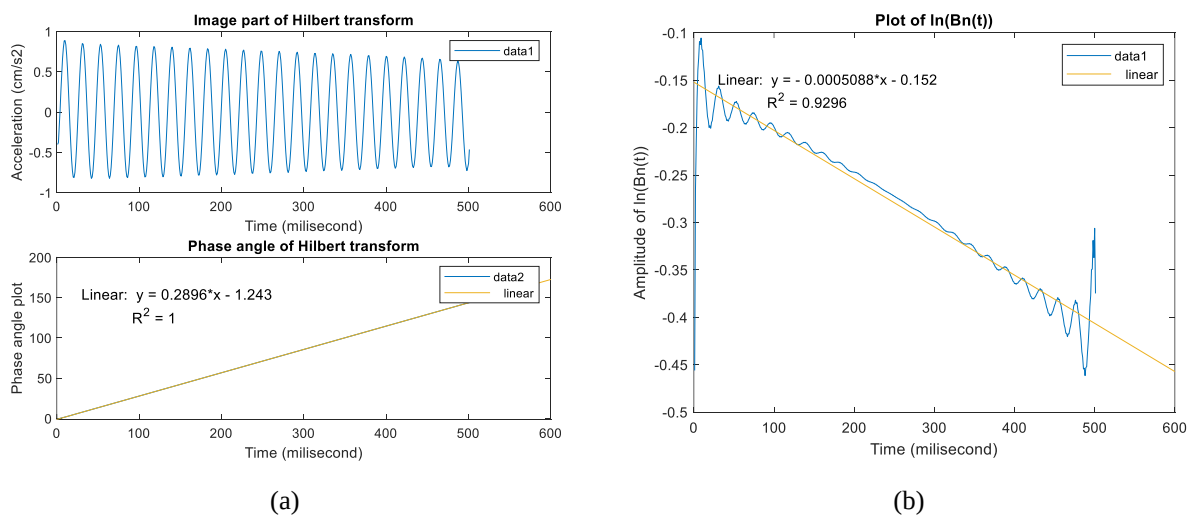


Fig. 13 – Its Hilbert transform and phase angle plot (a) Plot of $\ln(Bn(t))$ (b) of the signal first 0.5 second with frequency 46.05Hz

From the above figures, we can be identified $\omega_{nd}=289.6\text{rad/s}$, $f_{nd}=46.11\text{Hz}$ and $\xi_n=0.18\%$.

From equation 17, we can determine $\delta_n=1.11\%$.

For the last 0.5 second the data shows the following results:

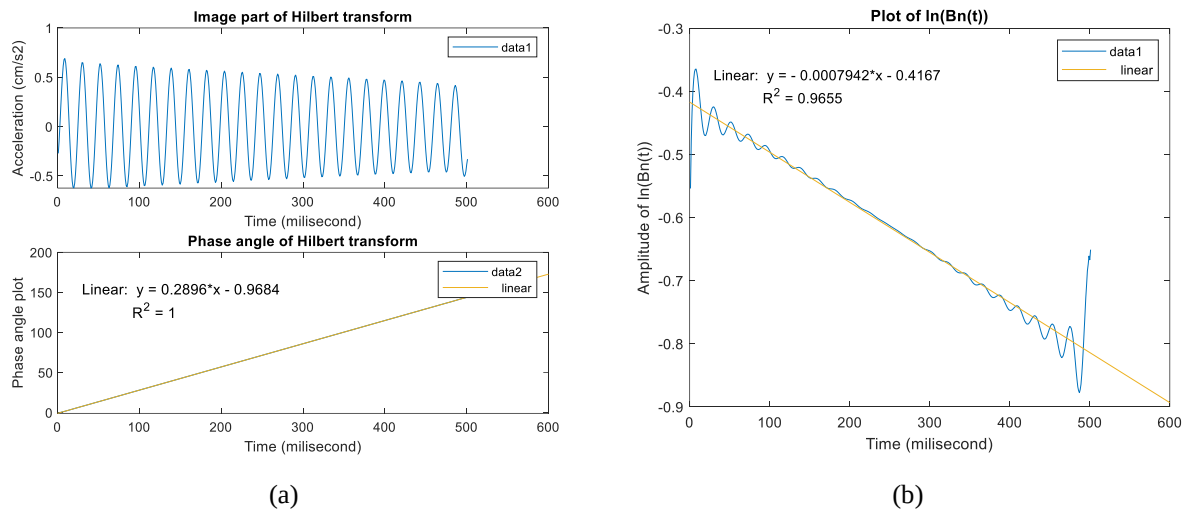


Fig. 14 – Its Hilbert transform and phase angle plot (a) Plot of $\ln(Bn(t))$ (b) of signal last 0.5 second with a frequency 46.05Hz

From the above figures, we can be identified $\omega_{hd}=289.6\text{rad/s}$, $f_{nd}=46.11\text{Hz}$ and $\xi_n=0.27\%$.

From equation 17, we can determine $\delta_n=1.72\%$.

Thus, when determining the damping ratio, a sufficiently short data segment is needed to obtain accurate results.

4 Conclusion

In the case of the experimental cable and the above analysis results, it is shown that the proposed method can accurately calculate the frequency and damping with the corresponding vibration mode of the cable.

The logarithmic decay changes over time during the vibration. To estimate the details, it is necessary to divide the data segment before calculating the logarithmic decay.

The results of calculating the vibration frequency using the Hilbert transform method are like the results of calculating the vibration frequency using the familiar FFT method. Therefore, it can be affirmed that the proposed method gives accurate results and is highly reliable.

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