



Journal of Materials and Engineering Structures

Research Paper

Numerical and experimental studies on vibration-based damage detection methods in beam structures

Duong Huong Nguyen ^{a,*}, Nhu Mai Thi Nguyen ^a

^a Faculty of Bridges and Roads, Hanoi University of Civil Engineering, Hanoi, Vietnam

ARTICLE INFO

Article history :

Received 29 July 2024

Revised 15 September 2024

Accepted 29 September 2024

Keywords:

vibration based damage detection,
measurement data,
damage detection,
modal curvature.

ABSTRACT

Vibration-based damage detection (VBDD) methods have emerged as powerful, cost-effective, and non-destructive methods for detecting and assessing damage in structures. These methods utilize the natural dynamics of structures to identify the presence, location, and extent of damage. Analyzing changes in modal parameters such as natural frequencies, mode shapes, and modal curvatures can detect structural anomalies that indicate damage. This paper describes the flexibility matrix method, the modal curvature method, and the gaped smoothing methods and proposes a new method called the square of modal curvature method. Initially, these damage detection methods are studied using a numerical model of a simply supported beam. The results indicate that the square of modal curvature (SMC) method can effectively detect damage using only a few low-frequency modes. The proposed SMC method is further tested using experimental data obtained from a free-free beam model. Two cases are considered: one with experimental vibration data from an intact beam, and the other without. In the absence of data from an intact beam, the SMC method is combined with the Gapped Smoothing Method (GSM). Both test cases yield reliable results.

1 Introduction

Vibration-based damage detection (VBDD) methods are among the popular and very important methods for structural health monitoring (SHM). The basic idea of these methods is that structural damage causes changes in structural properties (mass, damping, stiffness) and will lead to obtainable changes in vibration properties. The vibration of structures in the times domain can be collected from different kinds of sensors: strain, acceleration, deformation, etc. Vibration data in the times domain can be converted to the frequency domain by Fourier transform and the dynamic characteristics of the structure can be found. Many analysis methods have been proposed to eliminate the noise effect in measurements [1-3]. VBDD methods used in damage detection are based on modal parameters and can be categorized into different approaches including frequency, mode shape, curvature mode shape, flexibility matrix, and other methods.

The VBDD method using changes in frequencies was originally used in the 1970s to detect damage in structures [4]. Salawu summarized 65 works up to 1995 using the differences in natural frequencies to identify the damage in structures and

* Corresponding author. Tel.: +084 973092239.

E-mail address: duongnh2@huce.edu.vn

discussed the factors that could limit the success of the application [5]. The summary showed that the frequency change at only one sensor can help to identify the damage. However, if the damage was located in low-stress regions, the detection might be unreliable. It was suggested that the sensors should be placed at points where all the modes are well presented to get better detections. Salawu also commented that the frequency changes alone may not be sufficient to determine the location of the damage because similar cracks at different locations may cause the same frequency change and frequency itself does not contain location information. The natural frequencies are not very sensitive to damage [6]. Natural frequencies are global parameters and it is shown that significant damage may cause very small changes in natural frequencies [7, 8]. The environmental influence on dynamic responses can also be significant and can mask the frequency changes due to damage, resulting in false detections [9]. For example, the frequency changes due to temperature in the 1-year monitoring period of Z24 bridge is over 10% [10]. Meanwhile, when the cross-section at the center of a girder of the I-40 Bridge over Rio Grande was cut from the web center to the bottom flange, the frequencies of the first three modes reduced only 7.2%, 4%, and 0.3% [11], may be smaller than changes due to ambient condition. These above reasons have caused the frequency-based VBDD method to be no longer popular in recent years.

VBDD methods based on mode shape are more sensitive to damaged locations and less influenced by ambient conditions than the frequency-based method. These methods require installing a series of sensors along to the structure, especially in cases that higher modes are required. Mode shapes can be extracted based on vibration data in the frequency domain and can be used directly or indirectly for SHM [1, 12]. In direct methods, two indicators: Modal Assurance Criterion, MAC [13], and the Coordinate Modal Assurance Criterion, COMAC [14] are often used to evaluate the identical between two mode shapes of intact and damaged structures. Evaluating the first seven mode shapes of a concrete bridge, Williams [15] showed that although differences of natural frequencies before and after retrofit are less than 3%, the MAC indicator can identify the damage. Direct modal methods are applicable to both numerical models and experimental data [16, 17]. MAC and Modified Total Modal Assurance Criterion (MTMAC) serve as effective objective functions for addressing structural damage detection and model updating [18]. Furthermore, Ghannadi and Kourehli [19] have evaluated the sensitivity of these objective functions and have demonstrated that the combination of the Slime Mold Algorithm (SMA) and MTMAC yields highly accurate results for damage detection. In recent times, the utilization of VBDD with optimization techniques has gained significant popularity and widespread usage [20–22]. However, it is essential to acknowledge that while employing these direct methods, the presence of noise in experimental data, errors in equipment, and measurement methods can potentially impact the accuracy of the outcomes.

Modal curvature (MC) is a well-known indirect modal method. When local damage occurs, it causes a change in mode shape derivatives at the damage location. It means that mode shape curvature - the second derivative of mode shape - is more sensitive to local damage than the mode shape itself. Pandey et al. used modal curvature to identify and locate the damage in FEM beams [23]. The modal curvatures were calculated from displacement mode shapes using central finite difference method. It was shown that the changes in modal curvature increase with increasing size of damage, and modal curvature can be used to determine the quantity of damage. Wahab and De Roeck [24] applied the modal curvature method to real Z24 bridge and concluded that MC of lower modes is more accurate than higher modes and MC method is a promising method to detect damage in structures.

One of the requirements of the MC method is the mode shape of the intact structure must be known. In practice, intact mode shape is not always available. To solve this problem, Ratcliffe [25] introduced the gapped smoothing method (GSM), which uses only the mode shape of the damaged structure to locate the damage without needing the intact structure's mode shape. GSM has been effectively developed and applied in various types of structures [26, 27].

Flexibility matrix is the inverse of the stiffness matrix and therefore can be used to detect damage. Pandey and Biswas [28] used the changes in the measured flexibility to successfully identify and locate the damage in both numerical and experimental examples. Li et al. [29] applied this method to find damage in a cantilever structure and slender structure. Zhao and DeWolf [30] researched three methods: natural frequencies, mode shape, and flexibility matrix to detect damage in a spring that had 5 degrees of freedom, and concluded that flexibility matrix is the most sensitive method for damage detection.

This work researches natural frequencies, flexibility matrix, and mode shape curvature methods applied on both numerical model and experimental data. It introduces a new damage indicator called the SMC method, which is used to determine the location of damage in a structure. The paper is divided into four sections. The first section is the introduction. Section 2 describes the details of VBDD damage detection methods. Sections 3 and 4 provide the results of VBDD applications on numerical and experimental examples, respectively. The last section, Section 5, contains some conclusions.

2 Damage detection methods

2.1 Damage detection method using flexibility matrix (FM)

The homogeneous dynamic equilibrium equation for a structure can be expressed as:

$$K\Phi = M\Phi\Omega \quad (1)$$

where K and M are the stiffness and mass matrix, respectively. $\Phi = [\Phi_1, \Phi_2, \dots, \Phi_n]$ is the mode shapes matrix, n is number of modes. $\Omega = \text{diag}(\omega_i^2)$ is the modal stiffness matrix, ω_i is the i th modal frequency. If the mode shape normalized to unity as $\Phi M \Phi^T = I$, where I is an unit matrix, the flexibility matrix FM and stiffness matrix K can be written as:

$$K = M\Phi\Omega\Phi^T M = M(\sum_{i=1}^n \omega_i^2 \Phi_i \Phi_i^T) M \quad (2)$$

$$FM = K^{-1} = \Phi\Omega^{-1}\Phi^T = \sum_{i=1}^n \frac{1}{\omega_i^2} \Phi_i \Phi_i^T \quad (3)$$

Observing Eq (3), the flexibility matrix decreases as frequency increases and rapidly converges to a good approximation with a few low-frequency modes [28, 31]. The presence of damage in a structure reduces the stiffness. Since the flexibility matrix is the inverse of stiffness, the reduction of stiffness leads to an increase in flexibility. Therefore, changes in the flexibility matrix can be utilized to identify and locate damage in a structure [28].

Using this method, two sets of measurements need to be taken. One measurement for intact structure and another for the damaged structure. The changes in two flexibility matrices used to detect damage can be obtained as below

$$\Delta = FM_d - FM_u = \sum_{i=1}^n \frac{1}{\omega_{di}^2} \Phi_{di} \Phi_{di}^T - \sum_{i=1}^n \frac{1}{\omega_{ui}^2} \Phi_{ui} \Phi_{ui}^T \quad (4)$$

where FM_d is the flexibility matrices for the damaged and FM_u is for intact structures. Φ_{di} and Φ_{ui} are the i th mode shape, corresponding to the natural frequency ω_{di} and ω_{ui} , for the damaged and intact structure, respectively.

In reality, the measurements for the intact structure are not always available. Therefore, this is one of the limitations of this method.

2.2 Damage detection method based on Modal Curvature

2.2.1 Modal Curvature (MC)

The relationship of modal curvature and the bending stiffness at a point along the beam is expressed in the formula below

$$\phi''(x) = \frac{d^2\phi(x)}{dx^2} = -\frac{M}{EI(x)} \quad (5)$$

where: $\phi(x)$ is the displacement mode shape, $\phi''(x)$ is the modal curvature; M is the bending moment; $EI(x)$ is the bending stiffness.

The central difference method is often used to calculate the modal curvature from the displacement mode shapes

$$\phi_j'' = -\frac{\phi_{j-1} - 2\phi_j + \phi_{j+1}}{\Delta x^2} \quad (6)$$

where ϕ_j is the displacement mode shape at point j ; Δx is the distance between two measurement points.

The modal curvature is correspondent with the inverse of bending stiffness. The damage leads to a decrease in local stiffness, therefore increases the local modal curvature. The changes in modal curvature have been used as a damage index [24].

$$\delta_j = \frac{1}{N} \sum_{i=1}^N |\phi''_{0ij} - \phi''_{dij}| \quad (7)$$

where: ϕ''_{0ij} and ϕ''_{dij} are the curvature of the intact and damaged structure at point j , respectively. N is the total number of considered modes and i is the mode number.

This damage index has been proved useful to detect the local damage in a structure, but the data of intact structure needs to be known.

2.2.2 Gapped Smoothing method (GSM)

The gapped smoothing technique assumed that the modal curvature of a healthy structure is a smooth shape. Local damage creates the local change in mode shape, and slope of mode shape as well. Therefore, a jump appears in the modal curvature. By smoothing the modal curvature of the damaged beam, the modal curvature of the intact beam can be found. The approximated smooth curve can be estimated from the original curve by using a cubic polynomial [25]. Currently, Matlab R2018b (Natick, Massachusetts: The Math Works Inc.) provides the “fit function” to create an approximate Fourier smooth curve fitting any original curve. The fitting modal curvature ϕ''_{fi} at mode number i is defined below

$$\phi''_{fi}(x) = a_0 + \sum_{k=1}^n (a_k \cdot \cos(k\omega x) + b_k \cdot \sin(k\omega x)) \quad (8)$$

where: a_0 models a constant (intercept) term in the data, ω is the fundamental frequency of the signal, n is the number of terms (harmonics) in the series ($1 \leq n \leq 8$). The coefficients appearing in this equation will be chosen by Matlab to create a smooth curve fit the original MC curve of damaged beam.

Assuming the fitting modal curvature ϕ''_{fi} as the modal curvature of intact structure, the damage index δ_j at the point number j is calculated as the squared differences between the damaged curvature and the fitting modal curvature

$$\delta_j = \frac{1}{N} \sum_{i=1}^N (\phi''_{dij} - \phi''_{fij})^2 \quad (9)$$

2.3 Square of modal curvature method (SMC method)

A new modified flexibility matrix is proposed as below

$$\text{MFM} = \sum_{i=1}^n \frac{1}{\omega_i^2} \Phi_i'' \Phi_i''^T \quad (10)$$

where: Φ_i'' is the modal curvature of the structure at mode i th

Damage is then detected and located based on the changes in modified flexibility matrix as:

$$\Delta = \text{MFM}_d - \text{MFM}_u = \sum_{i=1}^n \frac{1}{\omega_{di}^2} \Phi_{di}'' \Phi_{di}''^T - \sum_{i=1}^n \frac{1}{\omega_{ui}^2} \Phi_{ui}'' \Phi_{ui}''^T \quad (11)$$

where: Φ_{di}'' and Φ_{ui}'' are the modal curvature of the damaged and intact structure of mode i th, respectively.

ω_{di} and ω_{ui} are the natural frequency of the damaged and intact structure of mode i th, respectively.

If the measurement data of the intact structure is not available, the natural frequencies of damaged structure are proposed to use for both cases and damage is detected based on the equation below

$$\Delta = \text{MFM}_d - \text{MFM}_u = \sum_{i=1}^n \frac{1}{\omega_{di}^2} \Phi_{di}'' \Phi_{di}''^T - \sum_{i=1}^n \frac{1}{\omega_{di}^2} \Phi_{ui_GSM}'' \Phi_{ui_GSM}''^T \quad (12)$$

Φ_{ui_GSM}'' is reconstructed from Φ_{di}'' using the gapped smoothing technique which detailed in section 2.2.2. The changes in natural frequencies are very small, especially at low modes. Natural frequencies in this equation have the meaning of weight factor. Therefore, Eq (12) combined with the gapped smoothing technique can be used to detect damage, in case the data from the intact structure cannot be reached.

Both FM and MFM matrixes are square matrix, size corresponding to the number degree of freedom of structure. We use δ_j to measure the changes in the flexibility matrix for each location. δ_j are the diagonal elements in matrix Δ .

$$\delta_j = \text{diag } \Delta \quad (13)$$

In order to improve the visualization of changes δ_j is normalized and used as the damage index.

3 Numerical example

To validate the proposed method, a simply supported beam with a rectangular cross-section is used. The beam length is $l = 1\text{m}$, rectangular cross section $b \times h = 0,1 \times 0,7$ (cm). The materials are as follow: Young's modulus $E = 200\text{GPa}$, density $r = 7800\text{ kg/m}^3$. The beam is modeled in Matlab with 50 elements as in Fig. 1. The damage was generated in Matlab by reducing the stiffness of each damaged element.

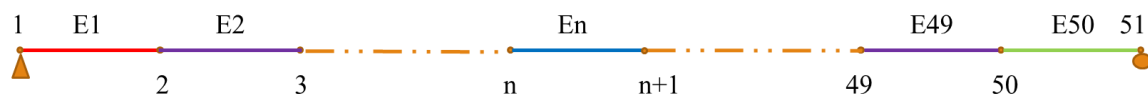


Fig. 1- The simply supported beam model

3.1 Single damage

The natural frequencies for the intact case and two typical damaged cases (e.g. 30% reduction in E in elements number 10 and elements number 20, respectively) are presented in Table 1. For the first 5 modes, the differences in natural frequency between intact and damaged cases are very small, less than 1%. Moreover, in Eq (11), the changes in the modified flexibility matrix depend on the inverse square of natural frequency. Therefore, in the proposed method, using the frequency of damaged structure in Eq (12) when the intact data is not available is acceptable.

Table 1. Natural frequencies of intact and two damaged cases (EI reduced 30%) for the simply supported beam

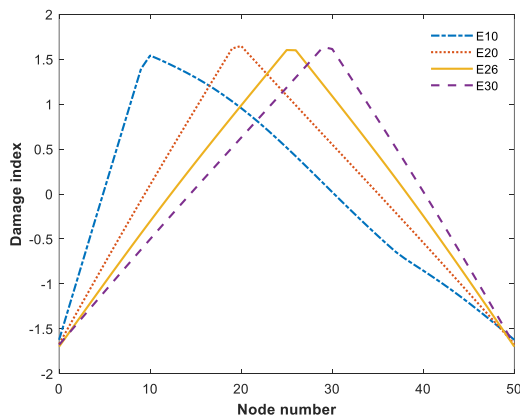
Mode	Natural frequency (Hz)			Differences (%)	
	Intact	Damage at element 10	Damage at element 20	Damage at element 10	Damage at element 20
1	53.170	53.026	52.770	0.270	0.750
2	212.652	211.097	211.925	0.731	0.342
3	478.369	474.592	477.319	0.789	0.219
4	850.190	846.924	843.434	0.384	0.795
5	1327.937	1327.581	1327.579	0.027	0.027

The damage indexes of the three detection methods FM, MC and SMC used to detect single damage at element 10, 20, 26 and 30 (30% reduction in EI) are shown in Fig. 2. All three methods successfully detected the locations of the damage. However, regarding to the value of damage indexes, the MC method and SMC method have higher values and more clear peaks at the damage locations. The SMC method provides a narrower damage location than MC method. Furthermore, in the SMC method, the neighbor points of the damage location (the point with the highest damage index) have the lowest local damage index, resulting to clearer damage identification.

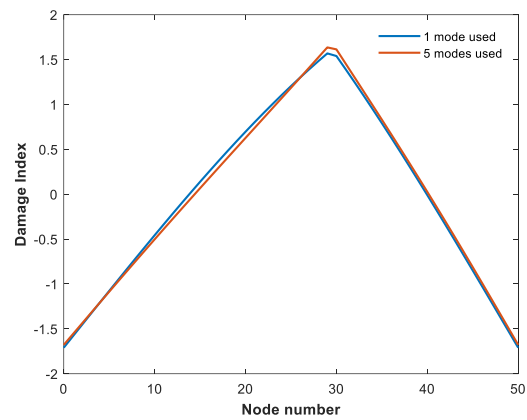
Fig. 3 shows the comparison of the three methods using different number of modes to determine the damage index at element 30. The changes in FM method and MC method converged rapidly. These two methods showed the potential to converge when using only the two first modes. The SMC converged slower. However, using the two first modes still provides a high value of damage index.

The sensitivity of damage detection methods to noise effect was evaluated by adding white Gaussian noise to mode shape data using 'awgn' Matlab function. Fig. 4 shows that the changes in the FM method, and the MC method are more stable with noise than in the SMC method. However, the damage index at the damaged location calculated from the SMC method

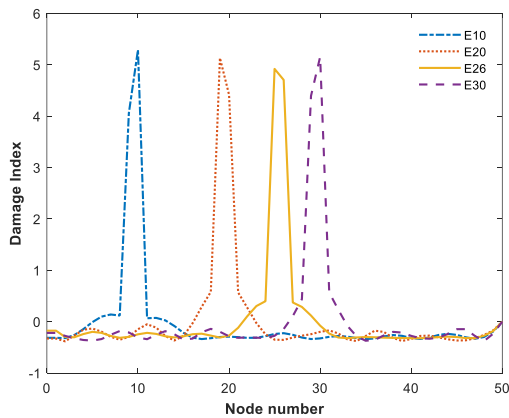
still is a very high value. Despite the noise-induced fluctuations in the SMC method's plot, the proposed method effectively identifies the location of damage, as evidenced by the high calculated damage index.



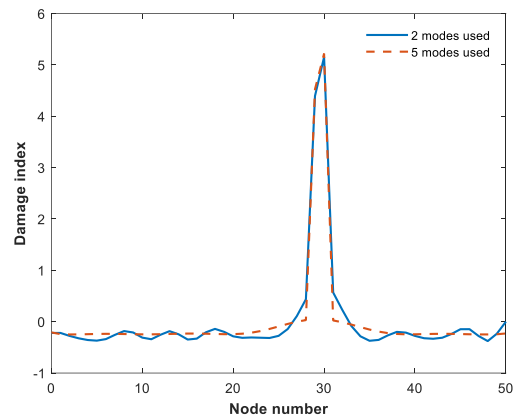
(a) Flexibility method



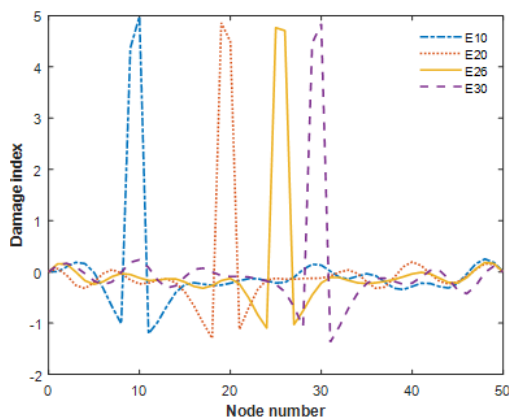
(a) Flexibility Matrix method



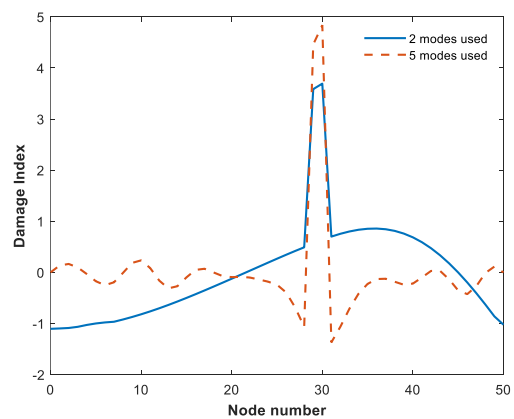
(b) Modal curvature method



(b) Modal curvature method



(c) Square modal curvature method



(c) Square of modal curvature method

Fig. 2- Damage indexes of four single damage location (30% reduction in EI)

Fig. 3 – Damage index plots as a function of number of modes used (damaged at element 30, 30% reduction in EI)

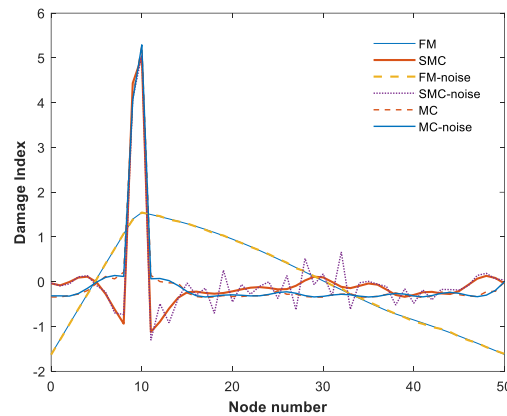


Fig. 4– Damage index plots as a function of noise (damaged at element 10, 30% reduction in EI)

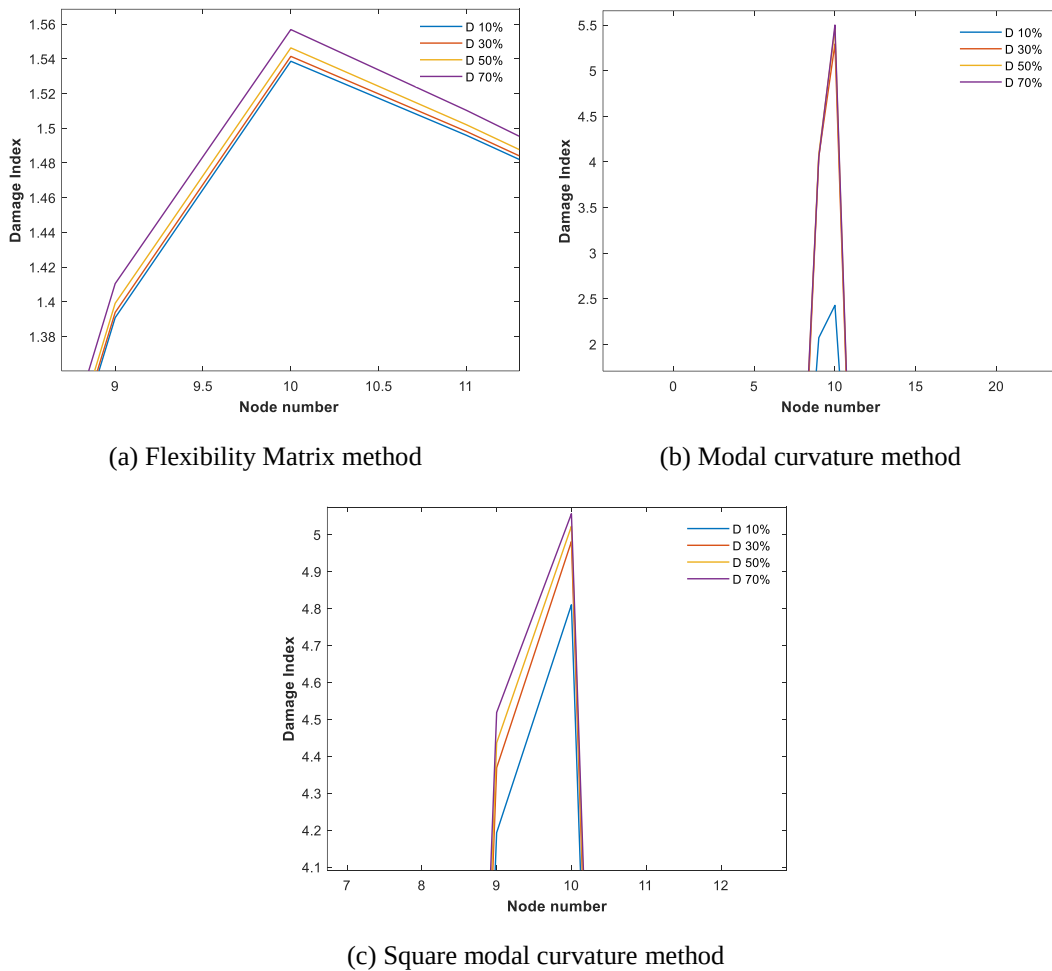


Fig. 5 – Damage index plots as a function of damage severity (damage at element 10)

To evaluate a damage detection method, the relationship between the damage indexes and the damage severities is very important. Fig. 5 shows the peaks between node numbers 9 and 10 for the case damage that appeared in element 10 for different severities: reduced stiffness 10%, 30%, 50%, and 70%. The damage indexes calculated from FM and SMC show the good corresponding with damage severities. The higher the damage severity is, the higher the damage index is. In the MC method, the damage index when the stiffness is reduced by 10% is much smaller than that of other reduced amounts. Moreover, the damage indexes when the stiffness reduces by 30%, 50%, and 70% are almost the same. Therefore, the damage

index calculated from the MC method after normalization shows that the damage severity is not as good as in the SMC method.

3.2 Multiple damage

Fig. 6 shows the numerical results for multiple damages in two cases: EI reductions in 2 elements (element 8 – 50% and element 12 – 50%) and EI reductions in 3 elements (element 10 – 30%, element 20 – 50% and element 35 – 50%). The damage index based on the SMC method shows the advantage of detecting multiple damage with high value and clear peaks. The peaks from the changes of SMC were clearer and narrower than from MC, especially in the case where damage occurs in 3 elements. Moreover, the damage indexes from SMC presented good correspondence with the damage severity, higher severity causes higher damage index for both multiple damage cases. The plots from the changes of FM shows that this method fails when detecting the multiple damage case. Upon closer inspection of the FM plots, slight changes in the slope curve at the damaged locations were observed; however, these changes were minimal and not clear.

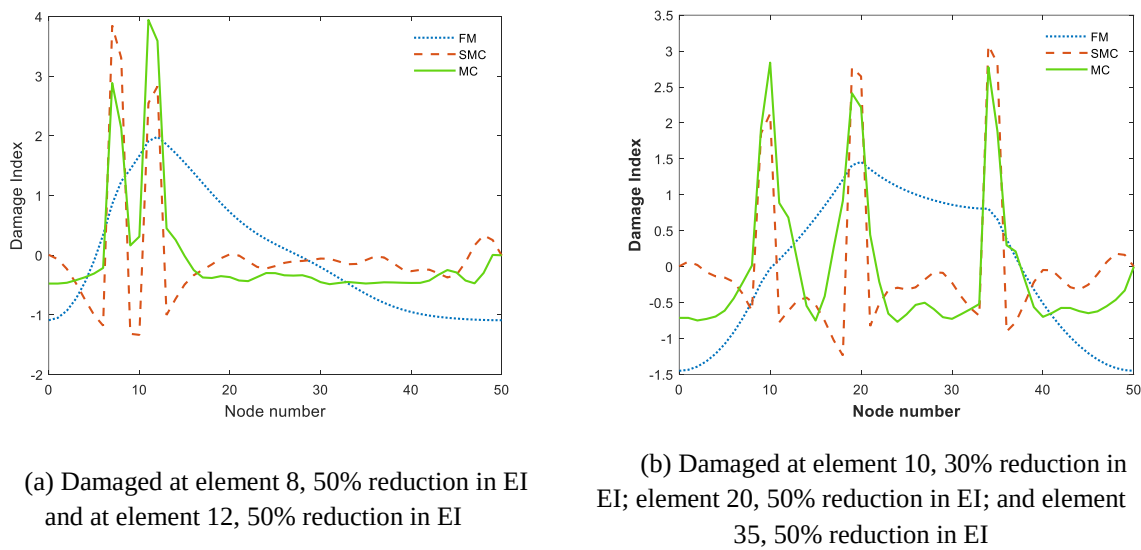


Fig. 6 – Two cases of multiple damage (30% reduction in EI)

4 Experimental example

4.1 Experimental setup

A studied steel beam shown in Fig. 7 with rectangular cross-section dimensions of 70 mm × 10 mm was used for experimental verification. The experimental beam was 1m long and the boundary conditions were set to be free-free by suspended by two thin filaments. To simulate the damage, two notches were introduced on both sides of the beam between two sensors number 5 and 6. Two notched dimensions are 12.5×0.5 (mm) and 13.3×0.5 (mm). Fig. 8 shows the details of these damage.

For data collection, fifteen accelerometers, sensitivity ranges from 10.13 – 10.50 mV/m/s², were attached along the beam in order to obtain the beam dynamic responses. Fig. 9 shows the location of these accelerometers. The source of excitation was a hammer, pushed once a time at the location of accelerometer 8. Data acquisition time was around 5 minutes, and the sampling frequency was 2560 Hz. Therefore, the frequency range of 0- 1200 Hz can be obtained. The collected data was analyzed by using the covariance-driven stochastic subspace identification algorithm (SSI-COV). The preset criteria are 1% for frequencies, 5% for damping ratios, and 1% for mode shape vectors. Based on the stabilization diagram, the first five mode shapes are identified. Moreover, modal parameters of intact and damaged beam can be found. More information about the experiment setup and data analysis can be found in Ref [32, 33]. Fig. 10 shows the flowchart of the damage detection method. The changes in the square of modal curvature are calculated based on experimental mode shapes and used as the damage index.



Fig. 7 – Overview of experimental beam



Fig. 8 – Two notches in experimental beam

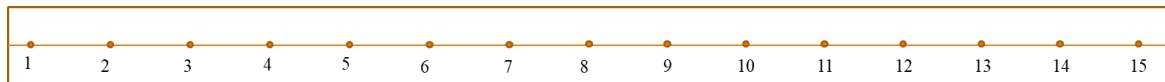


Fig. 9 – Location of accelerometers on the top of the beam

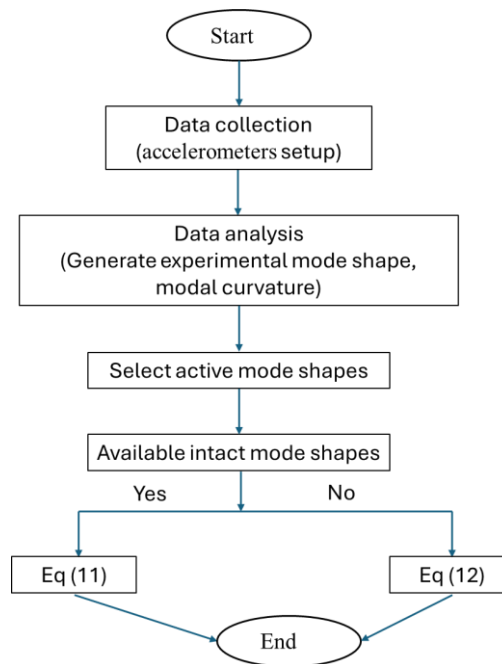


Fig. 10 – The flowchart of SMC method

4.2 Experimental results

The proposed method was successfully found out the damage location for both cases. The damage index is the highest at sensor number 5, decreases at sensor number 6, and shows two lowest peaks at sensor number 4 and 7. However, some small false peaks appear at other nodes along the beam. The reasons may come from the small number of installed sensors and the noise effect in the measurement data. The results from Case 1 are better than from Case 2. This is expected because Case 1 used the data from the intact beam and Case 2 did not use this data.

Table 2. Natural frequencies for the intact and damaged tested beam (Hz)

Mode	Natural frequency (Hz)		Differences (%)
	Intact	Damaged	
1	50.83	50.36	0.92
2	140.40	138.64	1.25
3	274.74	273.53	0.44
4	456.94	454.60	0.51
5	678.90	672.58	0.93

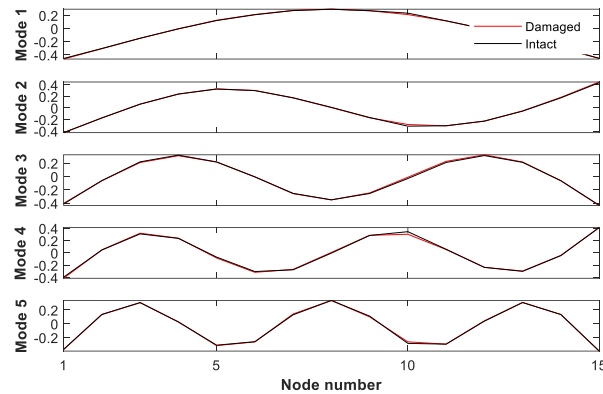
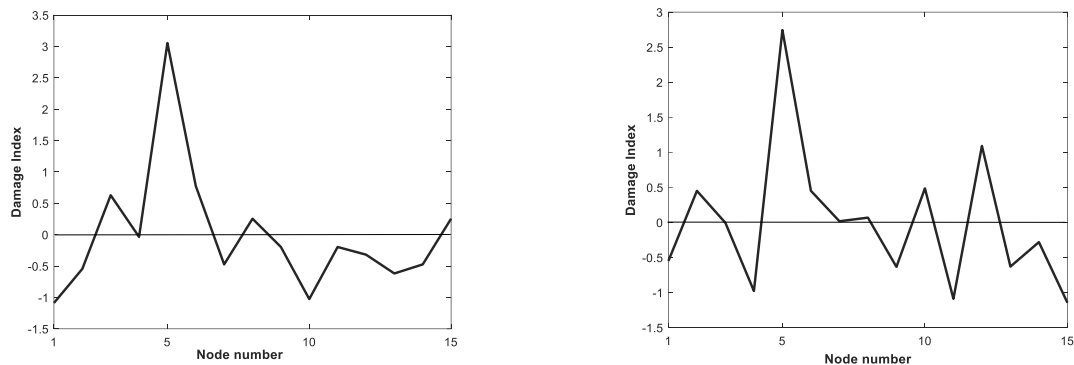
**Fig. 11 – First five mode shapes of intact and damaged tested beam****(a) The measurement data of intact structures is available****(b) The measurement data of intact structures is not available****Fig. 12 – The damage indexes of the tested beam using SMC method**

Table 2 lists the natural frequencies of the first five modes for both intact and damaged beams. The corresponding mode shapes are plotted in

Fig. 11. The mode shapes of intact and damaged beams are quite similar. The MAC value that measures the correlation between intact and damaged mode shapes is very close to 1 for all five modes. The differences in natural frequency are very small and do not provide any information about the state of the tested beam. The mode shapes for damaged and intact tested beam indicate nothing about the damage. However, from the mode shape data, modal curvature can be computed using the central difference method. Fig. 12 plots the damage index of the tested beam using the changes in square modal curvature. In this figure, two cases are considered. In Case 1, the experimental data of the intact beam were considered. Eq (11) was used to calculate the changes of modified flexibility matrix. The diagonal values of this matrix then normalized and used as a damage indexed and plotted in Fig. 12a. In Case 2, the data of intact tested beam were assumed not available. Therefore, Eq (12) were used instead of Eq (11). The gapped smoothing technique was used to reconstruct the modal curvature of the intact tested beam. The damage index can be found and presented in Fig. 12b. The proposed method was successfully found

out the damage location for both cases. The damage index is the highest at sensor number 5, decreases at sensor number 6, and shows two lowest peaks at sensor number 4 and 7. However, some small false peaks appear at other nodes along the beam. The reasons may come from the small number of installed sensors and the noise effect in the measurement data. The results from Case 1 are better than from Case 2. This is expected because Case 1 used the data from the intact beam and Case 2 did not use this data.

5 Conclusion

The paper introduces a novel VBDD method known as the SMC method. It presents a numerical analysis of a simply supported beam, comparing the effectiveness of various VBDD methods, including the FM method, MC method, and SMC method. The study demonstrates that the SMC method proficiently identifies single and multiple damages of varying severity in the beam, with clearer and narrower peaks than the MC method, especially in cases of damage to 3 elements. Additionally, the method allows for the estimation of damage severity through the use of damage indexes, which remain stable even with multiple damages. SMC method is tested with experimental data from a free-free beam. If data for an intact beam is not available, the SMC method can be combined with the GSM method to accurately identify the location of damage. The proposed SMC method does not require measurement data from the intact structure for damage detection, unlike the FM method. Furthermore, compared to the MC method, the SMC method provides a narrower band for predicting damage and can detect damage using only a few low-frequency modes. This feature has a special meaning, especially for some structures when the high-frequency modes may be difficult to obtain. The disadvantage of the proposed method is that this method is more sensitive to noise than other methods. Therefore, noise should be carefully considered and eliminated in the measurement data.

REFERENCES

- [1]- N.M.M. Maia, J.M.M. e Silva, Theoretical and experimental modal analysis. Research Studies Press, 1997.
- [2]- M. Friswell, J. Penny. Is damage location using vibration measurements practical. in Euromech 365 international workshop: Damas. (1997), 351-362.
- [3]- U. Lee, J. Shin, A frequency response function-based structural damage identification method. Computers & Structures, 80(2) (2002) 117-132.
- [4]- P. Cawley, R.D. Adams, The location of defects in structures from measurements of natural frequencies. The Journal of Strain Analysis for Engineering Design, 14(2) (1979) 49-57.
- [5]- O. Salawu, Detection of structural damage through changes in frequency: a review. Engineering Structures, 19(9) (1997) 718-723.
- [6]- E. Manoach, J. Warminski, L. Kloda, A. Teter. Vibration based methods for damage detection in structures. in MATEC Web of Conferences. EDP Sciences. (2016), 05007.
- [7]- W. Ostachowicz, M. Krawczuk, Vibration analysis of a cracked beam. Computers & Structures, 36(2) (1990) 245-250.
- [8]- E. Manoach, S. Samborski, A. Mitura, J. Warminski, Vibration based damage detection in composite beams under temperature variations using Poincaré maps. International Journal of Mechanical Sciences, 62(1) (2012) 120-132.
- [9]- H. Zhang, M. Gül, B. Kostić, Eliminating temperature effects in damage detection for civil infrastructure using time series analysis and autoassociative neural networks. Journal of Aerospace Engineering, 32(2) (2019) 04019001.
- [10]- B. Peeters, G. De Roeck, One - year monitoring of the Z24 - Bridge: environmental effects versus damage events. Earthquake engineering & structural dynamics, 30(2) (2001) 149-171.
- [11]- C.R. Farrar, W. Baker, T. Bell, K. Cone, T. Darling, T. Duffey, A. Eklund, A. Migliori, Dynamic characterization and damage detection in the I-40 bridge over the Rio Grande, Los Alamos National Lab.(LANL), Los Alamos, NM (United States). (1994).
- [12]- D. Ewins, Modal testing, theory testing and practice, Hertfordshire, England: Research Studies Press Limited. (1984).
- [13]- R.J. Allemang. A correlation coefficient for modal vector analysis. in Proc. 1st Int. Modal Analysis Conference. (1982), 110-116.
- [14]- N. Lieven, D. Ewins. Spatial correlation of mode shapes, the coordinate modal assurance criterion (COMAC). in Proceedings of the 6th international modal analysis conference. Kissimmee Florida, USA. (1988), 690-695.

-
- [15]- O.S. Salawu, C. Williams, Bridge assessment using forced-vibration testing. *Journal of Structural Engineering*, 121(2) (1995) 161-173.
- [16]- W.-X. Ren, G. De Roeck, Structural damage identification using modal data. I: Simulation verification. *Journal of Structural Engineering*, 128(1) (2002) 87-95.
- [17]- W.-X. Ren, G. De Roeck, Structural damage identification using modal data. II: Test verification. *Journal of Structural Engineering*, 128(1) (2002) 96-104.
- [18]- P. Ghannadi, S. Khatir, S.S. Kourehli, A. Nguyen, D. Boutchicha, M.A. Wahab. Finite element model updating and damage identification using semi-rigidly connected frame element and optimization procedure: An experimental validation. in *Structures*. Elsevier. (2023), 1173-1190.
- [19]- P. Ghannadi, S.S. Kourehli, Efficiency of the slime mold algorithm for damage detection of large - scale structures. *The Structural Design of Tall and Special Buildings*, 31(14) (2022) e1967.
- [20]- P. Ghannadi, S.S. Kourehli, S. Mirjalili, The application of PSO in structural damage detection: An analysis of the previously released publications (2005–2020). *Frattura ed Integrità Strutturale*, 16(62) (2022) 460-489.
- [21]- P. Ghannadi, S.S. Kourehli, A. Nguyen, The Differential Evolution Algorithm: An Analysis of More than Two Decades of Application in Structural Damage Detection (2001–2022), in *Data Driven Methods for Civil Structural Health Monitoring and Resilience*. CRC Press. (2024), 14-57.
- [22]- P. Ghannadi, S.S. Kourehli, Multiverse optimizer for structural damage detection: Numerical study and experimental validation. *The Structural Design of Tall and Special Buildings*, 29(13) (2020) e1777.
- [23]- A. Pandey, M. Biswas, M. Samman, Damage detection from changes in curvature mode shapes. *Journal of sound and vibration*, 145(2) (1991) 321-332.
- [24]- M.A. Wahab, G. De Roeck, Damage detection in bridges using modal curvatures: application to a real damage scenario. *Journal of sound and vibration*, 226(2) (1999) 217-235.
- [25]- C.P. Ratcliffe, Damage detection using a modified Laplacian operator on mode shape data. *Journal of Sound and Vibration*, 204(3) (1997) 505-517.
- [26]- M.K. Yoon, D. Heider, J.W. Gillespie, C.P. Ratcliffe, R.M. Crane. Local damage detection using a global fitting method on mode shape data. in *IMAC-XIX: A Conference on Structural Dynamics*. (2001), 231-237.
- [27]- M. Yoon, D. Heider, J. Gillespie Jr, C. Ratcliffe, R. Crane. Spatial damage detection in composite structures using a global fitting method on curvature operating shapes. in *American Society for Composites 16th Annual Technical Conference*. (2001), 9-12.
- [28]- A. Pandey, M. Biswas, Damage detection in structures using changes in flexibility. *Journal of sound and vibration*, 169(1) (1994) 3-17.
- [29]- G.-Q. Li, K.-C. Hao, Y. Lu, S.-W. Chen, A flexibility approach for damage identification of cantilever-type structures with bending and shear deformation. *Computers & Structures*, 73(6) (1999) 565-572.
- [30]- J. Zhao, J.T. DeWolf, Sensitivity study for vibrational parameters used in damage detection. *Journal of Structural Engineering*, 125(4) (1999) 410-416.
- [31]- A. Yan, J.-C. Golinval, Structural damage localization by combining flexibility and stiffness methods. *Engineering Structures*, 27(12) (2005) 1752-1761.
- [32]- D.H. Nguyen, L.V. Ho, T. Bui-Tien, G. De Roeck, M.A. Wahab, Damage Evaluation of Free-Free Beam Based on Vibration Testing. *Applied Mechanics*, 1(2) (2020) 142-152.
- [33]- L.V. Ho, D.H. Nguyen, M. Mousavi, G. De Roeck, T. Bui-Tien, A.H. Gandomi, M.A. Wahab, A hybrid computational intelligence approach for structural damage detection using marine predator algorithm and feedforward neural networks. *Computers & Structures*, 252 (2021) 106568.