



Journal of Materials and Engineering Structures

Research Paper

Development of P-Y curves for single piles based on full-scale lateral load tests and the cone penetration test (CPT) in clayey soils

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ARTICLE INFO

Article history :

Revised : 31 October 2023

Received : 25 March 2024

Accepted : 27 March 2024

Keywords:

P-Y curves

CPT test

Pile

Clay

ABSTRACT

It is nowadays recognized that the load-transfer P-Y curves methods offer a powerful framework of analysis of the pile response under lateral loading. The aim of this paper is to present a new formulation the P-Y curves to analyze the response of a single pile embedded in clayey soil, on the basis of the CPT test data. The methodology of work consists of analyzing 8 lateral loading tests on fully instrumented piles driven into a homogeneous saturated clayey soil. The P-Y curves were formulated by the PARECT (parabola-rectangle) function, and successful correlation of their parameters, namely the lateral reaction modulus and the lateral soil resistance, with the cone resistance as well as with the lateral pile/soil stiffness ratio was made. After a comparison with the existing P-Y curves methods, a methodology of analysis by the proposed P-Y curves for piles in normally consolidated to slightly over-consolidated clays was suggested. Moreover, a pile classification according to the pile/soil stiffness ratio was suggested, and the concept of the critical deflection corresponding to the threshold of domain of large deflections of piles was introduced. Validation process was launched by applying this methodology to a centrifuged scale model of a single bored pipe pile in a saturated slightly over consolidated clay. Direct comparison of the load-deflection curves showed an excellent prediction of the small deflections up to about 2% of the pile diameter. Beyond this value, the boundary conditions at the pile tip have an influence on the results, but showed a relatively pessimistic prediction.

1 Introduction

Design of pile foundations under lateral loads is prescribed by the modern geotechnical standards. This requirement should be fulfilled within the framework of a serviceability limit states design (SLSD) by pile displacement analysis. This latter is carried out either by measurement of pile deflection in a full-scale loading test or by computation on the basis of some commonly used methods, such as those based on the elasticity theory [1-4], or the numerical methods [5], or at last the methods based on the P-Y curves [6-11].

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Understanding the pile response under lateral loads is increasingly gaining interest among the geotechnical engineers due to the development of audacious structures, such as the monopiles-supported wind turbines and the offshore platforms working under severe conditions of lateral loading. In this regard, an ultimate limit states design (ULSD) should be undertaken accounting for the lateral soil resistance around the pile [10-12].

It is nowadays recognized that the load-transfer P-Y curves methods offer a powerful framework of analysis of the behavior of pile foundations. Compared to other methods of pile design, the P-Y curves concept has the advantage to take account of the variability of the soil properties and the non-linear behaviour of the pile/soil system. As illustrated in Figure 1, the pile/soil interface is simulated by a series of infinite nonlinear springs along the pile. At a given depth z , if the spring exhibits a lateral displacement Y due to lateral load applied at the pile head, it undergoes a soil reaction P . Consequently, the P-Y curve, as illustrated in figure 2, describes the pile/soil interface response as function of the lateral loading level at the pile head.

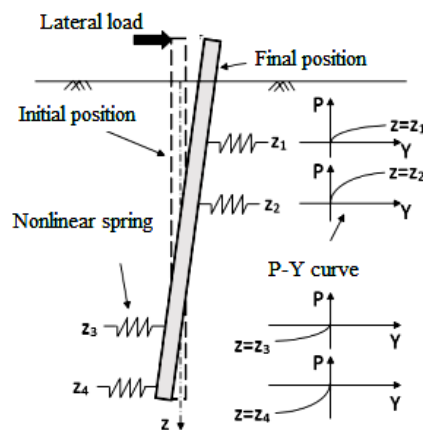


Fig. 1 – Scheme of lateral response of a single pile

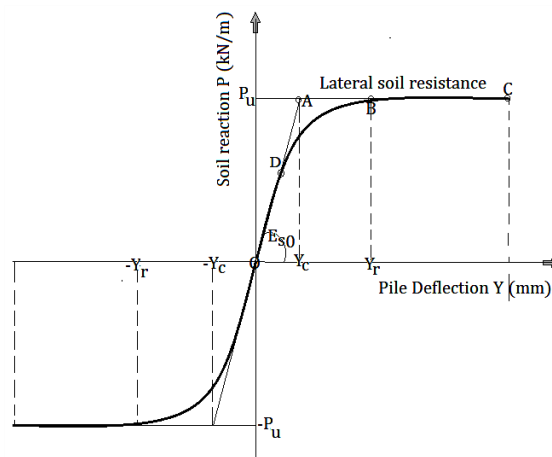


Fig. 2 – Typical P-Y curve

The main P-Y curves parameters are the initial reaction modulus E_{s0} representing the spring stiffness, and the lateral soil resistance P_u representing the lateral reaction mobilized at the limit equilibrium of the soil around the pile.

It can be noticed from the figure 2 that the P-Y curve has an inherent non-linear shape and encompasses three portions: initial linear portion OD, corresponding to the small pile deflections, a nonlinear portion DB corresponding to a gradual loss of the soil stiffness, and a horizontal segment BC corresponding to the full mobilisation of the soil resistance around the pile. Such a statement is in conformity with the observations of the pile response during a full-scale loading test.

Due to the simplicity of the experimental procedure and the continuous measurement of the soil resistance, the cone penetration test (CPT) is a widely used in-situ geotechnical test. However, compared to the pressure meter test (PMT) which

provides a complete stress-strain curve at a given depth, the CPT data is limited to provide only the soil penetration resistance. Nevertheless, some modern theories of soil mechanics, like the theory of expansion of spherical cavities [13-15], and the theory of strains path [16, 17], showed that the soil compressibility is a key factor affecting the mobilisation of the cone resistance. This latter may consequently be seen as an indirect way to evaluate the soil stiffness. Moreover, Jamiolkowski [18] and Lunne et al. [19] showed, on the basis of on experimental research works, that the cone resistance q_c may be correlated to the stiffness of sandy soils.

Based on this statement, it is possible to correlate the P-Y curves parameters to the cone resistance. The dimensional analysis will here after be used to highlight the main factors affecting the P-Y curve parameters. Let us assume that, at a given depth z , the reaction modulus E_{s0} and the lateral soil resistance P_u depend on q_c and the pile properties, as formulated as follows:

$$f(q_c^*, E_p I_p, D, B, P_u, E_{s0}) = 0 \quad (1)$$

q_c^* is the net cone resistance, taking into consideration the initial stress state at the depth z as follows:

$$q_c^*(z) = q_c(z) - \sigma_{v0}(z) \quad (2)$$

q_c and $\sigma_{v0}(z)$ are respectively the cone resistance and the initial vertical overburden pressure at the depth z .

$E_p I_p$, D , and B are respectively the flexural stiffness, the embedded length, and the diameter of the pile. In non-circular pile section, B is the dimension perpendicular to the lateral load direction).

By using the Buckingham's theorem, the dimensional analysis of the equation (1) reduces it as function of only 4 dimensionless variables as follows:

$$g\left(\frac{E_{s0}}{q_c^*}, \frac{P_u}{q_c^* B}, \frac{D}{B}, \frac{E_p I_p}{q_c^* D^4}\right) = 0 \quad (3)$$

The first variable, denoted K_E , is called hereafter "the modulus number" such as:

$$E_{s0} = K_E q_c^* \quad (4)$$

The second variable is denoted K_c and called "the lateral resistance factor". The lateral soil resistance may then be computed as follows:

$$P_u = K_c q_c^* B \quad (5)$$

The third and fourth terms are respectively the pile slenderness ratio and "the lateral pile/soil stiffness ratio" denoted K_R . It is to be noted that the coefficients K_E and K_c correspond respectively to the small pile displacements and large displacements, which implies they are independent, and consequently each of them depends on q_c^* , K_R and D/B . In this regard, the equation (3) may be uncoupled to the two following equations:

$$K_E = \frac{E_{s0}}{q_c^*} = h\left(q_c^*, K_R, \frac{D}{B}\right) \quad (6)$$

$$K_c = \frac{P_u}{q_c^* B} = j\left(q_c^*, K_R, \frac{D}{B}\right) \quad (7)$$

The aim of any research work on the correlation of the P-Y curves with CPT data consists in determining the functions h and j . In this paper, it is aimed to define these functions by interpretation of lateral loading of full-scale piles.

2 Brief review of the existing P-Y curves in clay

2.1 P-Y curves and the CPT test

First generation of P-Y curves was born thanks to Reese and Matlock [20] who were based on the concept of the lateral reaction modulus due to Winkler (in [21]). An elastic-perfectly plastic function, as described by the portion ODABC in figure

2, was proposed, and the P-Y curve parameters were defined only from the laboratory tests. Subsequently, Schmertmann [22] was the first to recommended to construct the P-Y curve based on the CPT data in sandy soils. With an elastic- perfectly plastic shape, the modulus E_{s0} was given as function of q_c and the pile diameter B , and P_u is equal to $(0.11q_cB)$ in loose sand, and to $(0.22q_cB)$ in dense sand. Other studies, limited to sandy soils, and based on physical modelling in centrifuge [23, 24] or on FEM modelling [25] were published.

Within the framework of the European geotechnical standard Eurocode 7, it is recommended by the French standard NF P94-262 for deep foundations, to construct the P-Y curve in clay as anelastic-perfectly plastic function whose first portion is defined by a lateral reaction modulus E_{s0} equal to $(6q_c)$, and the second one is an asymptote representing the lateral soil resistance P_u equal to $(q_cB/5)$. Under short duration loads, E_{s0} is doubled and P_u is kept same as under permanent loads [26].

It seems these methods are applicable for both short rigid piles and long flexible piles, ignoring the pile/soil stiffness ratio K_R as expected by equation (6). Such a statement is in contrast with the results of many full-scale pile tests published in literature [7].

2.2 Theoretical models of K_E and K_c

As reported by Bouafia [27] and summarized in table 1, some theoretical models of pile/soil interaction were proposed to evaluate the modulus number K_E according to the following general formula, E being the soil elastic modulus:

$$K_E = \frac{E_{s0}}{E} = a(K_R)^b \left[\frac{D}{B} \right]^c \quad (8)$$

According to table 1, the elastic models proposed by Baguelin, Banerjee, Randolph, and Poulos show that K_E is a decreasing function of the relative stiffness K_R , with an exponent about of -0.1. In contrast, the models of Terzaghi, Matlock, and Scott show that K_E is rather independent of K_R .

Comparison of the values of the coefficient a shows a wide variation from 0.55 to 2.92. Moreover, some contradictory statements are noticed regarding the effect of the slenderness ratio D/B . In fact, according to the Terzaghi, Matlock, Scott, and Banerjee & Davis models, there is no effect, whereas the other models show the contrary.

It is to be noted that the soil elastic modulus strongly depends on the stress path, among other physical parameters, which makes it one of the most difficult mechanical parameters to evaluate in geotechnical engineering. Nonetheless, its margins are usually roughly estimated by correlation with some geotechnical tests. In this regard, the use of equation (8) seems to be unreliable for practical purposes.

Table 1 – Compilation of the values of K_E for flexible piles in an elastic homogeneous soil ($\nu=0.5$)

Method	a	b	c	Model
Terzaghi [21]	0.74	0.00	0.00	Plate on an elastic half-space
Matlock [28]	1.80	0.00	0.00	embedded strip footing in soft clay
Baguelin et al [6]	2.77	-0.12	-0.53	Disk moving within an elastic medium
Banerjee & Davis [1]	0.55	-0.09	0.00	Elastic model
Randolph [29]	2.92	-0.14	-0.57	Finite element model
Scott [30]	1.00	0.00	0.00	Rigid cylinder moving into a finite elastic medium
Poulos & Hull [3]	2.08	-0.11	-0.45	Mindlin's equations of an embedded load

When considering an undrained response of the clayey soil, the following formula is usually used to determine the lateral soil resistance P_u as function of the undrained shear strength C_u :

$$P_u(z) = N_p \cdot C_u(z) \cdot B \quad (9)$$

N_p is a dimensionless factor of lateral resistance, depending on the depth and the roughness of the pile/soil interface.

The API standard recommends the values of N_p in soft clays equal to 3 and 9 respectively at surface and at a particular depth beyond which a deep lateral resistance occurs [31], whereas Broms [32] proposed a profile of $P_u(z)$ in clayey soil with N_p equal to 9.

Randolph and Houlsby [29] used the limit analysis theory to find an exact solution of N_p for a deep soil resistance, depending on the roughness of the pile/soil interface, and taking the values 9 and 12 respectively for smooth and rough interfaces.

On the basis of a laborious model derived from the theory of limit equilibrium, Hansen [33] proposed the values of 2.5 and 8.14 for N_p respectively at surface and at an infinite depth.

Reese & Van-Impe [11] were based on a mechanism of limit equilibrium to derive the value of N_p in a normally consolidated clay as follows:

$$N_p(z) = \min \left\{ 3 + 0.5 \frac{z}{B} + \frac{\gamma z}{c_u}, 9 \right\} \quad (10)$$

In an over-consolidated clay, N_p is computed as follows:

$$N_p(z) = \min \left\{ 2 + 2.83 \frac{z}{B} + \frac{\gamma z}{c_u}, 11 \right\} \quad (11)$$

Accordingly, the margins of values of N_p are 3-9 in normally consolidated clays and 2-11 in over-consolidated clays, the maximum values 9 and 11 corresponding to a deep lateral resistance.

Some discrepancy is to be noticed in the values of N_p , mainly depending on the mechanism of soil resistance adopted. According to the models of Hansen, the API, and Reese & Van-Impe based on two mechanisms of soil failure around the pile: shallow failure or deep failure, N_p depends on the depth, contrarily to the Broms model based on a unique soil failure mechanism where N_p does not depend on the depth. Note that the margins of N_p for a deep failure mechanism are comparable and lying between 9 and 12. However, all the theoretical models simply ignore the effect of the pile/soil stiffness ratio.

Many researchers interpreted the soil response to the penetration of a conical tip in order to derive the geotechnical properties. Such an approach was initially undertaken within a theoretical framework, but the studies were subsequently diversified by using the physical modelling based on centrifuge models or in calibration chamber, or the numerical modelling usually based on the finite elements method (FEM) or the finite difference method (FDM), in order to correlate the soil strength and stiffness properties with the cone resistance q_c .

According to Yu & Mitchell [34] and Lunne et al [19], despite important efforts devoted to explore this field, it still does not have a rigorous solution, and the difficulty of this problem is mainly due to the large deformations involved during the penetration and the complexity of the soil stress-strain behaviour [35-37].

The cone resistance q_c mobilized during the CPT test, at a depth z within a saturated fine soil, is described by the fundamental equation involving the cone factor N_k , the undrained shear resistance C_u , and the total initial vertical overburden stress σ_{v0} , as follows:

$$q_c = \sigma_{v0} + C_u \cdot N_k \quad (12)$$

The cone factor is defined as the ratio:

$$N_k = \frac{q_c - \sigma_{v0}}{C_u} \quad (13)$$

Among the theoretical methods, the classical theory of bearing capacity of piles was applied to the CPT test by modelling the cone penetrometer as a slender pile [38]. Moreover, modern theories like the theory of spherical cavities expansion [13-15] and the theory of strains path [16, 17] were used within this framework to interpret the CPT test. If considered as an elastic-perfectly plastic material, a fine saturated soil is usually characterized by the soil rigidity index I_r which is defined as the ratio of the soil shear modulus G to the undrained shear strength C_u , as follows:

$$I_r = \frac{G}{C_u} = \frac{E}{2(1+\nu)C_u} \quad (14)$$

It was theoretically shown that in the most theoretical approaches, N_k is a logarithmic function of I_r as follows:

$$N_k = a + b \cdot \ln(c \cdot I_r) \quad (15)$$

Table 2 summarizes some of the usual values of N_k according to these theoretical models, and the values of the constants a , b and c specific to each model. Further feedbacks showed the theory of bearing capacity of a rigid-plastic soil leads to a simplistic and insufficient interpretation of the CPT test. In addition, the multitude of parameters affecting the tip resistance mechanism led many researchers to undertake more realistic approaches based on numerical modelling (finite element methods, finite differences, etc).

As it can be seen in table 2, values of N_k strongly depend on the model used, and no clear accordance is noticed between them. N_k values derived from the theory of bearing capacity lie within the margin 7-11 which are the lowest and independent on the soil rigidity quantified by I_r . Moreover, the strains path theory as well as the FEM method show a dependance on the soil rigidity and the in-situ stress quantified by Δ .

Another category of methods of determination of N_k encompasses the empirical correlations developed on the basis of the physical modelling of the CPT test, namely by scale models in the centrifuge or by prototype cone in the calibration chamber.

Table 2 – Summary of some theoretical values of the cone factor N_k

Author	Theory	a	b	c	Reference
Terzaghi (1943)	Bearing capacity	7.41	0		[21]
Caquot & Kérisel (1956)	Bearing capacity	7	0		
Meyerhof (1951)	Bearing capacity	9.34(smooth cone) 9.74 (rough cone)	0		Cited in [19]
De-Beer (1974)	Bearing capacity	9.94	0		
L'Herminier (1967)	Bearing capacity	11	0		Cited in [39]
Meyerhof (1951)	SCE	2.33	1.33	1	
Gibson (1950)	SCE	3.06	1.33	1	Cited in [19]
Vesic (1972)	SCE	1.33	1.33	1	[15]
Baligh (1975)	CCE	12	1	1	[16]
Wittle (1992)	Strain path (smooth cone)	1.51	2	1	[40]
Yu (1993)	CCE (smooth cone)	4.18	1.15	0.87	Cited in [34]
	CCE (rough cone)	9.40	1.15	0.87	
Ladanyi & Johnson (1974)	SCE (smooth cone)	1.33	1.33	1	[13]
	SCE (rough cone)	3.06	1.33	1	
Teh&Houlsby(1991)	Strain path (smooth cone)	1.25-2 Δ	1.84	1	[41]
	Strain path (rough cone)	3.25-2 Δ	1.84	1	
Boufrina (2012)	FEM	3.79	4.13	1	[42]
Mir (2018)	FDM (FLA) (smooth cone)	10.73	0.46	1	[43]
	FDM (FLA) (rough cone)	11.60	0.68	1	
Lu et al (2004)	FEM (smooth cone)	3.4-1.9 Δ	1.60	1	[44]
	FEM (rough cone)	4.7-1.9 Δ	1.60	1	
Liyanapathirana (2009)	FEM (smooth cone)	1.0-1.7 Δ	1.83	1	[36]
	FEM (rough cone)	1.5-1.7 Δ	2.15	1	
Walker & Yu (2005)	FEM	0.21	2.0	1	Cited in [36]

SCE: Spherical Cavity Expansion, CCE: Cylindrical Cavity Expansion, FEM: Finite Elements Method, FDM: Finite Differences Method, FLA: Fast Lagrangian Analysis, Δ : In-situ stress ratio= $(\sigma_v-\sigma_{h0})/(2C_u)$

Moreover, a pragmatic approach to determine N_k is to correlate q_c obtained from the CPT test to the undrained cohesion measured from a direct shear test, a triaxial compression test, or a vane shear tests (VST). Table 3 summarizes some of these correlations, most of them were reported by many laboratories and research organizations during the 1st international symposium on the CPT, held in 1995 in Linköping, Sweden, on the basis of analysis of local databases. In the column (Site), if the study covered a large number of sites, only the country is indicated. This table clearly shows that the cone factor depends on the degree of over-consolidation of the clayey soil. It is remarkable that for soft or normally consolidated clays, the

empirical values of N_k fluctuate around an average value of 14 with a coefficient of variation (standard deviation/mean) of 20%. For stiff or over-consolidated clay, N_k is rather around 22 with a coefficient of variation equal to 27%.

Table 3 – Summary of some empirical values of the cone factor N_k

Author	Site(s)	Margin of N_k	Recommended value of N_k	Soil type	Reference
De Ruiter (1982)		10-15 15-20		NCC OCC	Cited in [39]
Barakat et al (1983)	Egypt		10	NCC	Cited in [39]
Mtynarek et al (1983)		20-24		OCC	Cited in [39]
Sanglerat (1965)	Annecy (France)	10-20	15	Soft clay	[45]
Amar et al (1972)	France		12 30	$C_u < 50$ kPa $C_u > 600$ kPa	[46]
Thom et al (1995)	Australia	12-15			
Nuyens et al (1995)			15		
Rocha et al (1995)	Bresil	10.5-15.5	14	Soft clay	
Denver (1995)	Danemark		10		
Tanaka (1995)	Japan	8-16			
Formonavicius (1995)			15-20	Soft clay	
Georges et al (1995)	Nigeria	12-20		NCC	
Lunne et al (1995)			15		
Ming-Fang (1995)	Malaysia		10	Marine clay	Cited in [47]
Ajdic et al (1995)			16	NCC	
Manas et al (1995)			15		
Moller et al (1995)	Sweden		16.3		
Nguen-Truong (1995)	Vietnam	14-30	20	Soft silty clay	
Olsen (1995)	USA	9-13 14-17		NCC OCC	
Desai et al (1995)	India	14-16 22-30		NCC OCC	
Bouafia (2002)		17.9		OCC	Cited in [43]

NCC: Normally consolidated clays, OCC: Over-consolidated clays

By equalizing the equations (5) and (9), the lateral resistance factor may be written as follows:

$$K_c = \frac{N_p}{N_k} \quad (16)$$

If one adopts the usual margins 14-22 for N_k and 9-12 for N_p , as mentioned above, K_c takes according to equation (16) the values of 9/14 and 12/22, which corresponds to a margin of 0.54-0.64, the lower and upper bounds corresponding respectively to over-consolidated and normally consolidated clays. The value of 0.2 prescribed by the French standard is rather out of the margin and equal about one third of these values.

3 Problem statement

The above literature review has shown the insufficiency of the theoretical models to evaluate the coefficients K_E and K_c of the P-Y curve based on the CPT test. Some contradiction was noticed when analyzing the effects of some key parameters, such as the slenderness ratio D/B and the pile/soil stiffness ratio K_R , on the E_{s0} and P_u . Some methods show a direct effect whereas the other ones infirm it, leading to a dependence of the factors K_E and K_c on the theoretical model adopted. Moreover, except for the French method, there is no direct method to construct the P-Y curves based on the CPT test. In fact, although the CPT is widely used as a powerful foundation design tool, to the knowledge of the author, the literature does not provide any practical recommendation regarding the direct use of the CPT test to analyze the pile load-deflection response in clayey soils [48].

A pragmatic approach of development of the P-Y curves, avoiding such a shortcoming, is the experimental analysis of instrumented pile behaviour, which may lead to construct the P-Y curves and correlate their parameters with the cone resistance measured by the CPT test. The present study falls within this research framework by focusing on the experimental determination of the P-Y curves, based on a thorough interpretation of many full-scale lateral loading tests in clay. Hereafter is described the methodology of such an approach, where a series of lateral loading tests on full-scale fully instrumented piles, driven into a clayey soil, was interpreted. Once the profile of bending moment, for a given lateral load applied at the pile top, was measured from the strain gauges, a process of double differentiation and double integration of the profiles of the measured bending moment is performed to obtain respectively the profiles of lateral reaction P and the pile deflection Y as function of depth, which allows deriving the P-Y curves at each depth.

4 Methodology of the study

4.1 Description of the experimental sites

Located in the county of Romainville, at the commune of Bagnolet, 8 km north-east of Paris, France, the experimental site belongs to the geological age of the Sannoisien, and is composed of a green saturated clayey layer, thick of 8 to 11 m, and classified according to the USCS system as a clay of high plasticity (CH).

Typical profiles of the cone resistance q_c are remarkably uniform with depth and characterized by a mean value of 3 MPa. The drained shear strength parameters are as follows: C' (cohesion)=55 kPa and $\phi'=22^\circ$, and the unconfined compressive strength of many samples was within 167-265 kPa, to which corresponds a margin of the undrained shear strength C_u of 84-133 kPa. According to the standard ISO 14688-2, the shear strength of this clay is rather high. Moreover, PMT tests performed in the site showed a uniform limit pressure profile with depth, characterized by an average value of 800 kPa and a coefficient of variation of 11%, indicating a remarkable homogeneity of resistance. According to Ménard's classification based on the PMT test, this clay is described as a normally consolidated clay becoming over-consolidated with depth [38].

4.2 Description of the experimental piles

A total of 8 experimental piles were designed by using rectangular steel Larssen's caissons, having a margin of width B of 0.43-0.6 m, a flexural stiffness $E_p I_p$ varying between 16 and 60 MN.m², a margin of slenderness ratio D/B of 5.8-11.8, and installed by vibratory driving. This set of test piles encompasses well foundations ($4 < D/B < 10$) as well as long piles ($D/B > 10$). In table 4 are summarized the main geometrical and mechanical properties of the test piles, where their initial nomenclature was kept as in the original document of testing.

To ensure a total dissipation of the stresses caused by the vibratory process, which may exceed 500 to 600 kPa, the lateral loading tests were carried out a few days after the installation phase. Each pile was instrumented by 11 pairs of strain gauges regularly spaced along the pile and distributed along two diametrically opposite axes, in order to measure the bending moment profile for each lateral load applied at the pile head.

The lateral load is applied at a level of 0.3 to 1.15 m above the ground surface, by means of a hydraulic jack and measured by a load cell fixed at the pile head. By using a computerized acquisition unit, the experimental results were obtained at real-time during the test. Two dial gauges were fixed at two points above the ground surface level to obtain the deflection Y_0 and the rotation Y_0' at surface, which will be used as boundary conditions for the double integration process of the bending moment profile. Moreover, load cell was used to measure the lateral load applied at the pile head, and Glotzl's pressure cells were mounted along each test pile to check out the experimental lateral soil pressures mobilized. During each loading test, a real-time data acquisition was performed by using an acquisition chain system [27].

4.3 Construction of the P-Y curves

For a given lateral load, the pile deflection profile $Y(z)$ is obtained by a double integration of the corresponding bending moments profile $M(z)$, z being the depth of the strain gauge, as described by the following equation:

Table 4 – Summary of the geometrical and mechanical properties of the test piles

Pieu	Test	B (m)	D/B	K _R
D	#3	0.60	5.83	13.47x10 ⁻²
D	#4	0.60	7.50	4.94x10 ⁻²
B	#5	0.43	6.16	17.7x10 ⁻²
B	#6	0.43	5.83	1.29x10 ⁻²
B	#7	0.43	9.65	2.96x10 ⁻²
A	#9	0.43	8.14	3.59x10 ⁻²
A	#10	0.43	10.8	1.15x10 ⁻²
B	#11	0.43	11.4	1.53x10 ⁻²

$$Y(z) = \frac{1}{E_p I_p} \int_0^z \left(\int_0^t M(t) dt \right) + Y'_0 \cdot z + Y_0 \quad (17)$$

The soil reaction profile $P(z)$ is obtained by two successive differentiations of $M(z)$ as follows:

$$P(z) = -\frac{d^2 M(z)}{dz^2} \quad (18)$$

The P-Y curve is then constructed at the depth of strain gauge. However, equation (18) shows that $P(z)$ is the curvature of the bending moment at the depth z , and is consequently highly sensitive to any fluctuation of the bending moment. The values of $P(z)$ strongly depend on the function chosen to fit the bending moment. The experimental profiles $M(z)$ were then fitted by quintic spline functions or by least-squares polynomial functions, the static balance of forces and moments along the pile, within a given tolerance, was used as a choice criterion [42, 49].

5 Results and discussion

5.1 Formulation of the P-Y curves

In figure 3 are illustrated typical profiles of the bending moment, from which were derived those of the soil reaction and the pile deflection. From the profiles $P(z)$ and $Y(z)$, the experimental P-Y curves were then constructed point-by-point for a given depth along the test pile. The moment profile $M(z)$ is characterized by an inflexion point at 2.5 m (70% of the embedded length) which implies, as shown by the profile $P(z)$, the presence of two zones of passive earth pressures mobilized above and below this depth. At this depth, both the soil reaction P and the pile deflection Y are null and then change of sign, which is in accordance with the fundamental hypothesis of Winkler (in [21]) postulating that P is proportional to Y [7].

Moreover, for all the tests, the P-Y curves were expectedly found non-linear shaped, showing a trend to a horizontal asymptote for large deflections, describing the full mobilization of the lateral soil resistance. Within a deflection of 1 to 2 mm, the P-Y curves exhibit a regular increase with depth of the initial slope of the curve (which is the lateral reaction modulus E_{s0}).

According to some geotechnical standards, as well as in literature, the P-Y curves are formulated by simple analytical functions (Hyperbola, exponential, hyperbolic tangent, cubic function, etc). However, in such functions, the deflection corresponding to the mobilization of the lateral soil resistance P_u is infinite, leading to an unrealistic description of the pile response at large displacements. To account for such a statement, the experimental P-Y curves were fitted by a simple analytical function called PARECT (**PA**rabola-**RE**CTangle). This function takes into consideration the non-linear nonlinear shape of the curve (portion OB in figure 2), as well as the mobilization of the lateral soil resistance described by a horizontal asymptote (portion BC in figure 2) as from a threshold deflection $Y_R(z)$ called "Reference deflection".

The PARECT function is defined as follows:

$$P(z) = E_{s0}(z) \cdot Y(z) \cdot \left(1 - \frac{Y(z)}{2Y_R(z)} \right) \quad (19)$$

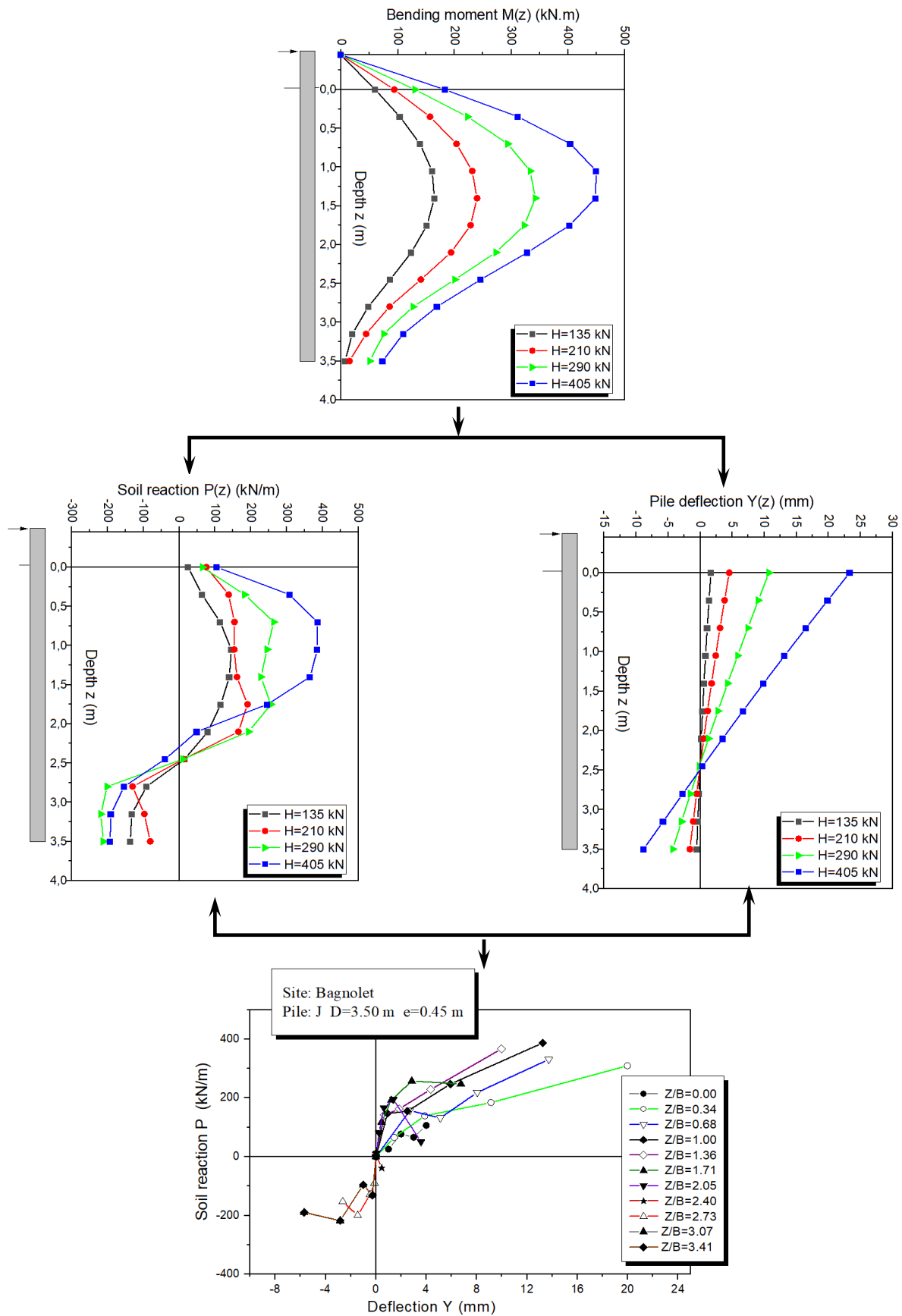


Fig. 3 – Typical construction of the experimental P-Y curves

The PARECT function is defined as follows:

$$P(z) = E_{s0}(z) \cdot Y(z) \cdot \left(1 - \frac{Y(z)}{2Y_R(z)}\right) \quad (19)$$

Another form of this function is as follows:

$$P(z) = 2P_u(z) \frac{Y(z)}{Y_R(z)} \left(1 - \frac{Y(z)}{2Y_R(z)}\right) \quad (20)$$

On the basis of equations (19) and (20), the reference deflection may be expressed as:

$$Y_R(z) = 2 \frac{P_u(z)}{E_{s0}(z)} \quad (21)$$

5.2 Definition of the lateral pile/soil stiffness ratio K_R

Due to an inherent non-homogeneity of the soil, it is expected that the profile of the cone resistance $q_c(z)$ exhibits an irregular distribution with depth. In this regard, based on the concept of the equivalent homogeneous soil, leading to analyze the response of an equivalent pile having an embedded length D_e into an equivalent homogeneous soil. This latter is characterized by an "equivalent net cone resistance" q_{ce}^* , conventionally defined as being a weighted average of $q_c(z)$ along D_e as follows:

$$q_{ce}^* = \frac{1}{D_e} \int_0^{D_e} q_c \cdot dz \quad (1)$$

D_e is called "the effective embedded length" of the pile, and corresponds to the depth beyond which no deflection is exhibited due to the lateral loading at the pile top, such as:

$$D_e = \min\{D, \pi L_0\} \quad (23)$$

L_0 is the "elastic length (or transfer length)", and usually formulated as:

$$L_0 = \sqrt[4]{\frac{E_p \cdot I_p}{K_E \cdot E_c}} \quad (24)$$

The lateral pile/stiffness ratio, already formulated in equation (3), may be rewritten as function of q_{ce}^* as follows:

$$K_R = \frac{E_p \cdot I_p}{q_{ce}^* \cdot D^4} \quad (25)$$

5.3 The lateral reaction modulus E_{s0}

The analysis of the function h formulated by equation (6) shows that the modulus number K_E decreases as a power function of the lateral pile/soil stiffness ratio K_R , as follows:

$$K_E = \frac{1}{2K_R^{3/4}} \quad (26)$$

This empirical equation is limited to piles with a slenderness ratio D/B is greater than or equal to 7.5.

Based on the previous equation, it is interesting to notice that since K_E decreases with K_R , a relatively flexible pile behaves with a greater value of K_E , and therefore mobilizes a greater lateral reaction modulus E_{s0} . According to equation (25), K_E decreases with the pile flexural stiffness $E_p I_p$ and increases with the pile length D .

5.4 Pile Classification using the relative stiffness

In engineering practice, the laterally loaded pile design and analysis requires the classification of the pile foundation with respect to the ratio of its lateral stiffness to that of the soil, by using the term K_R . A rigid pile, to which corresponds a non negligible value of K_R , behaves by mobilizing a linear profile of deflections and subjecting the surrounding the soil to important lateral pressures leading to the soil failure, whereas a flexible pile, to which corresponds a small value of K_R , behaves as a long beam and mobilizes important bending moment leading to the failure of the pile material.

According to the classical theory of beams on elastic foundations, a long flexible pile has an embedded length D greater than πL_0 . By combining equations (23) through (25) and arranging the terms, the condition of a flexible pile may be written as: $K_R > 2.5 \times 10^{-2}$. A short rigid pile is described as having an embedded length $D < L_0/2$. Similarly, by using the previous equations, this definition leads to $K_R > 1.50$. Consequently, a semi-rigid pile is classified as having K_R lying between 0.025 and 1.50.

Based on this classification, the test piles are characterized by a margin of K_R from 1.2×10^{-2} to 18×10^{-2} , and classified as flexible or semi-rigid piles, which implies that the proposed equation of K_E is applicable to flexible or semi-rigid piles.

As a practical consequence of the use of equation (26), and within the framework of a preliminary stage of design of a laterally loaded pile foundation, it is recommended to estimate the pile deflection under working lateral loads by estimating an average lateral reaction modulus based on equations (4) and (26), and then compute the pile ground deflection Y_0 . After classifying the pile with respect to the pile/soil stiffness K_R , for a flexible pile subjected to a lateral load H , combined to a bending moment M_0 , Y_0 is computed as follows [38]:

$$Y_0 = 2 \frac{H}{L_0 E_{s0}} + 2 \frac{M_0}{L_0^2 E_{s0}} \quad (27)$$

whereas for a semi-rigid pile with a free tip, Y_0 is computed as:

$$Y_0 = 2A_1 \frac{H}{L_0 E_{s0}} + 2A_2 \frac{M_0}{L_0^2 E_{s0}} \quad (28)$$

A_1 and A_2 are dimensionless functions of the ratio D/L_0 as follows:

$$A_1 = \frac{ch(\frac{D}{L_0})sh(\frac{D}{L_0}) - cos(\frac{D}{L_0})sin(\frac{D}{L_0})}{sh^2(\frac{D}{L_0}) - sin^2(\frac{D}{L_0})} \quad (29)$$

$$A_2 = \frac{sh^2(\frac{D}{L_0}) + sin^2(\frac{D}{L_0})}{sh^2(\frac{D}{L_0}) - sin^2(\frac{D}{L_0})} \quad (30)$$

The following example is an illustration of this practical recommendation. Let us assume that a steel pipe pile has an outer diameter B of 0.5 m, an embedded length $D = 5$ m, a flexural stiffness $E_p I_p = 56.6$ MN.m², and driven into a normally consolidated saturated clay, characterized by an average resistance cone q_c equal to 20 MPa. Based on equation (25): $K_R = 4.53 \times 10^{-3}$ corresponding to a flexible pile. Equations (26), (4), (24) give respectively $K_E = 28.64$, $E_{s0} = 572.88$ MPa, and $L_0 = 0.79$ m. Under a working lateral load of 50 kN applied at a height of 1 above the ground surface, the equation (27) gives a pile deflection Y_0 equal to 0.50 mm.

Let us consider the same pile/soil configuration but with a flexural stiffness $E_p I_p$ equal to 740 MPa, Equation (25) gives $K_R = 5.92 \times 10^{-2}$ corresponding to a semi-rigid pile. One obtains $K_E = 4.166$, $E_{s0} = 83.32$ MPa, and $L_0 = 2.44$ m. Finally, according to equation (28), the pile deflection Y_0 is equal to 0.77 mm.

5.5 The lateral soil resistance P_u

It was found that the function j , formulated by equation (7), increases as a power function with the lateral pile/soil stiffness ratio. Nonlinear fitting the experimental values of K_c , based on the least-squares technique, led to the following equation:

$$K_c = 0.1 + 8.6 K_R^{1.7} \quad (31)$$

Similarly, to K_E , the equation of K_c is applicable for D/B greater than or equal to 7.5 and limited to flexible or semi-rigid piles. This equation shows that a clayey soil around a rigid pile mobilizes more lateral soil resistance than around the same pile but flexible. Based on equations (25) and (31), since K_c increases with the $E_p I_p$ and decreases with the length D , to increase the lateral soil resistance, the pile should be designed with a shorter length and/or a higher flexural stiffness. By reference to the previous comparative example, the equations (31) and (5) lead respectively to $K_c=0.10$ and $P_u=1000$ kN/m for the flexible pile, whereas $K_c=0.170$ and $P_u=1703$ kN/m for the same pile when working as rigid.

It is to be noticed that for this particular example, although the flexural stiffness $E_p I_p$ was magnified 13 times, the lateral soil resistance increased only from 1.0 to 1.7 MPa/m, which highlights a relatively little effect of the pile stiffness on the mobilization of the soil resistance.

5.6 The reference deflection Y_R

As depicted in figure 2, the reference deflection Y_R at a given depth corresponds to the threshold of the ultimate limit state (ULS) of exceedance of the lateral soil resistance. By combining the equations (21), (26) and (31), one can express the normalized reference deflection Y_R/B by the following function:

$$\frac{Y_R}{B} = 0.4(K_R)^{0.75} + 34.4K_R^{2.45} \quad (32)$$

This equation shows that the normalized deflection Y_R/B regularly increases with the lateral pile/soil stiffness K_R and does not consequently take a constant value as conventionally prescribed in the literature. For a flexible pile, since $K_R \leq 2.5 \times 10^{-2}$, according to the previous equation, an upper bound for Y_R/B is 3%, which is less than the conventional value of 10%, prescribed by some methods in the literature. The critical deflection Y_c , as shown in figure 2, conventionally corresponds to the limit of the linear shape of the P - Y curve, and therefore to the limit of the linear pile/soil response. This value is half that of Y_R , and may then be bounded by 1.5% of the pile diameter (or the width) whatever the depth. To avoid any nonlinearity, this value is recommended to be used when checking the serviceability limit state (SLS), where the maximum pile deflection should not exceed a tolerable value.

5.7 Comparison of the parameters of the P - Y curve

According to the elastic models of Baguelin et al (1978), Banerjee (1978), Randolph (1981), and Poulos & Hull (1992), summarized in table 1, the modulus number K_E decreases as a power function of K_R with an exponent of about (-0.10), whereas the proposed value of K_E , formulated in equation (26), uses rather an exponent of (-0.75). The other elastic models, as well as the French standard, lead to conclude that K_E is independent of the pile/soil stiffness, in contrast with the experimental results.

Regarding the factor K_c , equation (31) gives for flexible piles an interval of 0.10 to 0.12, and 0.12 to 17.2 for semi-rigid piles. For comparison purposes, the values of K_c based on the cone factor N_k , lie within the margin 0.54-0.64, as stated above whatever the pile/soil stiffness ratio. Similarly, the French standard gives a value of 0.2 for K_c , ignoring the effect of K_R . Such contrasts, exhibited by these methods with respect to the experimental findings presented in this paper, may be explained by the necessity to derive simple equations of the P - Y curve parameters, which implies to neglect some effects (Bouafia, 2014).

6 Definition of a step-by-step procedure of computation based on the P - Y curves

The following methodology outlines a CPT-based method of analysis of a laterally loaded pile embedded into a normally consolidated to slightly over-consolidated clayey soil. The method necessitates the use of a software of computation based on the P - Y curves.

- 1 The CPT profile usually contains many peaks of resistance and is therefore irregular shaped, which leads to a complicated determination of the cone resistance q_c . In order to obtain a relatively regular profile, proceed to filter the profile of cone resistance by eliminating the peaks of resistance and simply replacing them by a linear interpolation,

- 2 The soil around the pile is subdivided into N thin horizontal sub-layers in such a way as q_c^* exhibits a linear variation within any sublayer. Consequently, the value of q_c^* at the mid-sublayer is representative of all the sub-layer,
- 3 The pile is first assumed to be semi-rigid, which means from the equation (23) that $D_e = D$,
- 4 Compute q_{ce}^* according to equation (22) by approximating the integration formula by the summation of trapezes of the $q_c(z)$ profile,
- 5 Compute K_R by using the equation (25),
- 6 Compute K_E on the basis of the equation (26),
- 7 Compute the elastic length L_0 by using the equation (24),
- 8 The equivalent embedded length D_e is computed from the equation (23). If $D_e < D$ (case of a flexible pile), then an iterative process is carried out by repeating steps 3 to 7 until convergence of K_R ,
- 9 The parameters E_{so} , K_c , and P_u are computed within each sub-layer respectively from equations (4), (31), and (5),
- 10 Compute the reference deflection Y_R for each pile segment by using the equation (32),
- 11 Use equation (19) to construct the P-Y curve point-by-point at mid-length of each pile segment,
- 12 Use a software based on the P-Y curves method to analyse the lateral pile response for a given series of lateral load on the pile top. Many softwares can do this task. Among them, the software SPULL (Single Pile Under Lateral Loads), developed at the University of Blida, may be used in this regard [7].

7 Validation of the proposed P-Y curves method

7.1 Description of the case study

Validation process was launched by applying this methodology to a centrifuged scale model of a single bored pipe pile in a saturated slightly over-consolidated clay. Khemakhem [50] conducted a series of monotonic lateral loading tests on small-scale piles in the centrifuge of the IFFSTAR (formerly: LCPC) laboratory in Nantes (France). According to the similitude laws, any small-scale test at a scale $1/N$ may ensure a rigorous conservation of the stresses and strains of the prototype, provided the macro-gravity field is N times g , g being the gravity acceleration ($g=10 \text{ m/s}^2$). The pile prototype is an Aluminum pipe having a diameter of 900 mm, an embedded length of 16 m, a slenderness ratio $D/B=17.8$, and a flexural stiffness of 895 MN.m^2 .

The test soil is a saturated Kaolinite characterized by a plasticity index $I_p=30\%$, a bulk unit weight of 17 kN/m^3 , and a linear profile of the undrained shear strength, with a margin of 1-20 kPa along the pile. This material is slightly over-consolidated with an OCR (Over-Consolidated Ratio) characterized by a margin of 1.7 to 4.3, a mean value of 2.4, a coefficient of variation of 34.7%, and a regular decrease with depth.

The CPT test was undertaken during the rotation of centrifuge by using a miniature penetrometer having a diameter of 12 mm penetrating with a velocity of 5 mm/s. This velocity was judged sufficiently high to mobilize an undrained response of the clayey soil. The net cone resistance $q_c^*(z)$ is characterized by a linear profile and a margin of 43-360 kPa, corresponding to a very soft clay. The lateral load was applied by a hydraulic jack prescribing a controlled-displacement loading to the pile model with a constant rate of 0.4 mm/s.

7.2 Discussion of the results

Equations (22) through (25) give respectively an equivalent cone resistance $q_{ce}^*=221.5 \text{ kPa}$, an effective embedded length $D_e=D=16 \text{ m}$, an elastic length $L_0=7.95 \text{ m}$, and a pile/soil stiffness ratio $K_R=0.062$. This test pile is therefore semi-rigid since $K_R>0.025$ and $D/L_0=2.0 < \pi$.

Equation (26) gives a modulus number equal to 4.04, which leads to compute the equivalent lateral reaction modulus E_{s0} according to equation (4) as 0.895 MPa. Manual computing of the pile deflection Y_0 at ground surface for very small lateral loading may be carried out by using the equations (28) to (30). For $H=10$ kN.m and $M_0=20$ kN.m, one obtains $A_1=1.133$, $A_2=1.129$ and $Y_0=5.2$ mm by assuming a free pile tip. The pile top deflection is thus equal to 5.2 mm, which is in very good accordance with the experimental value equal to 6.5 mm.

The normalized reference deflection Y_R is computed based on equation (32), and is equal to 8.7%, and the critical deflection Y_c , limiting the domain of the linear pile behaviour according to the PARECT function, is therefore equal to 4.3% of B . According to equation (31), the lateral resistance coefficient K_c is equal to 0.175.

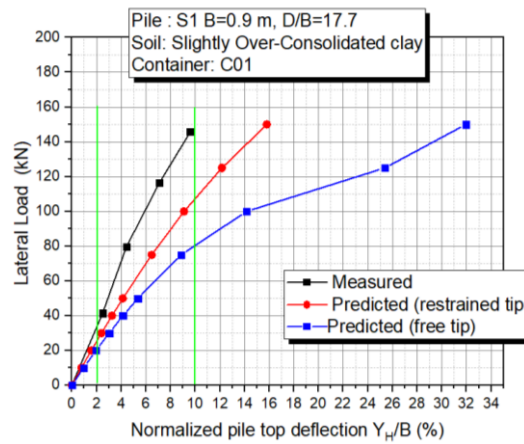


Fig. 4 – Comparison of the normalized pile top deflections

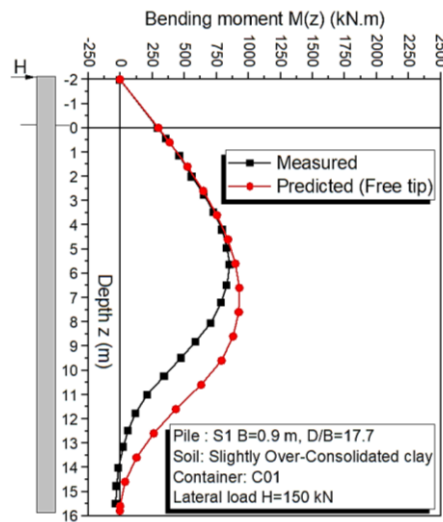


Fig. 5 – Comparison of the bending moment profiles

Numerical computing based on the proposed P-Y curves was undertaken following the methodology outlined in the previous section, by dividing the test pile into 17 segments, the P-Y curves being constructed and defined at mid-length of each segment. Since the test pile is semi-rigid, its response depends on the boundary conditions at its tip. The analysis by the software SPULL was carried out by taking the two extreme boundary conditions: free tip (bending moment and shear force equal to 0), and at-rest tip (deflection and rotation equal to 0).

As illustrated by figure 4, the direct comparison of the load-deflection curves showed an excellent prediction of the small deflections up to about 2% of the pile diameter. This is the domain where the lateral reaction modulus E_{s0} is a key parameter.

It is concluded that the proposed method is able to predict the small-deflections of the pile by a reliable formulation of E_{s0} based on the cone resistance (equations 4 and 26). Beyond 2% of B , as shown in figure 4, the boundary conditions at the pile tip have a non negligible influence on the results, and the predicted deflection are rather pessimistic. It may be possible that the lateral resistance P_u was underestimated by the equations 5 and 31 which leads during the iterative process to relatively larger deflections.

According to figure 5 showing an example of direct comparison of the measured bending moments and the predicted ones for a lateral load of 150 kN, a very good accordance is noticed particularly at shallower depths. To this level of loading corresponds a pile top deflection of about 10% of diameter.

8 Conclusions

This study presents the development of a new method to construct the P-Y curves for a single pile embedded in a normally consolidated or slightly over-consolidated clayey soil based on the cone penetration test (CPT). The interpretation of the lateral monotonic loading tests on 8 fully instrumented piles vibro-driven into a quite homogeneous saturated clayey soil led to construct the experimental P-Y curves following a process of double differentiation and double integration of the bending moment profile. The P-Y curve parameters were correlated to the cone resistance q_c .

The modulus number K_E and the lateral resistance coefficient K_c were found varying as power functions of the lateral pile/soil stiffness ratio K_R . The suggested CPT-based method is applicable for the analysis of the load-deflection behavior of flexible or semi-rigid piles with a slenderness ratio D/B greater than or equal to 7.5.

A methodology of analysis by the proposed CPT-based P-Y curves for piles in normally consolidated to slightly over-consolidated clays was proposed. Moreover, a pile classification according to the pile/soil stiffness ratio was suggested, and the concept of the critical deflection corresponding to the threshold of domain of large deflections of piles was introduced.

A validation process was launched by applying this methodology to a centrifuged scale model of a single bored pipe pile in a saturated slightly over consolidated clay. Direct comparison of the load-deflection curves showed an excellent prediction of the small pile deflections up to about 2% of the pile diameter. Beyond this value, the boundary conditions at the pile tip have a non negligible influence on the results, but showed a pessimistic estimation of the pile deflections. The bending moments were however very well predicted.

The proposed method is limited to the pile/soil configuration characterized by a pile slenderness ratio not less than 7.5 and to saturated normally consolidated to lightly over-consolidated clays. Further improvements of the proposed method will be carried out to extend it to the case of heavily over-consolidated clays, by carrying out the analysis of lateral loading tests of full-scale piles in such soils.

Acknowledgements

The author is grateful to Mr. Jacques Roudier, General Manager of the IFFSTAR (formerly Laboratoire Central des Ponts & Chaussées) Laboratory for giving him the authorization to interpret the experimental data of lateral loading tests of piles carried out at the site of Bagnolet.

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