Revue d'Economie et de Statistique Appliquée
Volume 23, number 01, June 2026
ISSN : 1112-234X EISSN : 2600-6642
Pages :102-116

COMPARATIVE ANALYSIS OF EXTREME VALUE THEORY APPROACHES FOR VALUE AT RISK (VaR) ESTIMATION

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Received: 12/04/2026 **Accepted :** 08/06/2026 **Published online**: 28/06/2026

ABSTRACT:
This study evaluates statistical techniques for Value at Risk (VaR) estimation, focusing on Extreme Value Theory (EVT) approaches compared to the Historical VaR benchmark. Using 249 daily log returns of AAPL.INC stock (NASDAQ) from May 9, 2022, to May 5, 2023, we applied GEV, GPD, Hill, Pickands, and Moments estimators. The GPD approach showed the closest alignment with Historical VaR, yielding a Mean Squared Error (MSE) of nearly zero (4.2×10^{-6}) and consistent results across confidence levels (e.g., VaR at 99%: -0.053 for Historical vs. -0.051 for GPD). Pickands estimator provided reasonable accuracy ($MSE = 2.2 \times 10^{-5}$), while Hill, Moments and GEV exhibited higher MSE values (1.9×10^{-4}, 2.1×10^{-4} *and* 6.5×10^{-4}, respectively). These findings highlight the potential of EVT for accurate tail risk modeling, with GPD emerging as the most robust approach.

Keywords: Value-at-Risk(VaR), Extreme value Theory (EVT), semi parametric estimators, risk management, financial modeling.
JEL Classification: G11, G32, C02, C13.

I. INTRODUCTION

In 1993, Value-at-Risk (VaR) was recommended as an indispensable risk measurement criterion. VaR is a risk analysis tool that measures a company's maximum potential loss, an investment portfolio over a given period at some specific confidence level (Jorion 1997). Statistically speaking, VaR is an extreme quantile of the distribution of the studied random variable. In recent years, both researchers and market practitioners have increasingly focused on Value at Risk (VaR) as a tool for analyzing market risk. The International Monetary Fund (IMF) reports that the 2008 financial crisis resulted in a total

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loss of $3.4 trillion across major financial institutions worldwide, representing the most significant economic downturn since the 1930s.

Extreme value theory (TVE) is a branch of statistics that appeared between 1920 and 1943 due to Frechet (1927), Fisher and Tippet (1928) and Gnedenko (1943).It provides statistical models for studying rare events. It allow to study the behavior of the tail of a distribution (the analysis of the largest or smallest observation of a sample) which is closely linked to a real parameter called the index of extreme values. Over the last 20 years, extreme value theory (EVT) has received much attention and growing interest for its applications to extremes event modeling and particularly in risk management, which is a common procedure in different areas, for example, hydrology, geology, insurance and finance, and Meteorology (McNeil 1999; Embrechts et al. 1997; Reiss and Thomas 2001; Naima et al. 2017; Hassen 2021; Friederichs 2010; Coles and Pericchi 2003; Coles and Tawn 1996).

However, TVE comprises two models: generalized extremes value (GEV) and Generalized Pareto Distribution (GPD). Several methods can be used to estimate the extremal index; The Maximum likelihod (ML) method (Coles 2001; Smith 1987), the Moments method (Hosking 1990; Greenwood et al. 1979; Christopeit 1999) and other semi-parametric methods (Hill 1975; Pickands 1975).

A significant challenge in modeling Value at Risk (VaR) lies in the common assumption of normality for the return data series of assets or portfolios. This assumption becomes invalid when the data exhibit heavy tails. To address this issue, extreme distribution-based methods, such as Extreme Value Theory (EVT), can be applied. In recent years, numerous studies have employed EVT to calculate VaR in the context of financial markets, particularly in response to currency crises, stock market turbulence, and credit defaults. This approach has been explored in works by Odening and Hinrichs (2002); Singh et al. (2011), and Baran and Witzany (2011).

The aim of this work is threefold. First, we intend to focus on traditional VaR calculation models. Secondly, we want to explore the applicability of EVT to VaR calculation through an application on the "closing price of AAPL" financial series. Finally, a comparison of the performance of EVT and standard VaR techniques shall support the decision as to whether more or less sophisticated methods are appropriate for assessing financial market risks. The evd (Stephenson 2002), extRemes (Gilleland and Katz 2011) and ismev (Stephenson 2014) packages of R (R Core Team 2015) were used for the data analysis.

This paper is organized as follows. Section II presents a brief overview of extreme value theory and shows how this concept can be used to estimate the tails of return distributions. Section III briefly describes the VaR concept and its different methods of estimations. In section IV, we apply EVT to VaR. The results are compared with the outcome of standard procedures, namely historical simulation method. The paper ends with a discussion of the results and a conclusion.

II. THEORETICAL BACKGROUND ON EXTREME VALUE THEORY

2.1. The Generalized Extreme Value (GEV) Distribution

Theorem 1. Theorem of Fisher and Tippet (1928): Suppose $x_1, x_2, \dots, x_n$ is a sequence of independent and identically distributed (IID) random variables with the common distribution cdf $F(x) = \mathrm{P}(\mathrm{X} \leq \mathrm{x})$. Let $M_n = Max\{x_1, x_2, \dots, x_n\}, n \in N$. If there exist sequences of normalizing constants $\{a_n > 0\}\ and\ b_n \in R$ and a non-degenerate distribution function G such that:

$$\lim_{n\to+\infty} P\left(\frac{M_n - b_n}{a_n} \leq x\right) \to F^n(a_n x + b_n) \to G(x)\,, \quad \forall\, x \in \mathrm{R} \qquad (1)$$

Then G belongs to one of the following families:

Type I: Gumbel (Λ)

$$G(x) = exp(-e^{-x})$$

Type II: Fréchet (Φ_α)

$$G(x) = \begin{cases} 0 & if\ x \leq 0 \\ exp(-x^{-\alpha}) & if\ x > 0 \end{cases} \quad ,\alpha > 0$$

Type III: Weibull (Ψ_α)

$$G(x) = \begin{cases} exp(-(-x)^{\alpha}) & if\ x \leq 0 \\ 1 & if\ x > 0 \end{cases} \quad ,\alpha > 0$$

Theorem 1 states that if there are suitable sequences a_n and b_n to stabilize M_n, then $\frac{M_n - b_n}{a_n}$ has a limiting distribution which is one of the types I, II and III. These three families of distributions are known as the Gumbel, Fréchet and Weibull families respectively and are collectively known as the extreme value distributions.

The three models can be combined into one family of models known as the Generalised Extreme Value (GEV) family as proposed by Von Mises (1954) and Jenkinson (1955) with distribution functions given by:

$$G_{(\xi,\mu,\sigma)}(x) = \begin{cases} exp\left\{-\left[1 + \xi\left(\frac{x-\mu}{\sigma}\right)\right]^{-\frac{1}{\xi}}\right\}, & if\ 1 + \xi\left(\frac{x-\mu}{\sigma}\right) > 0\,,\ \xi \neq 0 \\ exp\left(-exp\left(-\frac{x-\mu}{\sigma}\right)\right), & if\ \ \xi = 0 \end{cases} \qquad (2)$$

The parameters μ, σ and ξ are location, scale and shape parameters respectively. It is the value of the shape parameter, ξ, that distinguishes the three classes of extreme value distribution. Types II and III in Theorem 1 correspond to values of ξ > 0 (heavy

tail behavior) and $\xi < 0$ respectively. The case of $\xi = 0$ is interpreted as the limit as $\xi \to 0$ of Equation (2). This leads to the Gumbel family of distributions

2.2. The Generalized Pareto distribution (GPD)

Suppose $X_1, X_2 \ldots$ is a sequence of IID with a continuous distribution F(.) . suppose that $M_n = Max\{X_1, X_2, \ldots, X_n\}, n \in N$ and that F(.) also satisfies the condition in expression (1). Then, for suitably large threshold u, the distribution function F_u of $(X - u)$ condition on $X > u$ denote the distribution of the observations above the threshold u. It can be expressed by:

$$F_u(x) = P(X - u \le x \setminus X > u) = \frac{F_X(x+u) - F_X(u)}{1 - F_X(u)}$$

Where $0 \le x \le x_F - u$ and $x_F = sup\{x \in R: F(x) < 1\}$

Theorem 2.Gnedenko-Pickands-Balkema-de Haan: if F(.) is in the maximum domain of attraction (MDA) of the extreme value distribution with shape parameter ξ. The distribution F_u converge to the generalized Pareto distribution (GPD), denoted $G_{\xi,\sigma^*}(x)$ as u approaches to the end point x_F. The convergence can be described by the following expression:

$$\lim_{u \to x_F} sup_{0 \le x \le x_F - u} \left| F_u(x) - G_{\xi,\sigma^*}(x) \right| = 0 \qquad (4)$$

The GPD distribution is given:

$$G_{\xi,\sigma^*}(x-u) = \begin{cases} 1 - \left\{1 + \xi \frac{x}{\sigma^*}\right\}^{-1/\xi}, & if\ \xi \neq 0 \\ 1 - exp\left(-\frac{x}{\sigma}\right), & if\ \xi = 0 \end{cases} \qquad (5)$$

Where $\sigma^* > 0, x \ge 0\ for\ \xi \ge 0\ \ and\ \ 0 \le x \le {-\sigma^*}/{\xi}\ for\ \xi < 0$

Note that $\sigma^*\ and\ \xi$ are scale and the shape parameters respectively

2.3. Estimation methods of the GPD

2.3.1. Maximum Likelihood Estimation

The maximum likelihood method (Smith and Naylor 1987) is the most popular estimation method and has the advantage of allowing the addition to the fitting of co-variables (such as trends, cycles or physical variables). We use MLE to obtain the estimates of parameters σ and ξ .We choose the threshold u from the mean-excess plot, select the exceedances above u, and fit the GPD to excess returns. Recall that maximum likelihood estimate choose the estimates $\hat{\sigma}$ and $\hat{\xi}$ which maximize the likelihood function given by:

$$l(\sigma^*, \xi, x_1, x_2, \dots, x_{N_u}) = \prod_{i=1}^{N_u} g_{\xi,\sigma^*}(x) = \frac{1}{\sigma^{*N_u}} \sum_{i=1}^{N_u} \left(1 + \frac{\xi}{\sigma^*} x_i\right)^{-\frac{1}{\xi-1}}$$

Where N_u denote the number of exceedences above u

The log likelihood function is given by:

$$logl(\sigma^*, \xi, x_1, x_2, \dots, x_{N_u}) = \begin{cases} -N_u log\sigma^* - \left(1 + \frac{1}{\xi}\right) \sum_{i=1}^{N_u} log\left(1 + \xi \frac{x_i}{\sigma^*}\right), & \xi \neq 0 \\ -N_u log\sigma^* - \frac{1}{\sigma^*} \sum_{i=1}^{N_u} x_i & , \xi = 0 \end{cases} \quad (6)$$

By differentiating this Equation (6) with respect to the two parameters of interest, we obtain a system of two equations with two unknowns $\sigma^* and\ \xi$. it is by solving these equations that we obtain the ML estimates (using numerical methods ,e.g. using Newto Raphson Algorithm).

2.3.2. Semi parametric estimation:

The Hill estimator: the Hill estimator (1975) can only be used for Fréchet type distributions (therefore such that $\xi > 0$) for which it provides a more efficient tail index estimator. It is defined as follow:

$$\hat{\xi}_{k,n}^{H} = \frac{1}{k} \sum_{i=1}^{k} logX_{n-1+i,n} - logX_{n-k,n} \quad (7)$$

Where $X_{1,n} \leq X_{2,n} \leq \cdots \leq X_{n,n}$ are the order statistics of $X_1, X_2, \dots X_n$

The Pickands estimator ($\boldsymbol{\xi \in R}$): the pickands estimator (Pickands 1975) is valid whatever the distribution of the extremes (Fréchet, Gumbel and Weibull).it is given by

$$\hat{\xi}_{k,n}^{(P)} = \frac{1}{log2} log \frac{X_{n-k,n} - X_{n-2k,n}}{X_{n-2k,n} - X_{n-4k,n}} \quad (8)$$

The Moments estimator: in 1989, Dekkers et al proposed the moments estimator which is an extension of the Hill estimator. This estimator is valid for the entire index $\boldsymbol{\xi \in R}$. It is given by:

$$\hat{\xi}_{k,n}^{(M)} = M_n^1 + 1 - \frac{1}{2}\left(1 - \frac{(M_n^1)^2}{M_n^2}\right)^{-1} \quad (9)$$

Where : $M_n^r = M_n^r(k) = \frac{1}{k}\sum_{i=1}^{k}\left(logX_{n-1+i,n} - logX_{n-k,n}\right)^r, \quad r = 1,2$

III. VaR AND ITS ESTIMATING METHODS

Jorion (2002), Harlin et al (2007), and De Jonghe (2010) defined VaR as the maximal potentially of a portoflio of assets or an asset during a given time period for a given confidence interval p which will only be exceeded with a probability at most equal to $\alpha = 1 - p$. Statistically speaking the $VaR(h, \alpha) = F_h^{-1}(1 - \alpha)$ (10)

3.1. Classical Methods of VaR calculation

We will provide a brief overview about several methods of the VaR calculation : The historical method, the parametric method (Jorion 1997; Dowd 1998; Manfredo and Leuthold 1999), and the extreme value theory.

3.1.1. Historical Method

The Historical VaR is directly calculated from observed historical data. That means, we calculate VaR directly from past returns (the past is a good indicator of the near-future). The main characteristics of this method are: relatively simple to set up, does not assume any form of distribution, and depends on the quality and availability of data.

Accordingly, VaR estimates based on this approach are sensitive to changes in the data sample. In addition, it is not possible to predict events that are worse than the maximal loss during the sample period.The historical VaR is calculated using the following steps:

- Sort Historical Returns
- Decide the confidence level (e.g., 95% or 99%).
- Identify the corresponding percentile of the returns $(n \times \alpha)$
- The VaR is the return at the selected percentile

Or, directly finds the value at the desired percentile, making it a quick and accurate method for Historical Simulation. For a confidence level of $1 - \alpha$, the quantile at α corresponds to the VaR. we write : $VaR_h(\alpha) = F^{\leftarrow}(1 - \alpha)$ (11)

3.1.2. Parametric method (Variance-Covariance method)

It is a risk management technique for calculating the VaR of a portfolio of assets that first identifies the mean (the expected value), and standard deviation of an investment portfolio. It assumes that the returns distribution is Normal. We suppose that $R^p \sim N(\mu, \sigma^2)$, according to Equation (10) we have:

$$\Phi\left(\frac{VaR_h(\alpha) - \mu}{\sigma}\right) = \alpha$$

Where: Φ is the CDF of the standard normal distribution. Then the parametric VaR is given by:

$$VaR_h(\alpha) = \mu + \sigma\Phi^{-1}(\alpha)$$

The method is not recommended for long horizons, for heavy tailed portfolios distributions, or for assets with skewed distributions

3.2. VaR based on Extreme value theory (EVT-VaR)

3.2.1. VaR based on the MLE of the GEV

To estimate the Value at Risk (VaR) using the Generalized Extreme Value (GEV) distribution, the formula is derived from the quantile function of the GEV distribution. THE EVT-VaR is given by:

$$VaR(\alpha)_{GEV} = \begin{cases} \hat{\mu} + \frac{\hat{\sigma}}{\hat{\xi}}\left[(-\ln(1-\alpha))^{-\xi} - 1\right] & if\ \xi \neq 0 \\ \hat{\mu} - \hat{\sigma} ln(-\ln(1-\alpha)) & if\ \xi = 0 \end{cases} \quad (12)$$

Where $\hat{\mu}, \hat{\sigma}\ and\ \hat{\xi}$ are the MLE estimates of $\mu, \sigma\ and\ \xi$ respectively.

3.2.2. VaR based on the MLE of the GPD

According to (4), $F_u(x) \approx G_{\xi,\sigma^*}(x)$ for large u and. $\hat{\xi}\ and\ \hat{\sigma}$ are the MLE estimates. The GPD-VaR is given by:

$$VaR(\alpha)_{GPD} = \begin{cases} u + \frac{\hat{\sigma}^*}{\hat{\xi}}\left(\left(\frac{n}{N_u}\alpha\right)^{-\hat{\xi}} - 1\right) & for\ \xi \neq 0 \\ u + \hat{\sigma}^*\ ln\left(\frac{N_u}{n\alpha}\right) & for\ \xi = 0 \end{cases} \quad (13)$$

3.2.3. VaR based on semi parametric estimators

The Weissman Estimator: the Weissman Estimator is based on the assumption that the tail of the distribution follows a heavy tailed Pareto type. The Weissman estimator of the VaR is given by:

$$VaR(\alpha) = u\left(\frac{N_u}{n\alpha}\right)^{\hat{\xi}}$$

VaR based on the Hill estimator: Beirlant et al (1996b) proposed the Hill estimator of the VaR based on Weissman approach which is defined by the following formula:

$$VaR(\alpha) = X_{n-k_n,n}\left(\frac{k_n+1}{(n+1)\alpha}\right)^{\hat{\xi}^H_{k,n}} \quad (14)$$

Where : $\hat{\xi}^H_{k,n}$ is the Hill estimator defined in in Equation (7)

VaR based on the Pickands estimator: the Pickands estimator of the VaR is given by:

$$VaR(\alpha) = \frac{\left(\frac{k}{n\alpha}\right)^{\hat{\xi}^{P}_{k,n}} - 1}{1 - 2^{-\hat{\xi}^{M}_{k,n}}}\left(X_{n-k+1,n} - X_{n-2k+1,n}\right) + X_{n-k+1,n} \qquad (15)$$

Where : $\hat{\xi}^{H}_{k,n}$ is the Pickands estimator defined in in Equation (8)

VaR based on the moments estimator: Dekkers et al. (1989b) proposed to estimate VaR(α) as follows:

$$VaR(\alpha) = X_{n-k_n,n} + \hat{a}\left(\frac{k_n}{n}\right)\left(\frac{k_n}{n}\frac{\left(\frac{n}{k_n}\alpha\right)^{-\hat{\xi}^{M}_{k,n}} - 1}{\hat{\xi}^{M}_{k,n}}\right) \qquad (16)$$

Where

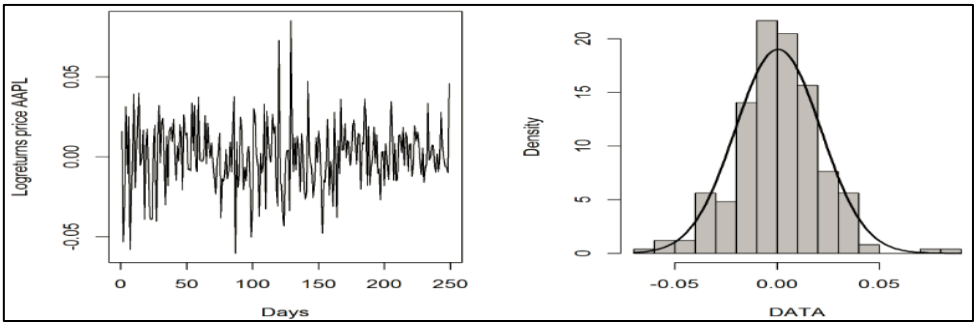
$$\hat{a}\left(\frac{k_n}{n}\right) = X_{n-k_n,n}\hat{\xi}^{M}_{k,n} max\left(1 - \hat{\xi}^{M}_{k,n}, 1\right)$$

$\hat{\xi}^{M}_{k,n}$ is the Moment estimator defined in Equation (9)

IV. APPLICATION

The data used in this study correspond to log returns closing price for AAPL.INC stock (NASDAQ stock market). We aim to forecast one day VaR at differents confidence levels (from 95% to 99%). The total data period is approximately 1 year (9 May-2022 to 5 May-2023) containing 249 daily log returns.

Figure 01: Plot (left) and Histogram (right) of AAPL log returns closing price



Source: By the author using R.4.1.2

4.1. Descriptive analysis

The series is given in figure 1 and its statistical characteristics are given in Table 1**.** It seems that the series is not affected by a trend, it appears stationary. From Table 1, it is found that our data is characterized by a mean of 0.0005. We note that 50% of the data are between -0.011 and 0.0138. The Histogram shows that the normality assumption could be rejected

for our series because its tails are heavier than the normality ones; this conclusion is supported by the skewness, which is greater than zero and the value of kurtosis (different from three).

Table 01: Descriptive statistics of AAPL log returns closing price

n	Mean	Q_1	Q_3	Min	Max	skewness	kurtosis	J-B
249	0.0005	-0.011	0.0138	-0.0604	0.0852	1.1775	0.08414	15.5

Source: By the author using R.4.1.2

4.2. The extreme value modelling

The EVT approaches (Block maxima, POT, semi parametric) model the right tail of the distribution, which corresponds to the largest positive values (extreme gains).In our study, we are interested in the left tail (extreme losses or minima), and we need to negate the log returns to effectively "flip" the tail. This ensures the tail modeling applies correctly to the extreme losses. We proceed to put DATA= - DATA

4.2.1. The Block Maxima Modelling

In this subsection, we will group the data into weekly blocks, and take the maxima of each block. Then we fit the GEV model to the series of weakly maximum that contains 36 observations. The statistical characteristics are given in the following table.

Table 02: Descriptive statistics of block maxima series

n	Mean	Q_1	Q_3	Min	Max
36	0.0255	0.0135	0.0381	0.0044	0.0604

Source: By the author using R.4.1.2

The maximum likelihood method was used to estimate the three parameters of the generalized extreme value distribution, the following results were found:

Table 03: Maximum likelihood estimation of the parameters of the GEVD

Location parameter			Scale parameter			Shape parameter		
μ	Sd	CI	σ	Sd	CI	ξ	Sd	CI
0.017	0.002	[0.012; 0.022]	0.011	0.001	[0.008; 0.015]	0.088	0.20	[-0.31 ; 0.48]

Source: By the author using R.4.1.2

As noted in table 03, the positive shape parameter ($\xi = 0.088$) implies that the GEV follows the Fréchet type.

4.2.2. The POT Modelling

To analyze maximum AAPL stock prices using the POT method, first a threshold value u_0 is determined and then the GPD is fitted to the price values above u_0. For our data, the threshold value of $u_0 = 0.016$ has been chosen using the mean excess plot approach and checked by the parameters stability plots**.** The statistical characteristics of the AAPL log returns price above selected threshold (44 observations) are given in Table 04.

Table 04: Descriptive statistics of POT series

N_u	Mean	Q_1	Q_3	Min	Max
44	0.0143	0.0041	0.0222	3.75×10^{-5}	0.0444

Source: By the author using R.4.1.2

The maximum likelihood method was used to estimate the two parameters of the GP distribution. The ML estimates of the scale and shape parameters, their associated 95% confidence intervals are given in Table 05.

Table 05: Maximum likelihood estimation of the parameters of the GPD distribution.

Scale parameter			**Shape parameter**		
$\boldsymbol{\sigma}$	Sd	CI	$\boldsymbol{\xi}$	Sd	CI
0.0196	0.004	[0.011; 0.027]	-0.353	0.169	[-0.686; -0.019]

Table 05 indicates that the shape parameter is negative ($\xi = -0.353$), suggesting that the GPD belongs to the Pareto type II limited tailed.

4.2.3. Semi-parametric estimating of the tail index ξ

The Hill, Pickands, and the Moments estimators of the extreme value index for the log returns for AAPL.INC stock price series are given in the following table:

Table 06: Semi-parametric estimators of ξ

Hill estimator		Pickands Estimator		Moments estimator	
k-optimal	$\hat{\xi}^H_{k,n}$	k-optimal	$\hat{\xi}^P_{k,n}$	k-optimal	$\hat{\xi}^M_{k,n}$
32	0.228	24	-0.049	40	-0.082

Source: By the author using R.4.1.2

The Hill estimator indicates that the AAPL log returns distribution has a heavy tail $\left(\hat{\xi}_{k,n}^{H} > 0\right)$ while the Pickands and Moments estimator shows that the distribution has a bounded tail $\left(\hat{\xi}_{k,n}^{P}, \hat{\xi}_{k,n}^{M} < 0\right)$

4.3. Estimation of the VaR

According to the equations 11, 12, 13, 14, 15 and 16, the estimation of both the standard VaR and the EVT-based VaR is summarized in the following table.

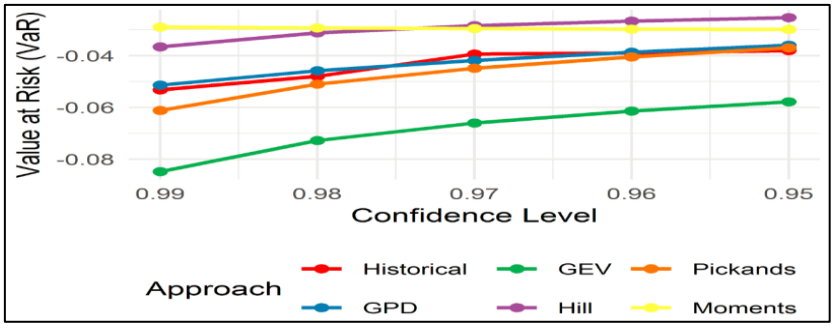
Table 07: Estimation VaR using EVT methods with MSE value between them and the historical approach.

	VaR (99%)	VaR (98%)	VaR (97%)	VaR (96%)	VaR (95%)	MSE
Historical-VaR	-0.053	-0.048	-0.039	-0.039	-0.038	-
GPD-VaR	-0.051	-0.046	-0.042	-0.039	-0.036	4.2×10^{-6}
GEV-VaR	-0.085	-0.073	-0.066	-0.061	-0.058	6.5×10^{-4}
Hill-VaR	-0.037	-0.031	-0.028	-0.027	-0.025	1.9×10^{-4}
Pickands-VaR	-0.061	-0.051	-0.045	-0.041	-0.037	2.2×10^{-5}
Moments-VaR	-0.030	-0.030	-0.030	-0.030	-0.030	2.1×10^{-4}

Source: By the author using R.4.1.2

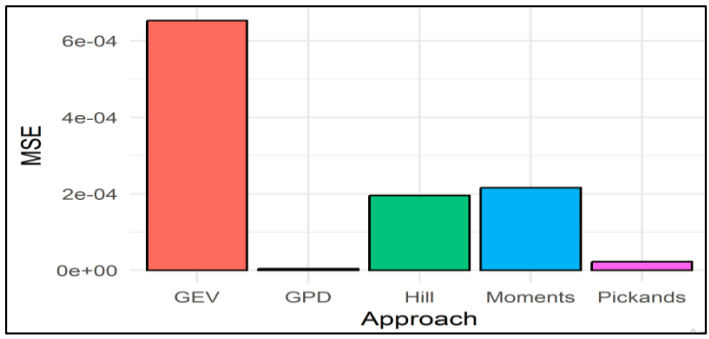
The results summarized in Table 07 are further illustrated in Figures 02 and 3, which provide a graphical comparison of the estimated VaR values.

Figure 02: Comparison of VaR across different approaches



Source: By the author using R.4.1.2

Figure 03: Mean squared error between Historical VaR and EVT approaches



Source: By the author using R.4.1.2

V. DISCUSSION OF THE RESULTS

The Historical VaR (red curve) represents the non-parametric method based on empirical quantiles. It shows a steady increase with decreasing confidence levels ($1-\alpha$), consistent with the expectation of higher risk at lower confidence levels.

The GPD VaR (ξ **= -0.353** bounded tail) aligns closely with the Historical VaR across most confidence levels, particularly for 97% and 96%. At higher confidence levels (98% and 99%), it slightly overestimates risk, which may result from sensitivity to threshold selection or parameter estimation. Notably, the GPD estimator has the lowest MSE, highlighting its excellent alignment with the Historical VaR and its effectiveness in capturing tail risks.

The GEV VaR ($\xi = 0.088$ slightly heavy tail) consistently estimates higher risk compared to the Historical VaR across all confidence levels. This reflects its ability to model heavy-tailed distributions effectively; however, it is overly conservative for this dataset. The GEV estimator has the highest MSE, indicating significant deviations from the Historical VaR and a potential misalignment for this type of data.

The Hill VaR ($\xi = 0.228$ Strongly heavy-tailed) underestimates risk compared to the Historical VaR across most confidence levels. This underestimation highlights the sensitivity of the Hill estimator to the choice of k and sample size. Its MSE is higher than those of GPD and Pickands, indicating less reliable performance for this dataset.

The Pickands VaR ($\xi = -0.049$ bounded tail but closer to zero than GPD) closely tracks the Historical VaR at moderate confidence levels (97% and 96%), demonstrating its robustness and suitability for tail risk estimation. While it has one of the lowest MSEs after GPD, it slightly underperforms GPD in terms of overall accuracy relative to the Historical VaR.

The Moments VaR (($\xi = -0.082$ bounded tail but closer to zero than GPD) shows nearly constant behavior across confidence levels due to its bounded-tail assumption. This pseudo-constant behavior indicates limited adaptability to varying confidence levels, leading to underestimation of extreme risks. The Moments estimator has a moderate MSE, making it less effective than GPD and Pickands but comparable to Hill.

VI. CONCLUSION

This study evaluated statistical methods for Value at Risk (VaR) estimation, applying Extreme Value Theory (EVT) techniques and benchmarking them against the Historical VaR. Using 249 daily log returns of AAPL.INC stock (NASDAQ) over one year (May 9, 2022, to May 5, 2023), EVT approaches including GEV, GPD, Hill, Pickands, and Moments estimators were analyzed. The results demonstrated that the GPD estimator exhibited the lowest Mean Squared Error (MSE), making it the most accurate EVT-based approach for estimating VaR.

The Pickands estimator also performed well, closely tracking the Historical VaR with low MSE. The GEV estimator provided robust tail modeling but demonstrated the highest MSE, reflecting significant overestimation of risk and misalignment with the Historical VaR., the Hill estimator underestimated risk due to parameter sensitivity, and the Moments estimator showed limited adaptability due to its bounded-tail assumptions.

Among the EVT methods, GPD emerged as the most reliable and effective approach for VaR estimation, followed by Pickands, providing valuable insights into financial tail risk management.

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