

DOWNWARD CONTINUATION OF AIRBORNE GRAVITY DATA

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ملخص

بأجهزة قياس الجاذبية المحمولة التي تصل بانتظام إلى دقة تقدر بـ 1-2 ميكروقال بدقة تميز 5-10 كلم ، فإن إنجاز متابعة تنازلية موثوقا فيها لشذوذ الجاذبية في الجو يعتبر تحديا. أجهزة قياس الجاذبية المحمولة مهيأة خاصة لتحديد الجيود في الجبال و المناطق القطبية، بسبب اختبار العينات على الميدان العشوائي، المتعلق بمسح الأسطح النموذجي. في هذا المقال، طرق عملية مختلفة معتمدة على التصنيف و على FFT يتم تخطيطها و تطبيقها على المتابعة التنازلية الخاصة بدمج معطيات السطح للمسح المنجز بالطائرة فوق Groenland و Svalbard، متراوحة في ارتفاع المسح من 150 م إلى 4100 م.

Résumé

Avec la gravimétrie aéroportée qui atteint régulièrement des précisions de 1-2 mGal à une résolution de 5-10 km, il serait un défi d'exécuter une continuation descendante fiable des anomalies de la gravité en altitude. La gravimétrie aéroportée est adaptée surtout pour la détermination du géoïde dans les régions montagneuses et polaires, dû à l'échantillonnage épuré du terrain, relatif aux levés de surface typiques. Dans l'article, des méthodes opérationnelles différentes basées sur la collocation et la FFT sont tracées et appliquées à la continuation descendante du fusionnement de données de la surface des levés aéroportés sur Groenland et Svalbard, variant en altitudes de levée de 150 m à 4100 m .

Abstract :

With airborne gravimetry routinely reaching accuracies of 1-2 mGal at 5-10 km resolution, it is a challenge to perform a reliable downward continuation of the gravity anomalies at altitude. Airborne gravimetry is especially suited for geoid determination in mountainous and polar regions, due to the random, alias-free sampling of the terrain, relative to typical surface surveys. In the paper different operational methods based on collocation and FFT are outlined, and applied to downward continuation of surface data merging of airborne surveys over Greenland and Svalbard, ranging in survey altitudes from 150 m to 4100 m.

Keywords: Airborne gravimetry, physical geodesy, harmonic downward continuation, geoid.

I. Introduction

Airborne gravity data over mountainous areas often need to be downward continued and merged with existing gravity field data in order to provide a uniform surface gravity grid, to serve both general gravity field mapping needs and geoid determination purposes.

The harmonic downward continuation of airborne gravity data is a classical unstable operation, and some kind of smoothing is necessary in practice, making the collocation approach or stabilized Fourier methods the method of choice for attacking this problem.

Downward continuation is essentially a high-pass filtering operation, which will amplify short-wavelength noise in the airborne data; However, if the primary purpose of the airborne survey is to generate a geoid model, the downward continuation amplification of high wavenumbers is offset by the gravity to geoid low-pass filtering operation; therefore airborne gravity is well-suited for geoid determination.

Direct formulations of the airborne gravity to geoid boundary value problems have recently been formulated by Li (2000); for more general studies on the downward continuation problem see e.g. Keller and Hirsch (1992) or Blaha et al. (1996).

The use of gravity terrain effects in the downward continuation process have significant advantages: It will – if properly applied – reduce the short-wavelength information in the airborne gravity data, and thus make the problem more stable; it will also diminish the topographic aliasing, especially important for surface gravity data, which typically do not have a random distribution relative to the topography (usually there is an over-weight of gravity data collected in valleys). However, the use of terrain effects may generate additional noise if terrain and density models are not sufficiently accurate.

Special terrain reduction problems occur e.g. in mountainous coastal areas, where gravity anomalies associated with deep fjords (negative local anomalies at 10-100 mgal typical magnitudes) may not be included in the computed terrain effects, e.g. due to unavailable bathymetry data. While airborne data may sample a whole coastal region uniformly, the typical land-only surface coverage will give a biased gravity field sampling, missing completely the negative anomalies of the fjords.

This gives the potential for long-wavelength systematic errors between gravity grids derived from surface data only and from the airborne data, which could propagate into erroneous geoid tilts.

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With airborne “random” sampling of topography the airborne geoid should theoretically be the best; in practice it is a trade-off problem, however, balancing the larger errors of the downward continued airborne data to the surface data. Similar aliasing problems will occur in glaciated areas with glaciers and ice caps of unknown thickness.

All of the above problems are fully present in the gravity field determination of Arctic areas. In connection with the need for a new Arctic-wide 5' gravity data set, prepared under the IGGC “Arctic Gravity Project” (Kenyon and Forsberg, 2001), it has been necessary to do new downward continuation computations of both Greenland and Svalbard, as significant new data have become available in recent years. This paper outlines these computations, and gives examples of results of different methods to make gravity and geoid grids.

II. Operational methods for downward continuation of airborne gravimetry

If the airborne gravity survey is carried out at constant altitude, downward continuation may readily be carried out by frequency domain methods. Let Δg and Δg^* be the gravity anomalies at altitude h and the geoid, respectively. By taking the two-dimensional Fourier transform of Δg

$$F(\Delta g) = \iint \Delta g(x, y) e^{-i(k_x x + k_y y)} dx dy \quad (1)$$

the gravity anomalies at the geoid are obtained by

$$F(\Delta g^*) = e^{kh} F(\Delta g), \quad k = \sqrt{k_x^2 + k_y^2} \quad (2)$$

It is clear that high wavenumbers (and thus noise) is strongly amplified by this operation. By assuming a Kaula rule decay of the power spectral density (PSD) of Δg , and assuming data noise to be white, the optimal Wiener filter downward continuation operator becomes (“frequency domain collocation”, cf. Nash and Jordan, 1978)

$$\Delta g^* = \Delta g \frac{e^{kh}}{1 + ck^4} \quad (3)$$

Here c is a resolution parameter, depending on the ratio of noise to gravity signal covariance (Forsberg and Solheim, 1988). In practice some stabilization is implicitly obtained by gridding of the airborne gravity data onto a regular grid by collocation, since the collocation (grid) estimate by default generates a smooth function, depending on assumed standard deviations of data.

The drawback of the FFT methods (and equivalent integral methods) is that varying altitudes of the airborne data are not easily taking into account, and existing surface data cannot be utilized. Therefore least-squares collocation with full spatial covariance models would be the method of choice for downward continuation in practice. In least squares-collocation the gravity anomaly signal “ s ” at a ground grid point is estimated from a vector “ x ” containing all available surface and airborne data by

$$\hat{s} = C_{sx} [C_{xx} + D]^{-1} \quad (4)$$

Covariances C_{xx} and C_{sx} are taken from a full, self-consistent spatial covariance model, and D is the (diagonal) noise matrix. Because the gravity field of the earth is known to follow Kaula rule, it is important to select covariance models which have an implied PSD decay in accordance with this.

An example of such a self-consistent covariance model on a spherical earth is the Tsherning-Rapp model (Tscherning and Rapp, 1974), and for a flat earth the much simpler planar logarithmic covariance model (Forsberg, 1987). In the latter model, the gravity covariance between gravity anomalies at two altitudes are of form

$$C(\Delta g^{h_1}, \Delta g^{h_2}) = - \sum_i \alpha_i \log(D_i + \sqrt{s^2 + (D_i + h_1 + h_2)^2}) \quad (5)$$

where α_i are weight factors combining terms relating to two depth value terms (D, T), taking the role analogous to the Bjerhammar sphere depth of spherical collocation and a long-wavelength “compensating depth” attenuation factor. The attenuation of long wavelengths in the model are necessary when a spherical harmonic reference model (EGM) is used.

III. Use of terrain and EGM information

Spherical harmonic geopotential models, such as EGM96 (Lemoine et al., 1996), have an implicit height dependency, and downward continuation of airborne gravimetry is therefore with advantage done using a remove-restore technique. Here the gravity anomalies are split into three parts

$$\Delta g = \Delta g_1 + \Delta g_2 + \Delta g_3 \quad (6)$$

where the first term is evaluated by spherical harmonics (as a function of latitude, longitude and height), the second term the (optional) gravitational attraction of a terrain model, and Δg_3 the residual anomalies.

In mountainous areas the use of rigorous terrain reductions, such as the residual terrain model (RTM) reduction (Forsberg, 1984), will stabilize the downward continuation significantly, since the removal of terrain effects will limit the short-wavelength variability of the gravity data significantly. Terrain effects are computed by e.g. prism integration or FFT methods, but for airborne gravity data one important aspect must be remembered: because airborne gravity data are invariably along-track *filtered*, the terrain effects computed at altitude must be filtered by the same filter algorithm as used in the airborne processing; otherwise short-wavelength terrain-effects may actually produce noise in the airborne data.

IV. Downward continuation of Greenland and Svalbard airborne gravity data

In connection with the preparation of on a uniform Arctic-wide gravity grid (“Arctic Gravity Project”) there has been a need to perform a new downward continuation and surface data merging process for Greenland and Svalbard. The Greenland area was covered by the first continental-scale airborne gravity survey of the US Naval Research Laboratory (NRL) 1991-92 (Brozina, 1992). The accuracy of the NRL airborne gravity data are estimated to be around 5 mgal r.m.s., verified by cross-over statistics and comparisons to upward continued surface data (Forsberg and Kenyon, 1996). The survey was flown at an elevation of nominally 4.1 km, with an along-track full-width filtering resolution around 18 km.

Since the pioneering NRL surveys, significant new surface, marine and low-level airborne gravity data have become available, as have much improved terrain models. The new data have been acquired with support from NIMA and oil companies.

The new data should make an improved downward continuation/merging of airborne data compared to earlier computations (Forsberg and Kenyon, 1996). The new KMS airborne data around the Greenland were flown at very low elevation (150-300 m) over marine areas. With an estimated accuracy around 2 mgal r.m.s., and a typical resolution of 6 km (Olesen et al., 2001) these data are therefore quite equivalent to marine gravimetry, and have been treated as such in the present computations.

For Svalbard a number of high-resolution airborne surveys was flown in a cooperative project of Statens Kartverk (SK), University of Bergen and KMS 1998-2001, using the same Twin-Otter aircraft system as in the Greenland surveys (Solheim et al., 2000). Some airborne lines were flown at up to 2 km elevation to allow clearance of high topography, and the downward continuation process was subsequently done on all airborne data. Fig 1 and Fig 2 shows the coverage of both surface and airborne gravity data in the two regions.

All gravity data from the two regions were EGM- and terrain-reduced using an RTM reduction of topography relatively to a mean height surface of approx. 90 km resolution. Basic DEM data used had 1 km resolution, and in Greenland a low-resolution (10 km) ice thickness model was used as well to model the terrain masses. No bathymetry data was used.

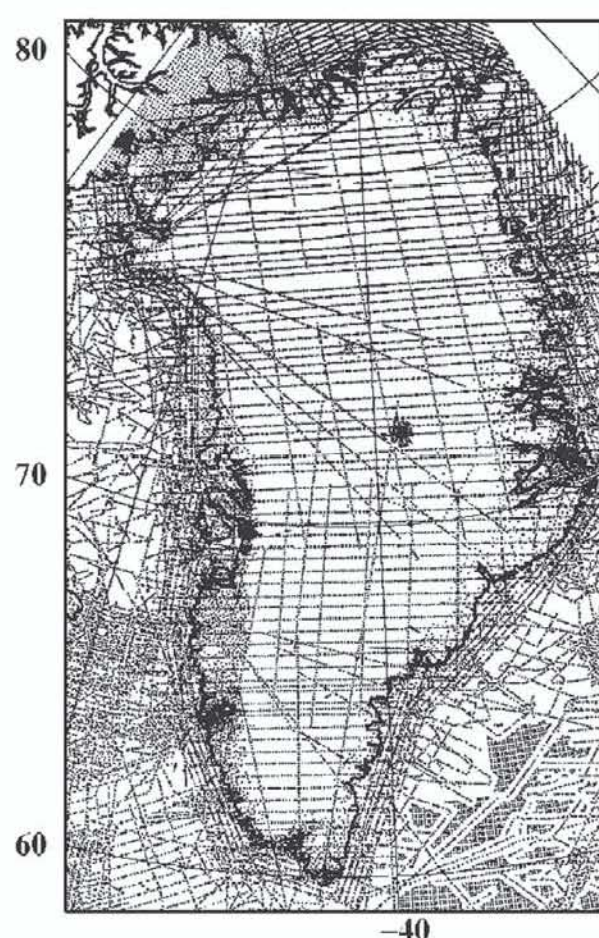


Fig. 1 : Gravity data coverage of Greenland. The interior of Greenland was covered by NRL 1991-92 high-altitude aerogravity, supplemented by KMS helicopter work in the ice-free areas 1991-97 and low-level aerogravity on continental shelf regions 1998-2001, supplementing older marine data and satellite altimetry gravity.

Table 1 summarizes the number of surface and airborne data used in the downward continuation process (thinned to 2.5' x 5' in latitude and longitude), as well as statistics of the EGM and RTM reductions. Spherical harmonic effects in three dimensions obtained by "sandwich" grid interpolation between reference levels of 0 and 4 km (Greenland) or 0 and 2 km (Svalbard).

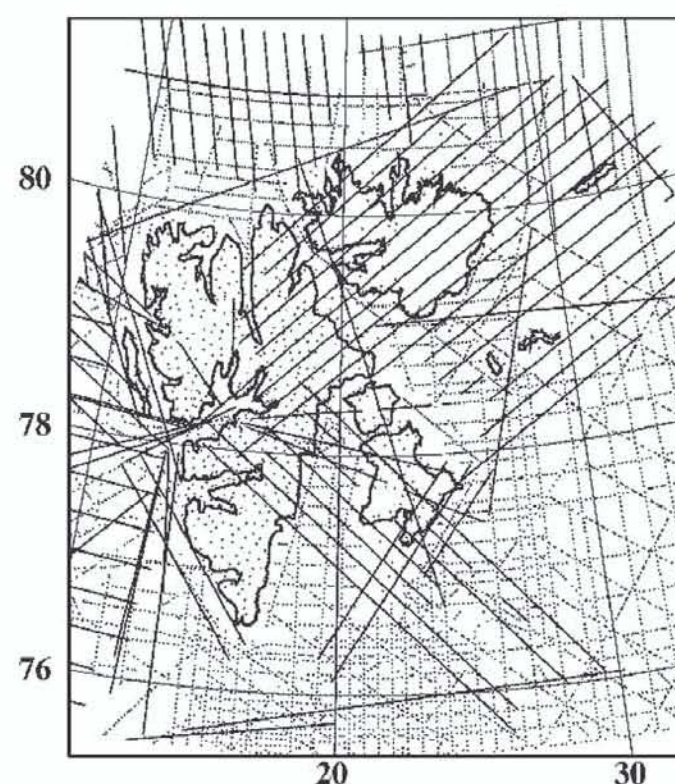


Fig. 2 : Gravity data coverage of Svalbard. SK/KMS 1998-2001 aerogravity lines supplement earlier land helicopter work and extensive marine gravity surveys south and east of the islands. Aerogravity lines north of Svalbard are part of NRL 1998 and 1999 Arctic Ocean airborne gravity data.

GREENLAND	Land data (13287 pts)		Marine data + low-level airb. (37514 pts)		NRL airborne gravimetry (32082 pts)	
	mean	σ	mean	σ	mean	σ
Original data	-4.1	46.6	16.1	35.6	16.3	38.0
- EGM96	-6.4	32.1	-1.1	19.2	0.0	13.7
- RTM	4.8	15.6	-1.0	15.9	-0.2	13.0

SVALBARD	Surface data (7925 pts)		Airborne data (13533 pts)	
	mean	σ	mean	σ
Original data	15.5	30.0	23.0	27.1
- EGM96	0.8	15.8	0.4	14.8
- RTM effects	-0.1	11.8	0.9	13.4

Table. 1 : Statistics (mean and standard deviation σ) of surface and airborne gravity data, reduced by EGM and RTM.

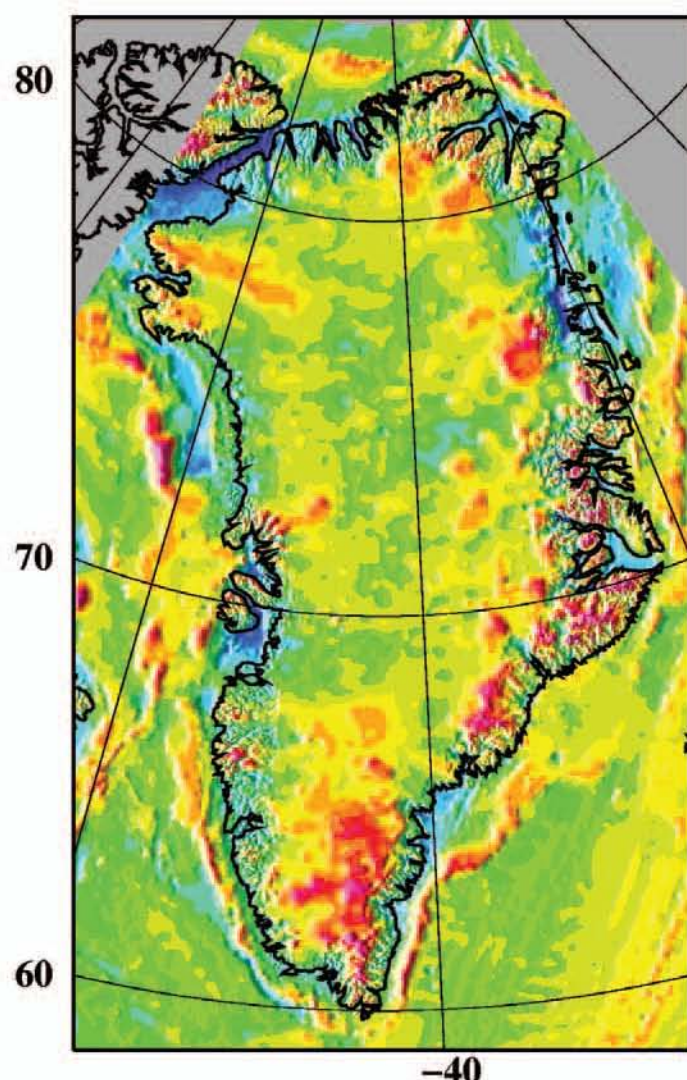


Fig. 3 : Free-air gravity field of Greenland from collocation downward continued airborne and surface data. Range of colour/grey scale from -150 to +150 mgal.

The reduced surface and airborne gravity data were subsequently combined by least-squares collocation using the covariance model (5). Standard deviations were assigned to data, and empirical D and T covariance parameters fitted to data (using constant values $D = 11$, $T = 41$ km in Greenland; and $D = 7$, $T = 26$ km in Svalbard). Due to the size of the computations, these were done in 3° blocks expanded with 1° overlaps to adjacent blocks (latitude spacing; longitude block size varied from 6° to 17° due to meridian convergence). The typical number of gravity observation points per block was in the range of 2500-4000.

A basic $2.5' \times 5'$ reduced free-air grid was predicted both at the geoid level and at the surface of the topography in each block, and all blocks subsequently merged into a combined uniform grid. The whole collocation job setup ran for nearly a week on a SUN Sparc 10 workstation. The final free-air grids were subsequently obtained by restoring the RTM terrain effects and the EGM96 gravity effects.

Fig. 3 shows the "optimal" free-air gravity grid thus produced for Greenland, and subsequently used for the ArcGP grid.

V. Comparison of collocation and FFT downward continuation in Greenland

The Greenland downward continuation have additionally been done by the outlined Fourier domain methods (3), based on the NRL airborne gravity data alone. This grid (the FFT "A" solution) has then been merged with existing surface data in a gridding process utilizing the NRL airborne data only in the interior of the Greenland ice sheet (FFT "B" solution). The two FFT grids thus corresponds in the ice-free parts of Greenland to either "full weight" to the NRL airborne data, or to a "surface-priority" weighting scheme. The main collocation solution by default mixes the two data sets, taking into account the different resolutions and accuracy, and should thus corresponds to some kind of mix of the FFT methods.

Due to the very rough topography in many regions along the Greenland coasts, very large discrepancies between the FFT grids and the collocation grids are evident, illustrating to a large degree the aliasing effects of topography, especially related to fjords and glaciers, and likely only to a lesser degree systematic errors in the airborne gravity data. To quantify the differences, Table 2 shows the statistics of comparisons of the downward continued gravity grids to available point data in some typical regions with good surface data coverage: the centre of the ice sheet (flat, smooth gravity field), a region on the west coast of Greenland (moderately mountainous, with few glaciers and fjords, and a central east Greenland area (very rugged topography with deep fjords and numerous glaciers).

Table 2 shows that the downward continued airborne gravity data alone (FFT "A") compare quite poorly to the point values, a natural consequence of the lack of resolution of the airborne data. However, the comparisons in the more "benign" areas on the centre of the ice sheet and in western Greenland are roughly consistent with the error estimate of the NRL airborne data aloft at 5 mgal r.m.s. (the downward continuation adding additional noise). The comparisons of the collocation and FFT "B" solutions only shows the internal consistency (and illustrates the very rough gravity field in East Greenland), since the surface data used for comparisons where also used for the computations.

Area	Lat	Lon	No of points	Surface gravity data		Fit to collocation grid		Fit to airborne (FFT "A")		Fit to combined (FFT "B")	
				mean	σ	mean	σ	mean	σ	mean	σ
Icesheet	72-76 N	45-35 W	221	-3.8	6.8	0.0	1.4	-2.6	7.7	-0.8	3.1
West coast	66-68 N	54 50 W	407	4.1	15.1	-0.0	1.6	-0.3	9.4	0.0	1.8
East coast	70-74 N	29-22 W	1105	4.6	22.9	-1.5	5.0	-7.3	21.6	-1.6	5.5

Table. 2 : Comparisons of downward continued gravity grids to point data (RTM- and EGM-reduced data). Unit: mgal.

A more independent comparison of the downward continued data gravity data is therefore to look on geoid information. Because of the large discrepancies between the collocation and FFT gravity grids, similar large discrepancies should be expected in the geoid domain. The various gravity grids were therefore converted into corresponding geoid models, using the multi-band spherical FFT method with a Wong-Gore modified Stokes kernel, cf. Forsberg and Featherstone (1998). In the method 5 bands were used in 800×1024 point FFT transformations, corresponding to use of 20 % zero padding. The optimal kernel modification degree turned out to be around $n=18$, implying that the EGM96 model is improved by the new gravity data down to about this harmonic degree.

The independent geoid control data were GPS stations along the Greenland coasts with a reasonably accurate GPS height and mean sea level height. Since no levelling exist in Greenland, the GPS-MSL heights are taken as a rough approximation of the geoid, and the noise in this estimate is likely many 10's of cm due to sea-surface topography. The GPS-MSL data covered most of the coast, including 19 observations in the fjord region of central of East Greenland (Fig. 4, same area as in Table 2), where especially large geoid discrepancies could be expected.

Table 3 outlines the geoid comparison results. As it can be seen there are large differences, and especially the downward continued NRL airborne data alone are not doing well in recovering the geoid, whereas the combinations with surface data are doing better. This could point to some long-range bias problems still present in the NRL data set. An example of the geoid differences in East Greenland is shown in Figure 5.

Solution	All Greenland (64 GPS pts)		East Greenland (19 points)	
	mean	σ	Mean	σ
GPS-MSL data	35.75	13.74	52.13	3.21
EGM96 difference	.07	.64	.38	.63
Collocation geoid diff.	-.21	.45	-.11	.26
FFT "A" geoid diff (NRL airborne only)	-.02	.76	.17	.80
FFT "B" geoid diff (airborne + surf. data)	-.26	.40	-.20	.25

Table 3. Comparisons of GPS-MSL geoid heights to geoids from gravity downward continuation in Greenland. Unit: m

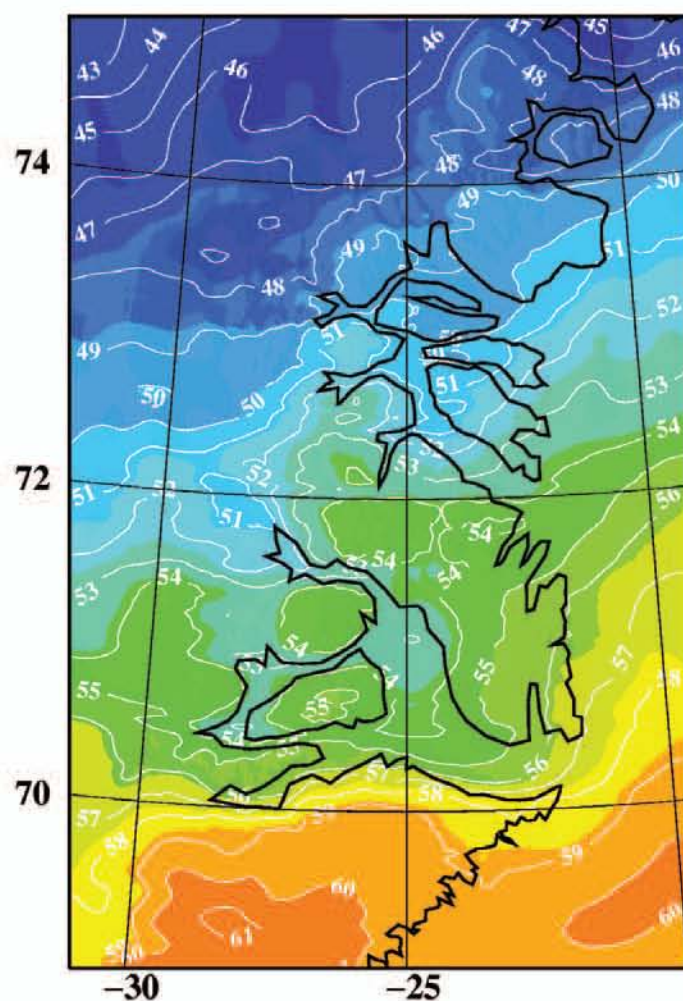


Fig. 4 : Geoid heights from downward continued airborne gravity data in East Greenland. The geoid shows a strong local dependence on topography (heights in region up to 3700 m). Contour interval 1 m.

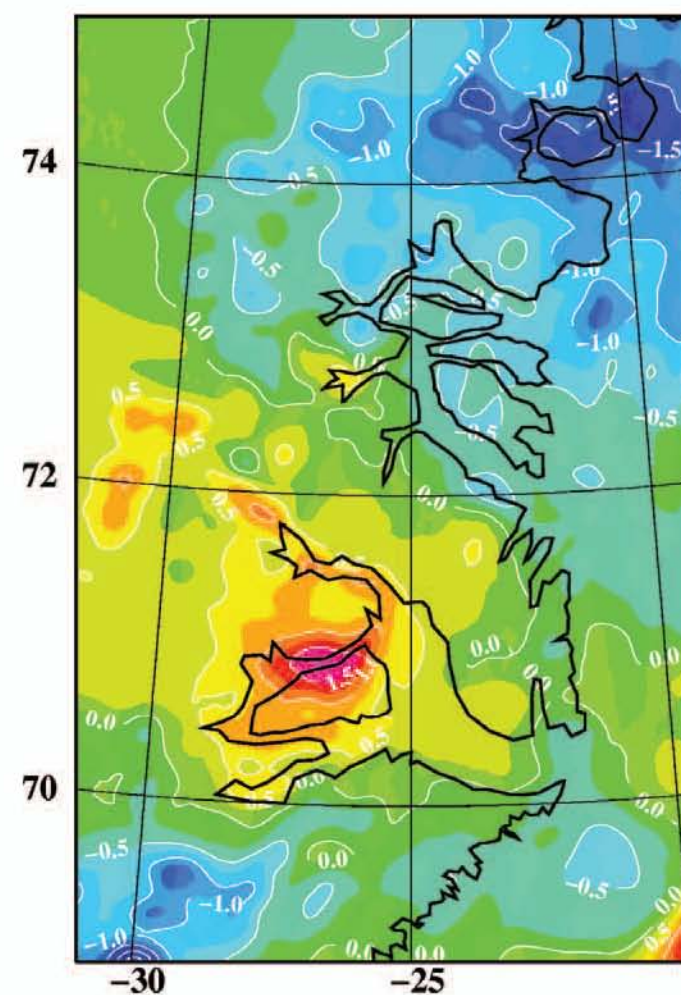


Fig. 5 : Geoid differences between collocation and "FFT A" downward continued fields. Contour interval 0.5 m. Differences both correlate with deep fjords and likely errors in airborne gravity tracks.

VI. Conclusions

The basic principles of operational downward continuation have been outlined, and the method of choice – least squares collocation – have been applied to merging of airborne gravity data in Greenland and Svalbard for the Arctic Gravity Project. Comparisons of methods and data in Greenland show large differences in the downward continued gravity grids, and the merging of high-altitude airborne gravity with surface data, with the joint utilization of digital terrain models, seems to be essential for obtaining reasonable geoid models in the very rugged topography regions.

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