



THE PROVISIONING RISK MEASUREMENT IN AUTOMOBILE INSURANCE USING VALUE AT RISK APPROACH

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Summary

This article focuses on the problem of quantifying uncertainty related to the provisioning risk in property and casualty insurance under the new "Solvency II" framework. In this context, the volatility of technical reserves must be included in their estimation by adding a risk margin. This measure can only be achieved by modeling future claims settlement flows. To this end, we apply two stochastic models to a portfolio of the Automobile branch of an Algerian insurance company "CAAT". These models can be used to estimate the outstanding reserves, while highlighting their capacity to determine the variability of the reserves, as well as their contribution to the calculation of the risk indicators imposed by the "Solvency II" reform, such as the Value at Risk (VaR).

Key words: Provisioning risk, risk measurement, Bootstrap, Mack, Value-at-Risk.

JEL Classification: C13, C15, C53, G22

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1 INTRODUCTION

In property and casualty insurance, and especially in Automobile insurance, the outstanding reserves remains the predominant of all technical reserves, it represents "the estimated value of the expenses necessary to settle all claims that have occurred not yet paid at the date of the inventory" (Nicolas, 1980). The estimation of liabilities to policyholders is one of the main sources of provisioning risk for non-life insurance companies, in particular their outstanding provisions. This uncertainty is mainly due to the fact that at the time of the valuation of the claims reserves, future payments for incurred claims are not known. So, a poor estimate of the latter will have a heavy impact on insurance companies.

With the development of the regulatory system at the global level, it should be noted that the "Solvency I" regulation requires insurers to value their liabilities as precisely as possible, through the prudent and continuous constitution of technical provisions.

The new European "Solvency II" standard imposes new regulatory requirements from a quantitative and qualitative point of view on insurers to enable them to estimate this provision as accurately as possible by integrating the notion of risk.

The basic principle is to measure the risk and determine the amount of provisions by reference to a previously determined level of risk. This requires the ability to model the ultimate loss and identify the risk. In the context of Solvency II, the volume of reserves is set by reference to a risk measure. For example, we will choose the provision that gives rise to a full payment of claims in 75% of cases (VaR at 75%).

For this reason, insurers who previously only used to estimate outstanding reserves using deterministic methods are now putting more emphasis on the precise quantification of prediction errors by stochastic models in order to select the most appropriate amount of provisions by the method best suited to their data.

Within this framework, we refer to the seminal contributions of (Mack & Munich, 1993), and (England & Verall, 1999).

Building on these ideas, the main objective of this article is to answer the following question:

"How can we measure the risk that lies in estimating the amount of outstanding reserves in order to have an adequate amount?"

The rest of this document is structured as follows: the second and third sections are devoted respectively to a review of the theoretical and empirical literature on non-life insurance provisioning. The third section outlines the methodology adopted. And finally, a last one dealing with the results obtained by the methods used and we will thus conclude this paper with a synthesis of the key findings.

2 THE PROVISIONING RISK MEASUREMENT: A LITERATURE REVIEW

Provisioning risk is defined as a poor assessment of the Best Estimate (or the average level) of the claims payable and its volatility (Riad et al., 2020).

The literature presents a variety of methods from which the actuaries can select for funding purposes, as highlighted by the (Faculty and institute of actuaries, 1997).

With the introduction of the "Solvency II" Directive, a fundamental change has taken place in the valuation of provisions for claims payable in the insurance sector. In this context, the literature mainly distinguishes between two types of provisioning methods (Riad et al., 2020): **Deterministic methods** and **stochastic methods**.

The provisioning methods used in practice are often deterministic. Claims reserves are often obtained depending on the case of the individual claims estimate by the claims handlers. The popular statistical method is the Chain Ladder method, which has been of deterministic origin (G. Taylor, 2000).

According to (Mc Clenahan, 2003), the concept of a "reasonable estimate" wasn't widely used in the process of evaluating claims reserves until the 1970s. The concept's development started with (Anker, 1973) and (Skurnick, 1973). The idea of creating an interval for estimated provisions discussed in this context. This can be carried out statistically or subjectively, with the latter employing stochastic methods.(Anker, 1973)

Consequently, deterministic models have been widely applied in practice. The Chain Ladder method, extensively used in actuarial literature, is employed for evaluating claims reserves. The theoretical understanding of the model's statistical foundation is significantly improved by the comprehensive documentation on the chain ladder method, including works by (G. C. Taylor & Ashe, 1983), (Mack, 1993)and (England & Verall, 1999).

In parallel to the efforts and research of (Skurnick, 1973), (Bornhutter & Ferguson, 1972) published a foundational study on the subject, establishing the deterministic Bornhuetter-Ferguson method, which is also frequently employed in practice. Historical experience is used to calculate a presumed loss ratio in this model.

Previous studies were crucial in establishing a comprehensive research trajectory on the estimate of claims reserves. The literature initially concentrated on deterministic methods like the chain ladder. Nonetheless, it was transitioning to stochastic methods due to later technological breakthroughs, which facilitated the implementation of more intricate and varied simulations.

Furthermore, various stochastic models have also been developed over the past few years to estimate the probability distributions related to the amount of the technical reserves, since they may obtain a variety of information that deterministic models cannot provide. (England & Verall, 1999)

Regardless its apparent simplicity, the chain ladder method relies on hypotheses that may quickly collapse when applied to the dataset under consideration. This approach is unsuitable when the data are unstable or highly sensitive to external influences, like inflation (Lemaire, 1985). In addition, it produces only a single point estimate of the outstanding reserves (Chase, 2015). Acknowledgment of these constraints has led the actuarial profession to pursue the development of stochastic models.

In this context, there are two main groups of stochastic methods: parametric and non-parametric. Parametric methods provide estimator sets whose primary assumption is that the input data adhere to a predetermined distribution. Since parametric methods are outside the scope of this study, they will not be discussed in this paper. In contrast, non-parametric methods start with the premise that the data does not a priori follow any particular probability distribution (Berrar, 2019).

The bootstrap, a non-parametric stochastic method, is a resampling technique that creates an empirical probability distribution for evaluating the precision of estimates and statistical conclusions regarding a dataset (Berrar, 2019). This methodology, founded on Pearson residuals, was introduced by (England & Verall , 1999). This study will employ the non-parametric bootstrap approach to assess the variations of claims reserves and to construct a confidence interval for outstanding reserves, using data from an insurer. The method will be detailed analytically in the subsequent section.

In terms of provisioning risk measures, it has been shown that stochastic models lead to a sharp increase in provisions, especially when VaR is considered as a risk measure, and that for a high $1-\tau$ quantile, the latter is all the more important.(Khordj et al., 2017).

The literature on the estimation of technical provisions in Algeria, though relatively recent, is growing steadily with several notable research contributions exploring critical aspects of this field. Among these, (Chouchaoui & Saidi, 2018) critically examined a common assumption in non-life insurance provision estimation—that of independence between insurance branches. They argued that this assumption can lead to underpricing and insufficient provisions, failing to cover the actual claim burden for a given occurrence year. Focusing on two sub-branches within Automobile insurance, they introduced Archimedean copulas, specifically the Joe copula, to model dependence between branches. Their findings demonstrated that acknowledging this dependence leads to better estimates of provisions for claims payable, thereby enhancing overall risk management in non-life insurance.

Building on the evolving regulatory landscape, (Assas & Saidi, 2019) tackled the computation of the Best Estimate of claims provisions under the European Solvency II framework. They employed both deterministic and stochastic methods while addressing the critical requirement of Article 77 regarding inflation integration. Their study highlighted that adjusting for inflation is vital for producing realistic valuations of technical provisions. However, they emphasized that correcting for inflation alone is insufficient. Proper discounting of future claim payment flows using a risk-free yield curve relevant at the analysis date is equally essential to capture the true economic value of provisions accurately.

In his article, (Wassila, 2017) highlights the importance of precise estimation of technical provisions in non-life insurance by combining classical deterministic methods such as Chain Ladder and London Chain with stochastic approaches, notably the Mack model and the Log-normal model. The latter proves particularly effective in modeling the asymmetric distribution and volatility of claims amounts to be paid. The study emphasizes that using the Log-normal model not only allows estimating the expected average provision but also evaluates the associated uncertainty, offering better risk management and enhanced compliance with regulatory requirements such as Solvency II. This stochastic approach thus emerges as a necessary evolution to improve the robustness of estimates in the Algerian context.

3 VALUATION METHODS AND MEASUREMENT OF PROVISIONING RISK: A BRIEF OVERVIEW

According to the literature, the Chain-Ladder method is the most classic method of calculating provisions. It will therefore serve as a reference throughout our study.

This method was proposed by (Astesan, 1938) which formalized the technique of development triangles, thus generating this very popular method. In summary, the principle of this method is as follows :

This method is characterized by the following two assumptions :(Charpentier & Denuit, 2005)

- **H1** : Independence between the years of occurrence ;
- **H2** : The years of development represent the explanatory variables of the future behavior of the claims.

For development factors, they are calculated using the following formula :

$$\hat{f}_j = \frac{\sum_{i=1}^{n-j} C_{i,j+1}}{\sum_{i=1}^{n-j} C_{i,j}} \text{ For } j = 1, \dots, n-1.$$

With the help of these factors, it will become possible to estimate future settlements. For we will have : $i + j > n$

$$\hat{C}_{i,j} = C_{i,n-i}(\hat{f}_{n-i} \times \dots \times \hat{f}_{j-1})$$

The ultimate loss is presented in the last column of the completed rectangle using the predefined factors. We write :

$$\hat{C}_{i,n} = C_{i,n-i}(\hat{f}_{n-i} \times \dots \times \hat{f}_{n-1}) \text{ For } i = 2, \dots, n.$$

The total outstanding provision to be constituted is therefore equal to the sum of the provisions calculated for each year of occurrence :

$$\hat{R}_i = C_{i,n} - C_{i,n+1-i} \text{ and } \hat{R} = \sum_{i=1}^n \hat{R}_i \quad (\hat{R}_i = C_{1,n} - C_{1,n-i} = 0)$$

✓ UTILITY OF THE TRIANGLE OF INDIVIDUAL FACTORS (D-TRIANGLE) :

The use of the D-triangle or also called the triangle of individual developmental factors makes it possible to verify the first hypothesis of independence of the years of occurrence. Its factors are calculated as follows:

$$f_{i,j} = \frac{C_{i,j+1}}{C_{i,j}} \text{ For } i + j \leq n - 1$$

If for j ranging from 1 to n-2, the individual factors for each jth column are substantially constant, then assumption (H1) is acceptable. If this is not the case, we will examine this triangle to identify atypical factors (e.g. seasonal effects) to remove the years of occurrence that have them from our calculation to have a certain stability.

3.1 MACK'S METHOD

In 1993, (Mack, 1993) introduced the first stochastic model that replicates Chain Ladder estimates. Indeed, the estimated amount of provisions is identical. However, this nonparametric model has the advantage of estimating an error in the prediction of provisions. It is also called the free chain-ladder distribution, because no distribution assumptions are made about the components of the triangle.

This model is based on three assumptions:

- **H1:** The independence by year of occurrence of the cumulative amounts of the triangle, i.e. and are independent for each $(C_{i,1})_{j=1,\dots,n} (C_{i,2})_{j=1,\dots,n} i_1 \neq i_2$.
- **H2:** The conditional expectation of is written: $C_{i,j+1} | C_{i,1}, \dots, C_{i,j}$

$$\mathbb{E}[C_{i,j+1} | C_{i,1}, \dots, C_{i,j}] = f_j \times C_{i,j}$$

- **H3:** The conditional variance of $C_{i,j+1} | C_{i,1}, \dots, C_{i,j}$ is related to the last observation, such that :

$$Var(C_{i,j+1} |) = C_{i,1}, \dots, C_{i,j} \hat{\sigma}_j^2 \times C_{i,j}$$

The same parameters estimated in the Chain Ladder method are used for the Mack model. This results in having exactly the same provisions constituted by this standard method.

In his reasoning, (Mack, 1994) also offers an unbiased variance σ_j^2 estimator that we will note here $\hat{\sigma}_j^2$:

$$\hat{\sigma}_j^2 = \frac{1}{n-j-1} \sum_{i=1}^{n-j} C_{i,j} \left(\frac{C_{i,j+1}}{C_{i,j}} - f_j \right)^2$$

$$\hat{\sigma}_{n-1}^2 = \min \left(\frac{\hat{\sigma}_{n-2}^4}{\hat{\sigma}_{n-3}^2}, \hat{\sigma}_{n-2}^2, \hat{\sigma}_{n-3}^2 \right)$$

$$\text{For: } i = 1, \dots, n \quad \text{and} \quad j = 1, \dots, n - 1.$$

$$mse(\hat{R}_i) = E \left[(\hat{R}_i - R_i)^2 | D \right] \text{ Where } D = \{C_{i,j} \setminus i + j < n + 1\}$$

This mean squared error is estimated by the following formula :

$$mse(\widehat{R}_i) = \hat{C}_{i,n}^2 \sum_{j=n-i+1}^{n-1} \frac{\hat{\sigma}_j^2}{\hat{f}_j^2} \left(\frac{1}{\hat{C}_{i,j}} + \frac{1}{\sum_{k=1}^{n-j} C_{k,j}} \right)$$

And for each variable $\widehat{R}_i$ the standard deviation is :

$$\widehat{se}(\widehat{R}_i) = \sqrt{mse(\widehat{R}_i)}$$

Similarly, it estimates the mean square error of the total reserves to be made by :

$$mse(\widehat{R}) = \sum_{i=2}^n \left(mse(\widehat{R}_i) + \hat{C}_{i,n} \left(\sum_{k=i+1}^n C_{k,n} \right) \times \sum_{j=n-i+1}^{n-1} \frac{2 \hat{\sigma}_j^2 / \hat{f}_j^2}{\sum_{k=1}^{n-j} C_{k,j}} \right)$$

This indicator allows us to see the error made to the reserve amount when estimating it. The main objective is to minimize this error, because the higher it is, the less adequate the amount of the provision is by this model

Mack's model is not based on any assumptions about the distribution of reserves. On the other hand, it is possible to formulate hypotheses to construct a confidence interval.

- **H1** : The r.v follows a normal distribution, we write : (R_i)

$$R_i \rightarrow N(\widehat{R}_i, se(\widehat{R}_i)^2)$$

The 95% confidence interval will then be :

$$[\widehat{R}_i - 1,96 \times se(\widehat{R}_i); \widehat{R}_i + 1,96 \times se(\widehat{R}_i)]$$

- **H2** : We assume that the provisions obey a log-normal distribution.

$$R_i \rightarrow \log N(\mu_i, \sigma_i^2)$$

With :

$$\begin{cases} \mu_i = \ln(\widehat{R}_i) - \frac{\sigma_i^2}{2} \\ \sigma_i^2 = \ln \left(1 + \left(\frac{se(\widehat{R}_i)}{\widehat{R}_i} \right)^2 \right) \end{cases}$$

So, at the 95% threshold, the new interval is written as follows:

$$[\exp(\mu_i - 1,96 \times \sigma_i); \exp(\mu_i + 1,96 \times \sigma_i)]$$

3.2 THE BOOTSTRAP METHOD

(Efron, 1979) in his research work entitled "*Bootstrap methods : Another look at the Jackknife*" introduced a resampling method known as Bootstrap. As its name suggests, it evokes the action of "hoisting oneself by using one's laces", where it is based on simulations of N new samples from an initial sample.

The different steps for building a "Bootstrap" method are as follows:

Step 1 : Determine $\hat{f}_j$ from the initial triangle of cumulative payments $\hat{C}_{i,j}$ for the estimation of the upper increments per recursion.

Step 2 : Calculation of Pearson residuals $\forall i + j \leq n$ $r_{i,j} = \frac{x_{i,j} - \bar{x}_{i,j}}{\sqrt{\bar{x}_{i,j}}} * \sqrt{\frac{\frac{n(n+1)}{2} - 2}{(\frac{n(n+1)}{2} - 2) - (2I - 1)}}$

End of common steps and start of simulation phase to get N amounts of the ultimate loss (next steps are performed for N simulations)

Step 3 : Resampling of residues $r_{i,j}$ and recovery of new residues $r_{i,j}^*$

Step 4 : Calculation of the new triangles of the increments: $\forall i + j \leq n \quad X_{i,j}^* = r_{i,j}^* \times \sqrt{\hat{X}_{i,j} + \hat{X}_{i,j}}$

Step 5 : Calculate individual reserves $\hat{R}_t^*$ and by applying the Chain Ladder method and replenishment N times.

The process thus results in a distribution of the ultimate loss from which the 99.5% VaR and the 99% TVaR will be calculated.

In addition to model recommendations, the literature favours Value-at-Risk (VaR) and Tail Value-at-Risk (TVaR) as risk measures proposed by the new Solvency II framework. The work of (Msiyah, 2016) showing that the projection of the distribution of provisions at the end of year $t + 1$ gives, for a 99.5% confidence interval, a capital requirement of a percentage of the initial amount of provisions applying the VaR.

✓ METHODOLOGICAL APPROACH

SAMPLING AND DATA COLLECTION:

The deterministic and stochastic methods mentioned in the previous part will be applied to real data of 11 years. Ranging from 2014 to 2021, from the technical assessments of the Automobile branch, granted by the Algerian Insurance Company (CAAT), they represent all the material settlements issued by the latter.

This data will be used in the form of a triangle called a liquidation triangle. With the help of XLSTAT software, we will summarize and process our raw data.

4 CASE STUDY: CASE OF AN ALGERIAN INSURANCE COMPANY "CAAT"

In this section, we will apply the different methods proposed in the previous section on real data, in order to see what advantage could be provided by the adoption of such methods by insurance companies when estimating their outstanding reserves.

LIQUIDATION TRIANGLE:

This development triangle represents all the payments made, plus management fees.

Table 1 : Non-cumulative settlement triangle

Unit : DZD

Année de survenance	Année de développement											
	année	1	2	3	4	5	6	7	8	9	10	11
	2 011	1 166 816 443	612 562 412	557 654 328	293 751 033	163 991 336	275 217 802	67 543 664	28 447 626	30 917 132	17 225 538	27 510 647
	2 012	1 116 501 000	1 943 547 373	713 811 448	378 690 074	425 934 365	144 070 420	59 656 490	47 408 051	24 079 477	20 111 154	
	2 013	2 600 654 771	2 217 427 376	751 990 520	371 926 855	534 653 681	160 721 847	60 890 958	2 764 490	49 621 901		
	2 014	2 923 474 970	2 217 075 708	741 295 615	374 619 907	573 026 943	123 802 704	52 566 294	57 213 688			
	2 015	2 777 993 454	2 063 415 626	629 949 778	308 891 553	576 862 356	85 858 873	58 829 091				
	2 016	2 543 030 005	1 703 419 931	577 248 109	313 358 160	162 903 014	134 856 982					
	2 017	2 572 144 474	1 573 720 960	517 019 188	267 499 563	214 678 083						
	2 018	2 799 245 371	1 638 841 414	467 460 431	298 144 076							
	2 019	3 090 506 433	1 640 070 353	592 938 659								
	2 020	2 727 512 603	1 398 654 367									
2 021	3 110 420 286											

Source : Developed by us using data from the CAAT

After adding up the present increments, we then apply the methods already described above.

4.1 THE CHAIN LADDER METHOD

Despite the simplicity of this method, it is important to first verify its assumptions:

$$(H_1) : \{C_{i,1}, \dots, C_{i,n}\}, \{C_{k,1}, \dots, C_{k,n}\} \quad i \neq j$$

The independence between the years of occurrence is verified by the D-triangle, it will only be acceptable if its individual factors are substantially constant for each year of development ($j=0, \dots, n-2$):

$$\frac{C_{1,j+1}}{C_{1,j}} = \frac{C_{2,j+1}}{C_{2,j}} = \dots = \frac{C_{n-j,j+1}}{C_{n-j,j}}$$

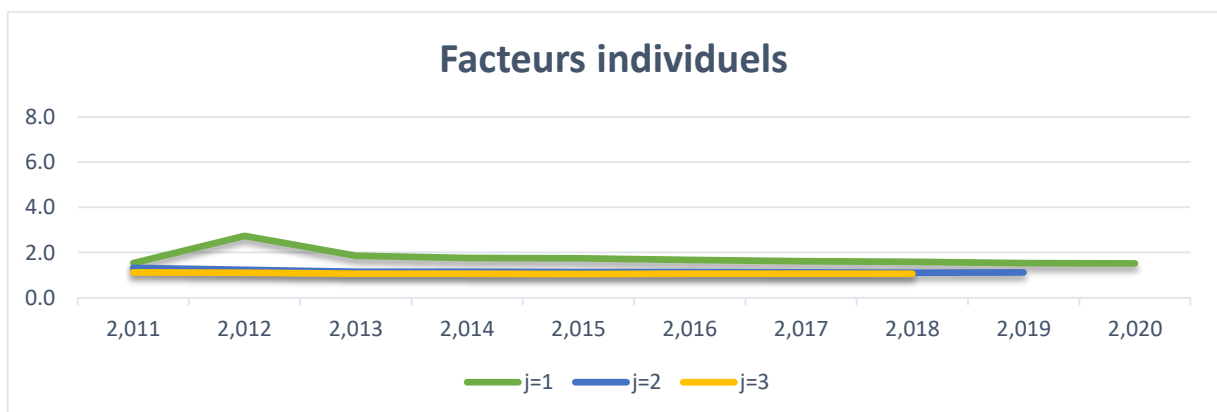
Table 2 : Individual developmental factors (the D-triangle)

Année de survenance	Année de développement										
	Année	1	2	3	4	5	6	7	8	9	10
Année de survenance	2 011	1,525	1,313	1,126	1,062	1,098	1,022	1,009	1,010	1,005	1,009
	2 012	2,741	1,233	1,100	1,103	1,031	1,013	1,010	1,005	1,004	
	2 013	1,853	1,156	1,067	1,090	1,025	1,009	1,000	1,007		
	2 014	1,758	1,144	1,064	1,092	1,018	1,008	1,008			
	2 015	1,743	1,130	1,056	1,100	1,014	1,009				
	2 016	1,670	1,136	1,065	1,032	1,025					
	2 017	1,612	1,125	1,057	1,044						
	2 018	1,585	1,105	1,061							
	2 019	1,531	1,125								
	2 020	1,513									

Source : Developed by us

We propose to graphically present the variations of the individual factors for the first j th columns, to verify this hypothesis.

Figure 1: Individual developmental factors for $j=1, 2$, and 3



Source : Developed by us

From this figure, we can see that the development factors can clearly be considered constant in relation to the years of occurrence.

The first assumption on which the Chain Ladder method is based is verified.

(H 2) : $E(C_{i,j+1} / C_{i,1}, \dots, C_{i,n}) = C_{i,j} \hat{f}_j$ for $i = 1, \dots, n$ et $j = 1, \dots, n - 1$

We can express this hypothesis as a linear regression model :

$$Y_i = C + X_i + \varepsilon_i$$

With :

- $Y_i = C_{i,j+1}$;
- $C = 0$;
- $X_i = C_{i,j}$;
- ε_i : is the error term such that $E(\varepsilon_i) = 0$.

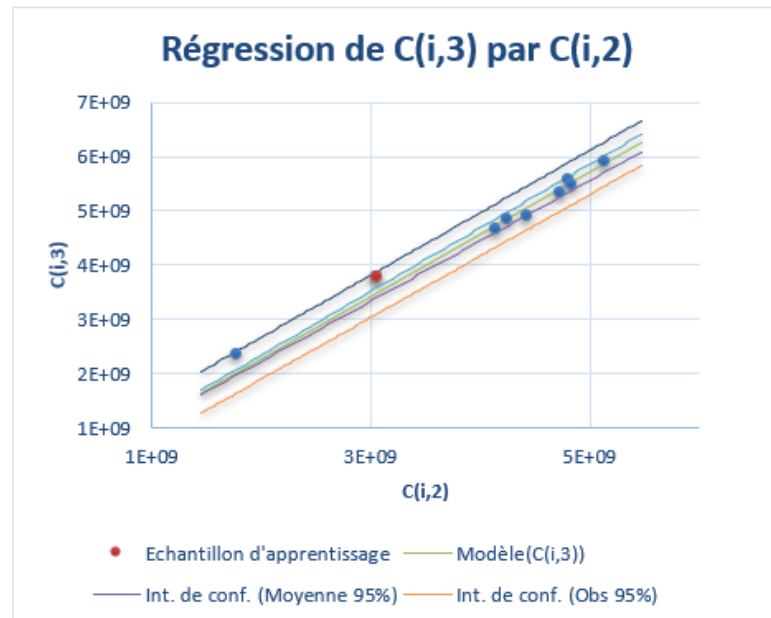
To verify this hypothesis, we draw a graph of $C_{i,j} \hat{f}_j$ as a function of $C_{i,j+1}$, and we look to see if the points are close to the line passing through the origin and slope.

Let's look at the case of $j = 2$ and $j = 3$, even if the number of data is not really enough to perform tests, the data are as follows :

$C_{i,2}, C_{i,3}$ et $\hat{f}_2 = 1,149$ Based on Table 7.

After using XLSTAT we get:

Figure 2 : $C_{i,3}$ Settlement by $C_{i,2}$



Source : Developed by ourselves.

Looking at this graph, we can see that the majority of the points ($C_{i,2}, C_{i,3}$) are aligned on a slope line $\hat{f}_2$ passing through the origin, so we consider that hypothesis (H2) is true.

Since both hypotheses are practically true, we can therefore validate the standard Chain Ladder method.

Result :

Table 3 : The Chain Ladder reserve

Years of occurrence	Ultimate Loss (Final Cost)	Final settlement	Provision for Claims
2011	3 241 637 961	3 241 637 961	-
2012	4 915 526 204	4 873 809 852	41 716 351
2013	6 840 009 017	6 750 652 399	89 356 618
2014	7 207 512 195	7 063 075 829	144 436 366
2015	6 676 436 609	6 501 800 730	174 635 879
2016	5 640 858 000	5 434 816 201	206 041 799
2017	5 492 797 820	5 145 062 268	347 735 552
2018	5 978 397 819	5 203 691 292	774 706 527
2019	6 542 067 374	5 323 515 444	1 218 551 930
2020	5 827 058 426	4 126 166 969	1 700 891 457
2021	7 464 930 736	3 110 420 286	4 354 510 450
Total	65 827 232 160	56 774 649 232	9 052 582 928

Source : Developed by us

The total provision is obtained either by subtracting the sum of the totals of settlements from the sum of the estimated ultimate losses or by summing the provisions to be made by year of occurrence. The total provision estimated by Chain Ladder is : 9,052,582,928 DA.

4.2 THE SEMI-PARAMETRIC MODEL : THE MACK MODEL

Before starting to determine its measurements, we first check its hypotheses. The first two were verified as part of the application of the "Chain Ladder" method.

We now recall the hypothesis (H3) concerning volatility, on which the Mack model is based:

- **H3 :** The conditional variance of $C_{i,j}$ is presented as follows :

$$Var(C_{i,j+1} | C_{i,1}, \dots, C_{i,j}) = \hat{\sigma}_j^2 \times C_{i,j}$$

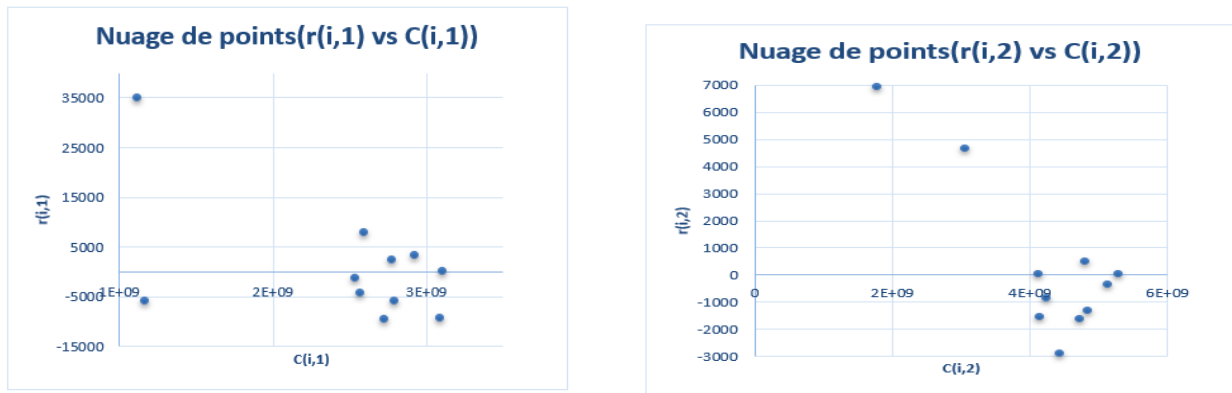
It is therefore possible to give a graphical interpretation to this hypothesis: to j give the points $r_{i,j}$ as a function of the $C_{i,j}$ where :

$$r_{i,j} = (C_{i,j+1} - C_{i,j}f_j) / \sqrt{C_{i,j}}$$

Corresponds to the residuals of a least-squares estimate, must be unstructured (random). (Charpentier & Denuit, 2005)

For $j=1, 2$, the following graphical representation of the points $(C_{i,j}, r_{i,j})$ allows us to have the appearance of the residues.

Figure 3 : Standardized residuals for j=1 and j=2



Source : Developed by us using XLSTAT software

For the first two values of j, the residuals show no trend. They are arranged in a disorderly manner around the x-axis. Their distribution seems totally random. Therefore, hypothesis (H3) is not rejected.

We note that all three hypotheses are validated. We can therefore a priori apply Mack's model to the data triangle.

4.2.1 RESULTS OF THE MACK MODEL

The following table summarizes the different parameters of risk, namely the mean squared error 'mse' and the standard error 'se'.

Table 4 : Risk parameters of the Mack model

Year of occurrence	$\hat{R}_i$	$\widehat{mse}(\hat{R}_i)$	$\widehat{se}(\hat{R}_i)$	$\widehat{se}(\hat{R}_i)$ en % from $\hat{R}_i$
2011	-	-	-	-
2012	41 716 351	4.89351E+12	2 212 127	5%
2013	89 356 618	4.71753E+13	6 868 427	8%
2014	144 436 366	2.92330E+14	17 097 652	12%
2015	174635879	1.25774E+15	35 464 665	20%
2016	206 041 799	1.77034E+15	42 075 435	20%
2017	347 735 552	2.22707E+16	149 233 705	43%
2018	774 706 527	5.43871E+16	233 210 473	30%
2019	1 218 551 930	8.03751E+16	283 504 948	23%
2020	1 700 891 457	1.38346E+17	371 948 533	22%
2021	4 354 510 450	1.44288E+18	1 201 200 731	28%
Total	9 052 582 928	1.74163E+18	2 342 816 696	26%

Source : Developed by us

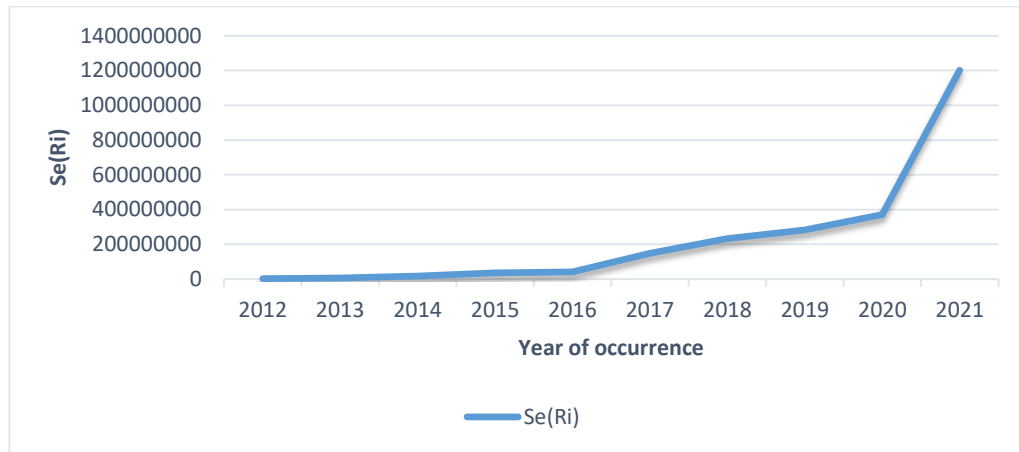
Looking at this table, the first remark that can be drawn is the high volatility of the amount of provisions. The latter is measured by the mean squared error of the prediction $mse(\hat{R}_t)$. It comes mainly from the last two financial years for which there is not much information.

As a second remark: the proportion of error in relation to the amount of provisions for each year is important, it represents 1/5 of the average estimated provisions, which reinforces the choice of stochastic models.

The standard error for all years amounts to 2,342,816,696, or 26% of the overall provision.

To better visualize the evolution of the standard error over all years, it is plotted in the following graph :

Figure 4 : Evolution of the standard error



Source : Drawn up by us on the basis of Table 4

We notice that the standard error is continuously increasing. For the last years of development, the curve increases exponentially. This is mainly due to the fact that the number of payments to be estimated to build up the reserve increases from year to year, the more recent the year, the higher the number of estimates and the greater the accumulated errors.

4.2.2 CONSTRUCTION OF A CONFIDENCE INTERVAL

Mack constructs prediction intervals for the provision under the assumptions about the choice of a conditional distribution for the "R" reserve. The parameters of the distribution will be the conditional mean and standard deviation of the reserve : $\hat{E}(R) = \hat{R}$ and $\widehat{se}(R)^2$.

Table 1: Construction of confidence intervals for the normal distribution and the log-normal distribution

			The normal distribution		The log-normal distribution	
			Confidence interval		Confidence interval	
Year	$\hat{\sigma}^2$	μ_i	Lower Bound	Upper Bound	Lower Bound	Upper Bound
2011	-	-	-	-	-	-
2012	0,0095	16,9289	18 284 613	26 922 434	18 592 452	27 221 233
2013	0,0118	17,9489	49 351 133	761 70 681	50 420 198	77 204 617
2014	0,0212	18,5621	83 030 612	149 792 805	86 600 930	153 205 092
2015	0,0549	18,7898	79 435 711	217 916 647	91 375 384	228 982 004
2016	0,0505	19,0058	101 961 497	266 255 892	115 549 510	278 893 065
2017	0,1885	19,5093	35 017 389	617 738 898	126 826 268	695 608 258
2018	0,0913	20,39	296145747	1 206 776 215	397 124 167	1 297 921 306
2019	0,0545	20,8726	639 605 581	1 746 623 803	734 694 903	1 834 778 450
2020	0,0476	21,2172	952 049 847	2 404 418 868	1 068 630 810	2 513 101 945
2021	0,0737	22,1509	1 980 285 780	6 670 683 984	2 448 409 129	7 098 446 437
Total			4 235 167 910	13 383 300 226	5 138 223 750	14 205 362 406

Source : Developed by us

The bounds of the 95% prediction intervals obtained by the log-normal distribution are higher than the normal distribution, which may cause the lower bounds of the normal distribution to be negative. Such a distribution is therefore not suitable. We will then prefer the log-normal distribution because it gives results closer to reality.

As we have seen, the standard error for the total amount of reserves is 26%, which means that the variance of the estimate ($\hat{E}\hat{R}_t$) represents 26% of the total estimated reserve.

We can then say that the uncertainty related to the amount of the provision calculated by the "Chain Ladder" method could be lower if it was not for the asymmetry or the lack of information related to the last few years, which led to an increase in the risk of error.

4.3 THE NON-PARAMETRIC METHOD : THE BOOTSTRAP

The Bootstrap mechanism for a single iteration, allows us to have the following results :

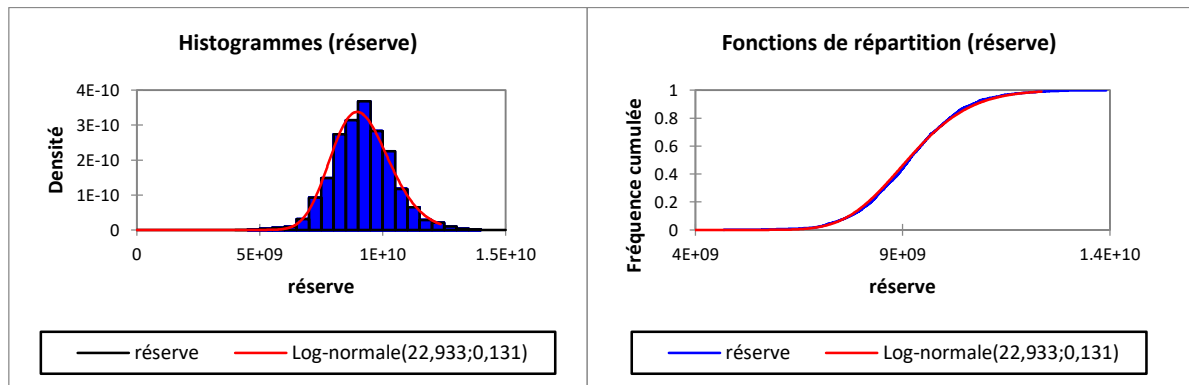
Table 6 : Provisions obtained by the Bootstrap

Years of occurrence	Ultimate Loss (Final Cost)	Final settlement	Provision for Claims
2011	3 340 294 637	3 340 294 637	0
2012	5 364 902 338	5 327 669 239	37 233 099
2013	8 017 210 999	7 996 132 938	21 078 061
2014	7 308 210 767	7 262 867 962	45 342 805
2015	7 456 958 314	7 356 128 908	100 829 406
2016	6 126 703 631	5 915 338 279	211 365 352
2017	5 173 665 568	4 871 704 755	301 960 812
2018	5 987 832 379	5 198 137 284	789 695 095
2019	7 000 794 260	5 578 609 180	1 422 185 080
2020	6 598 168 727	4 510 544 625	2 087 624 102
2021	6 832 185 127	2 792 586 727	4 039 598 400
Total	69 206 926 746	60 150 014 533	9 056 912 213

Source : Developed by us

To have more reliable results, we apply the Bootstrap mechanism by simulating 10,000 times by the XLSTAT software. We obtain 10,000 different amounts of the estimated total reserves, which allows us to deduce their distribution.

Figure 5 : Distribution and cumulative distribution function of reserves by Bootstrap



Source : Developed by us using XLSTAT software

This histogram gives us the shape of the distribution of the reserves from which we see that the empirical density of the latter is not symmetrical, which means that they do not follow the Normal distribution, nor the log-normal distribution by graphical fitting.

4.4 THE PROVISIONING RISK MEASURE 'THE RISK PARAMETER 'VAR'

- CASE OF MACK'S MODEL :**

For Mack's model, if we assume that the distribution adopted is Normal, the estimate of the quantile of order η of R is given by the following formula :

$$\hat{VaR}[R;\tau] = \hat{R} + \hat{se}(\hat{R}) \times q_{1-\tau}$$

With $\hat{R}_i$ and $\hat{se}(\hat{R}_i)$ the mean and standard deviation respectively, and $q_{1-\tau}$ the quantile of order $1 - \tau$ of the standard normal distribution.

- **CASE OF THE BOOTSTRAP :**

Given the asymptotic normality of the distribution of the amount of the reserve by the central limit theorem (TCL), the VaR is therefore equal to :

$$\hat{VaR}[R;\tau] = E^b(\hat{R}) + se_{boot}^B q_{1-\tau}$$

Where : $E^b(\hat{R})$, Se_{boot}^B and $q_{1-\tau}$ respectively : the mean, the standard deviation of the reserve, and the quantile of order $1 - \tau$ of the standard normal distribution.

The value at risk for each given confidence level, and for each stochastic model, is presented in the following table :

Table 7 : Value at Risk by Confidence Level

Level of confidence	75%	90%	95%	99%
Quantile $q_{1-\tau}$	0,7734	0,8159	0,8289	0,8382
VaR Mack	10 864 517 361	10 964 087 070	10 994 543 687	11 016 331 883
VaR Bootstrap	10 108 229 503	10 158 621 265	10 174 035 215	10 185 062 118

Source : Developed by us

After determining its VaR, which corresponds to the extreme loss it can incur, the insurance company on the entire balance sheet for a predefined probability. So the "CAAT" can then use this amount to set the capital necessary to hold in respect of the reserves of claims to be settled of the Automobile branch (material damage) so that it can manage its provisioning risk.

As we can see from the results presented in the table above, that for a larger η , the VaR is even higher, which reinforces the financial theory that for a higher risk the profitability is greater. Thus, in order to better prevent possible risks, the company must make more provisions for the Automobile branch.

5 CONCLUSION

The assessment of provisioning risk in Automobile insurance represents a major challenge within the framework of the "Solvency II" regulatory system. The uncertainty associated with the estimation of claims reserves, particularly those related to outstanding reserves, highlights the need for more robust approaches than traditional deterministic methods. While the latter, such as the Chain Ladder method, provide an initial approximation of technical provisions, they remain limited by their inability to capture the variability of estimates and the associated forecasting errors.

The introduction of the stochastic methods has enabled us to see that there is a risk of provisioning for the Automobile branch of the CAAT in terms of material damages. The standard deviations obtained by stochastic models constitute a first precautionary indicator against the risk of provisioning to be added to the assessment of the amount of claims to be paid to deal with a possible volatility of the claims experience.

The calculation of the VaR for each stochastic model and for different confidence thresholds allowed us to calculate the maximum loss that it can incur on its entire balance sheet, and this is a key risk management parameter in terms of non-life provisioning that the insurance company relies on.

By providing a probabilistic framework for extreme but plausible scenarios, the VaR allows insurers to better align their provisioning strategies with the solvency and capital adequacy requirements imposed by Solvency II.

6 BIBLIOGRAPHY

1. Anker, R. A. (1973). *Loss reserving methods*. 59–62.
2. Assas, H., & Saidi, G. (2019). Calcul du best estimate des provisions pour sinistres à payer sous solvabilité II en présence de l'inflation. *Revue Administration et Développement Pour Les Recherches et Les Études*, 08(01), 346–369.
3. Berrar, D. (2019). Introduction to the Non Parametric Bootstrap. *Encyclopédie de Bioinformatique et de Biologie Computationnelle*, 766–773.
4. Bornhutter, R., & Ferguson, R. (1972). The actuary and IBNR. *Actes de La Casualty Actuarial Society*, 59, 181–195.
5. Charpentier, A., & Denuit, M. (2005). *Mathématiques de l'assurance non-vie, Tome II: Tarification et provisionnement* (Economica).
6. Chase, T. R. (2015). *Analysis of bootstrap techniques for loss reserving*. Faculty of the North Dakota State University of Agriculture and Applied Science.
7. Chouchaoui, L., & Saidi, G. (2018). Estimation des provisions techniques en assurance automobile selon la théorie des copules. *Revue d'Economie et de La Statistique Appliquée*, 15, 90–104.
8. Efron, B. (1979). Bootstrap methods : Another look at the Jackknife. *The Annals of Statistics*, 7(1), 1–26.
9. England, P., & Verall, R. (1999). Analytic and bootstrap estimates of prediction errors in claims reserving. *Insurance : Mathematics Ans Economics*, 25(3), 281–293.
10. Faculty and institute of actuaries. (1997). *Claims Reserving manual* (Vol. 1).
11. Khordj, M., Haffar, A., & Teulon, F. (2017). Provisionnement et mesure de risque en assurance dommage dans le cadre de Solvabilité II. *La Revue En Management et Prospective "ICHEC - GESTION2000,"* 34, 137–168.
12. Lemaire, J. (1985). Automobile Insurance : Actuarial Models. *ASTIN Bulletin : The Journal of the IAA*.
13. Mack, T. (1993). Distribution-free calculation of the standard error of chain ladder reserve estimates. *ASTIN Bulletin : The Journal of the IAA*, 23(2), 213–225.
14. Mack, T. (1994). Measuring the variability of chain-ladder reserve estimates. *Casualty Actuarial Society*, 1.
15. Mack, T., & Munich, R. (1993). Measuring the variability of Chain Ladder reserve estimates. *Casualty Actuarial Society*.
16. Mc Clenahan, C. L. (2003). Estimation and application of Ranges of Reasonable Estimates. *Proceeding of Casualty Actuarial Society Forum*, 213–230.
17. Msiyah, C. (2016). Modélisation stochastique de risque technique d'assurance non vie : application a une ligne d'activité et mesure annuelle de risque. *Assurances et Gestion Des Risques*, 83.
18. Nicolas, J. (1980). *Les assurances* (2ème édition Dalloz).
19. Riad, M., Akhenak, A., & Djellouli, B. (2020). Analyse de la solvabilité en assurance dommages dans le cadre référentiel "Solvency II" : Méthodes de provisionnement. *The Future Journal of In-Depth Economic Studies*, 03(01).
20. Skurnick, D. (1973). A survey of loss reserving methods. *Proceedings of the Casualty Actuarial Society*, 16–58.
21. Taylor, G. (2000). *Loss Reserving-An Actuarial Perspective* (Kluwer Academic press, Ed.).
22. Taylor, G. C., & Ashe, F. R. (1983). Second moment of estimates of outstanding claims. *Journal of Econometrics*, 23(1), 37–61.

23. Wassila, B. (2017). Estimation des provisions techniques par les méthodes déterministes et les modèles stochastiques cas empirique du modèle log normal. *Abdelouahab, Latreche* , 14.