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Modeling and Simulation of Nonlinear Behavior of Planar Structures, of Reinforced Concrete and Fibers Concrete by the Finite Element Beams

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Abstract – *A model is formulated while being based on the theory of Navier _ Bernoulli for the cross-sections in the plans reinforced concrete and fibre reinforced concrete structures. This modelling is carried out using beams finite elements takes into account the nonlinearity had: with the materials concrete and steel, with the interaction steel concrete, the problem of cracking and also with the influence of fibres on the behaviour. A data-processing program «frame_2d» is elaborate in FORTRAN 90, which will allow the digital simulation. Lastly, several examples extracted the literature were tested by using the program «frame_2d ». The comparison of the results obtained with theoretical and experimental results, is very satisfactory.*

Keywords: *nonlinear Modeling, finite element beam, reinforced concrete, fiber concrete, cracking*

I. Introduction

In civil and industrial construction, structural concrete elements come in various forms: beams, columns, slabs and walls. In this field, the main concern of design engineers is with the application of adequate techniques to ensure the durability and the smooth operation of built works. To achieve this, it is indispensable to them to predict the actual behavior of the various components, and understand their responses under various modes of loading. This can be achieved by the use of numerical modeling, allowing the simulation of the real behavior of structures. This essential step in the structural analysis; will lead to the adoption of optimal models which can provide sufficient accuracy in predicting the actual behavior of a structure.

II. Theoretical modal

II.1. Hypotheses

In this study, we focus on a plane beam element oriented in accordance with the longitudinal axis x and the dimensions normal to the yz plane x are relatively small compared to the longitudinal dimension x (Figure 1). Use is made of the following assumptions ([1] , [2] , [3]) :

A. The longitudinal axis of the beam is straight.
B. The cross section is symmetrical with respect to xz plane.
C. Loads acting on the beam are applied in the xz plane (Figure 2).
D. The beam deforms in the xz plane of symmetry (membrane, bending and shear).
E. The plane beam transmits normal efforts NR X (x) according to X, shear forces Tz (x) according to z and bending moments My (x) around the local axis y orthogonal in the xz. plan
F. The plane and straight sections before deformation remain plane and orthogonal to the neutral axis after deformation. G. Displacements and deformations of the second order are neglected (linear relationship between strains and displacements).

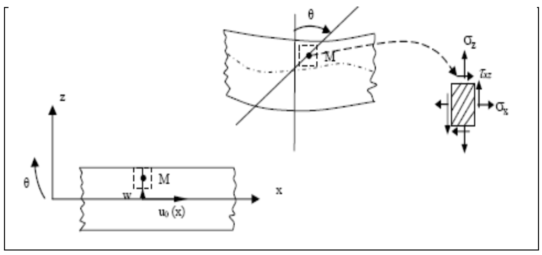

Figure 1: Definition of the kinematics of the beam in 3 functions of displacements u_0 (x), w(x) and θ (x).

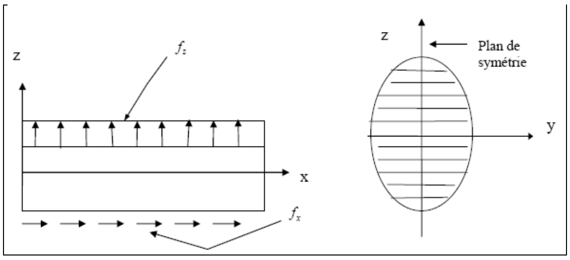

Figure 2: Representation of the loading acting in the xz plan and of the cross section of the beam.

II.2. Formulation

A beam section is considered before and after deformation (Figure 3) of the point M (x, z) in the not deformed configuration, after the deformation point M (x, z) is subjected to an axial displacement u (x, z) according to x and a transverse displacement w (x, z) according to z :

$$\left.\begin{array}{c} u(x,z) = u_0(x) + z \times \theta(x) \\ w(x,z) = w(x) \end{array}\right\} \dots\dots\dots(1)$$

$$\text{avec} \qquad \theta(x) = -\frac{dw}{dx} + \gamma \dots\dots\dots(2)$$

Deformations due to shear are neglected ($\gamma = 0$). So:

$$u(x,z) = u_0(x) - z\frac{dw(x)}{dx} \dots\dots\dots(3)$$

u_0 *(x):* Axial displacement at the reference axis of the beam.

θ(x), γ: Respectively, the normal rotation of the abscissa x cross section, and the rotation due to the transverse shear.

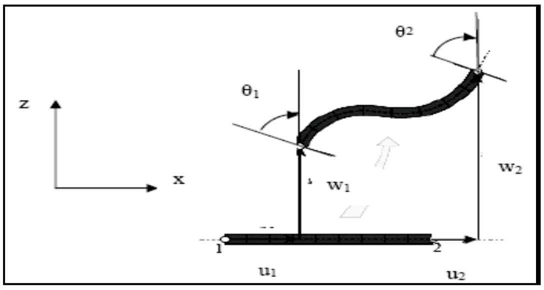

Figure 3: mapping the element before and after deformation

Consider the assumption of isotropic linear elastic behavior, with a plane stress state at point M (x, z). The stress-strain relation is written:

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xz} \end{Bmatrix} = \frac{E}{1-\nu^2}\begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix}\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xz} \end{Bmatrix} \dots\dots(4)$$

The strain-displacement relationship based on the assumptions is linear. Taking into account the relation (3) we obtain:

$$\varepsilon_x = \frac{du_0(x)}{dx} - z\frac{d^2w(x)}{dx^2} = \varepsilon_{0x} + z\phi \dots.(5)$$

$$\text{One posing} \quad : \begin{cases} \varepsilon_{0x} = \dfrac{du_0(x)}{dx} \dots\dots\dots(6') \\ \phi = \dfrac{d^2w(x)}{dx^2} \dots\dots\dots(6'') \end{cases}$$

Or ϕ the curvature and ε_{0x} is longitudinal deformation at the reference axis

Using the principle of virtual work, we arrive at the expression:

$$\int_0^l \left[\delta\widehat{\varepsilon}_x \int_x \sigma_x ds + \delta\widehat{\phi} \int_x \sigma_x z ds\right] dx - \int_0^l \delta\widehat{w} f_z dx - \int_0^l \delta\widehat{u} f_x dx = 0 \dots\dots\dots(7)$$

Expressions of the normal force and bending moments in the cross section of abscissa x are respectively written:

$$N = N(x) = \int_s \sigma_x ds = \varepsilon_{0x}\int_s E ds + \phi\int_s E z ds \dots(8)$$

$$M = \int_s \sigma_x z ds = \varepsilon_{0x}\int_s E z ds + \phi\int_s E z^2 ds ..(9)$$

$$\text{One posing :} \begin{cases} \overline{EA} = \int_s E ds \\ \overline{ES} = \int_s E.z.ds \\ \overline{EI} = \int_s E.z^2 ds \end{cases}$$

$\overline{EA}$ = stiffness to the normal force

$\overline{ES}$ = stiffness of the coupling normal force + bending.

$\overline{EI}$ = flexural stiffness.

The expression (7) can be rewritten as:

$$\int_0^l \langle \delta\hat{\varepsilon}, \delta\widehat{\phi} \rangle \begin{bmatrix} \overline{EA} & \overline{ES} \\ \overline{ES} & \overline{EI} \end{bmatrix}\begin{Bmatrix} \varepsilon_{0x} \\ \phi \end{Bmatrix} dx - \int_0^l \delta\widehat{w} f_z dx - \int_0^l \delta\widehat{u} f_x dx = 0 \qquad \forall\delta\widehat{\varepsilon_x}, \forall\delta\widehat{\phi} \dots\dots\dots(10)$$

II.3. Discrétisation

After discretization of the displacements, we obtain the following:

$$\int_0^l \langle \delta \widehat{u_n} \rangle [B]^t [D][B]\{u_n\} dx - \int_0^l \delta \widehat{w} f_z dx - \int_0^l \delta \hat{u} f_x dx = 0 \ldots\ldots\ldots\ldots(11)$$

The expression of the elementary stiffness matrix $[k]_e$ is written:

$$[k]_e = \int_0^l [B]^t.[D].[B] \ldots\ldots\ldots\ldots\ldots(12)$$

For the discretization of the cross section, one adopts the multi layers approach i.e. the subdivision of the total section in a finished number of the horizontal layers, to allow to evaluation of the state of stress – strain and to determine the matrix of rigidity and efforts in the cross-section and the element beam in its totality.

The quantities $\overline{EA}, \overline{ES}\ et\ \overline{EI}$ are evaluated by dividing the cross section into a certain number of trapezoids. Each trapezoid is subdivided in a certain number of horizontal layers of modules E_j, thickness h_j and width b_j, and which will be counted from bottom to top (see figure 4)

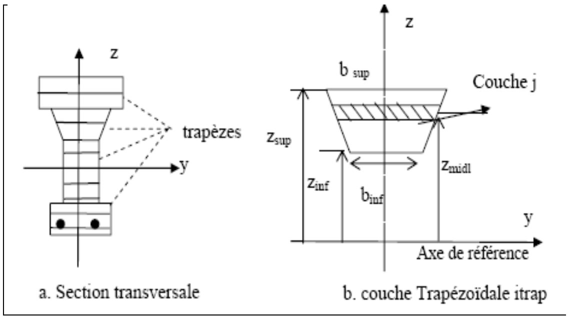

Figure 4: Discretization of the trapezoidal cross section layers

The residual forces are evaluated by:

$$\int_0^l [B]^t \begin{Bmatrix} N(x) \\ M(x) \end{Bmatrix} dx - \int_0^l [N] \begin{Bmatrix} f_x \\ f_z \end{Bmatrix} dx = 0$$

$$\text{Or simply } \{p\}^e - \{f\}^e = 0 \ldots\ldots\ldots\ldots\ldots(13)$$

With: $\{p\}^e$ Vector of nodal forces resulting from internal forces.

$\{f\}^e$: Vector of nodal forces resulting from efforts to spread applied the current element.

III. Nonlinear resolution

In the analysis of nonlinear behavior of the structure considered by the finite element method, we obtain a system of algebraic equations of the form:

$$\{F\} - [K(U)]\{U\} = \{\psi(U)\} \neq 0 \ldots\ldots\ldots\ldots(14)$$

With:

$[K(U)]$: Stiffness matrix of the structure-dependent vector $\{U\}$.

$\{F\}$: Nodal force vector applied to the structure.

$\psi(U)$: Vector of the residual forces or residues.

$\{U\}$: Vector of nodal displacements.

For the resolution of previous systems, it is contemplated an iterative calculation procedure which searches the solution $\{U\}$ making the residue $\psi(U)$ as close as possible to zero.

The external load $\{F\}$, is applied by successive increments $\{\Delta F\}$, in this study, an iterative method based on the incremental secant stiffness is adopted. It is best suited for the case of structures with a softening behavior [4], [5].

IV. Calculation organization chart

The algorithm of nonlinear resolution is thus presented as follows:

1. In the convergent step (j -1), corresponding to the loading $\{F\}^{j-1}$, the nodal displacement vector $\{U\}^{j-1}$ is known.
2. Incrementing the applied load:
   a. $\{F\}^j = \{F\}^{j-1} + \{\Delta F\}^j$
   b. Starting the iteration counter i=1
   c. Evaluation of the vector of the residual forces to balance at the current stage j :

$$\{\psi\}^i = \{\Delta F\}^j + \{\psi\}^{j-1}$$

3. Evaluation of elementary stiffness matrixes and then assembling the global stiffness matrix, depending on the solution $\{U\}^{i-1}$ of the previous iteration i-1.

$$[K]^i = [K(u^{i-1})]$$

* In the case of the first iteration (i=1), then: $\{u\}^{i-1} = \{u\}^{j-1}$ from the previous convergence step j-1.

4. Resolution of the system of equations :

$$[K]^i \{\Delta U\}^i = \{\psi\}^i$$

5. Accumulation vector of nodal displacements:

$$\{u\}^i = \{u\}^{j-1} + \{\Delta u\}^i$$

6. Calculate vector elementary nodal forces resulting from internal forces, at iteration i: $\{F_R\}_e^i$, and the overall assembly in vector $\{F_R\}^i$ resistant forces in the structure.

7. Evaluation of the vector of the Residual forces not balanced at iteration i :

$$\{\psi\}^{i+1} = \{F_R\}^j - \{F_R\}^i$$

8. Convergence test
   - If the current step j converges $\Rightarrow$ to pass the following load increments j+1 and returning to step 2.
   - Otherwise: move to the next iteration i+1 and return to step 3.

V. Used materials modelisation

To describe the behavior of concrete in compression, several approaches are used in literature [6], [7], [8], [9], [10], [11], [12].In the context of this study, the modal Sargin (Figure 5) which includes parabolic law and rectangle-parabola law.

As for to the behavior of concrete in tension, we consider the contribution of the concrete tended after cracking "effect of the tension stiffening". The modeling approach includes the fragile elastic law and the lenitive elastic law with parabolic linear decreasing branch

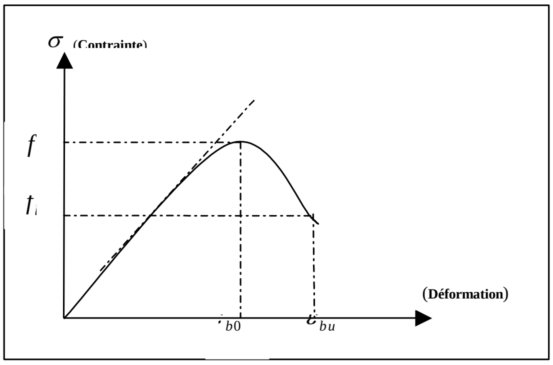

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Figure 5: Model of Sargin

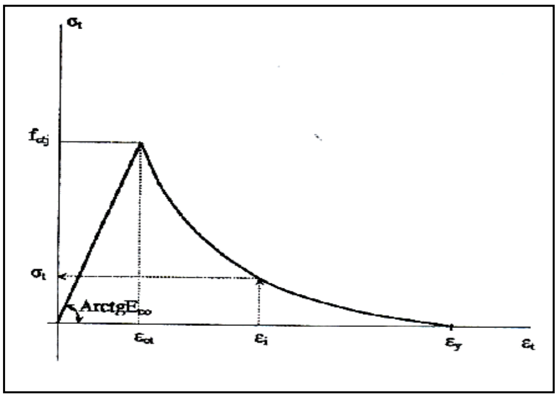

Figure 6: Model of Grelat

For modeling the behavior of reinforcement, a perfect elastoplastic model is used.

For modeling the behavior of fibers concrete in tension, model Kachi [13] (Figure 8) is adopted, the behavior of fibers concrete in compression is modeled by the law Sargin.

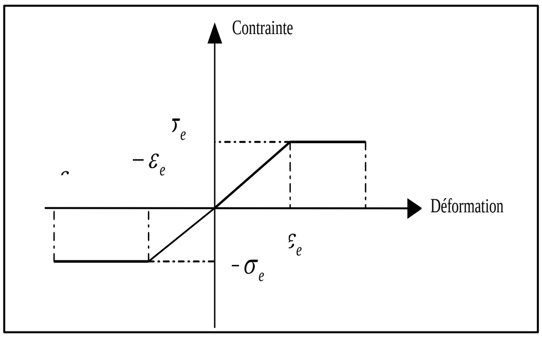

Figure 7: Law of behavior of reinforcement

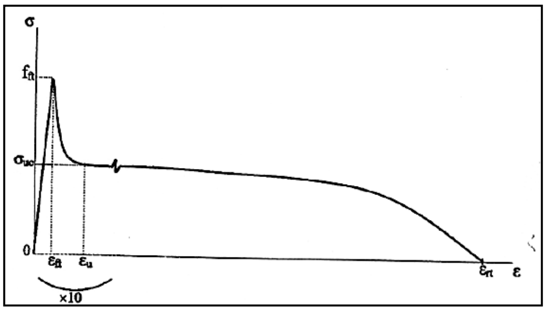

Figure 8: Behavior of the fibers concrete in tension [13]

VI. Application and validation

Based on modeling and calculation methods presented in this work, a computer program called “frame_2d”, written in FORTRAN 90 language, is developed. This program allows the numerical simulation of nonlinear static behavior until rupture of any planar structure of reinforced concrete or fibers concrete, consisting of elements of the beam type.

In order to check the aptitude and the reliability of the program developed, some examples of applications are presented. Several examples are treated. We retained an example of a concrete beam reinforced and an example with a metal gantry.

VI.1. Vecchio and Emara beam [14]

This is a two extremities built-in beam of reinforced concrete, has been the subjected of a theoretical study conducted by Vecchio and Amara [14]. The geometric data of the beam and reinforcement, as well as the numerical results are shown in the following figure 9:

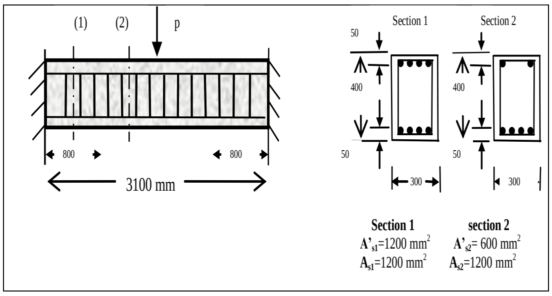

Figure 9: Geometrical data of the beam [14]

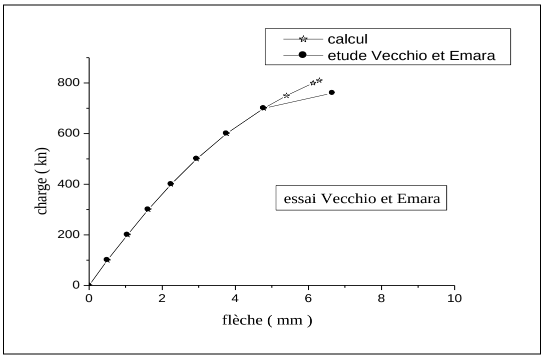

Figure 10: Evolution of the deflection according to the load

It is noted (figure 10) that the behavior of the beam in the study of Vecchio and Amara is well approached by calculation in this present study. Simulation shows a good estimate of the load of ruin and also of the corresponding maximum deflection. .

VI.2. Grimaldi and Rinaldi beam [15]

This beam was one of several beams tested by Grimaldi and Rinaldi [15], in order to determine the influence of the percentage of metal fibers in the concrete matrix on the overall behavior of the element (Figure 11).

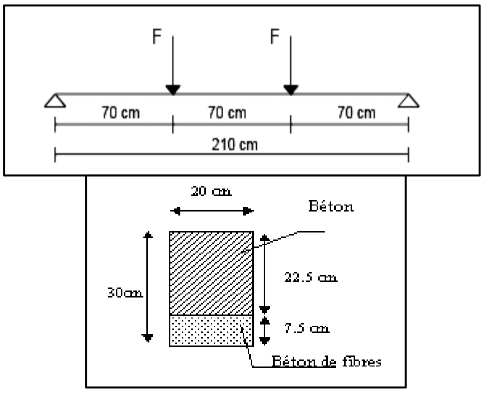

Figure 11: Grimaldi and Rinaldi beam [15]

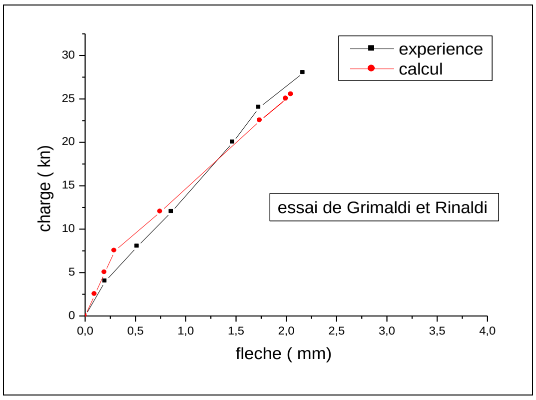

Figure 12: Evolution of the deflection according to the load

The calculation and experimental curves are plotted in Figure 12. For this test, the experimental behavior is well simulated until the approach of the load of ruin (difference $\leq$ 10%).

VI.3. JENNINGS and MADJID metalic gantry [16]

This is a built-in gantry, formed of square section profiling, and tested by JENNINGS and MADJID [16], it was modeled in 9 elements, taking into account, the considerations of symmetry.

The dimensions of the gantry and the loading system to which it is subjected and representative results are shown in Figure 13. Comparison calculation - test is given in Figure 14.

The breaking load is slightly overestimated; the difference is due to the effect of the second order which is not taken into account in calculation.

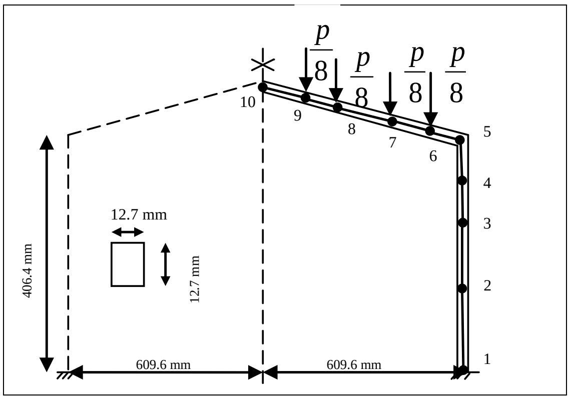

Figure 13: Characteristics of the gantry of JENNINGS and MADJID [16]

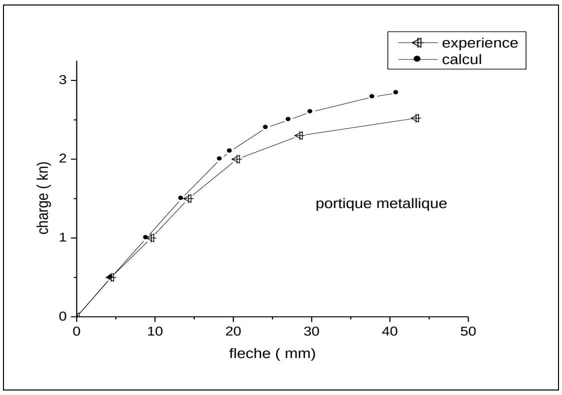

Figure 14: Evolution of the deflection according to the load

VII. Conclusion

This work consisted in the study and modeling of the nonlinear behavior of plane structures composed of elements, beam type, reinforced concrete or fibers concrete.

In this context, a model formulation of the “frame_2d”, based on the assumption of Navier-Bernoulli, was developed. Discretization by the finite element method was successful, then, the elaboration of a computational tool for modelling and simulating of nonlinear behavior until rupture, of plane structure consisting of beams and columns.

The comparison of calculation findings with experimental and theoretical results, carried out for several examples, helped to validate the computer program developed. The examples discussed show the ability of this program to simulate correctly the nonlinear behavior of reinforced concrete or fibers concrete structure and also structure planar metal frame.

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