Using the GARCH model to study the impact of bitcoin exchange rate volatility on the stability of the US dollar

استخدام نموذج GARCH لدراسة أثر تقلبات سعر صرف البيتكوين على استقرار الدولار الأمريكي

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Abstract:

This study aims to explore the impact of cryptocurrency price fluctuations on international exchange rates, focusing on bitcoin as the most widely traded and significant virtual currency in the market. Bitcoin was selected as the primary sample for analysis in this research. The researcher used the Augmented Dickey-Fuller (ADF) test and the Phillips-Perron (PP) test to verify the stationarity of the time series, in addition to applying conditional heteroskedasticity regression models, namely GARCH(1,1) and EGARCH(1,1), to analyse the volatility. Currency exchange rate data were used for the period from 1 January 2014 to 1 November 2024. After conducting the analysis, the study concluded that fluctuations in the exchange rate of bitcoin did not have a statistically significant impact on the stability of the US dollar during the period studied.

Keywords: Cryptocurrency, Bitcoin, GARCH Models, US Dollar.

JEL Classification Codes: C58,E42

ملخص:

تهدف هذه الدراسة إلى تحليل تأثير تقلبات أسعار العملات المشفّرة على أسعار الصرف الدولية، مع التركيز على عملة البيتكوين بوصفها العملة الافتراضية الأكثر تداولًا وأبرزها في الأسواق العالمية. وقد جرى اعتماد البيتكوين كعينة أساسية للتحليل. استخدم الباحث اختبار ديكي–فولر المطور (*ADF*) واختبار فيليبس–بيرون (*PP*) لفحص استقرارية السلاسل الزمنية، إلى جانب توظيف نماذج الانحدار ذات التباين الشرطي غير المتجانس، والمتمثلة في *GARCH(1,1)* و*EGARCH(1,1)*، بهدف قياس وتحليل التقلبات. واعتمدت الدراسة على بيانات أسعار الصرف خلال الفترة الممتدة من 1 جانفي 2014 إلى 1 نوفمبر 2024. وأظهرت نتائج التحليل أن تقلبات سعر صرف البيتكوين لم يكن لها تأثير ذو دلالة إحصائية على استقرار الدولار الأمريكي خلال فترة الدراسة.

كلمات مفتاحية: العملات المشفّرة، البيتكوين، نماذج GARCH، الدولار الأمريكي.

تصنيفات JEL : C58,E42

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Introduction:

Modern economics is characterized by the collection of various economic data and attempts to delve deeply into the economic environment to understand its variables and regulate its trends. Economic theory must be examined using specific methodologies to determine its ability to explain actual economic conditions. This can be achieved through econometric methods, which have evolved beyond merely showcasing different economic theories based on inference and logic. The primary task of economics is now to penetrate the economic environment to understand its variables and regulate its trends, which can be accomplished through econometric methods. These methods contribute to the development of new theories or the necessity of modifying existing ones.

Based on the above, the main research question of this study can be formulated as follows:

What is the impact of Bitcoin exchange rate fluctuations on the US dollar exchange rate?

In light of the general research question, the following hypothesis was formulated:

- Bitcoin exchange rate fluctuations do not have a significant impact on the stability of the US dollar.

1. The GARCH model:

Econometric models play an important role in economic studies because they are used in analytical forecasting and decision-making processes to design appropriate economic policies based on the needs of various economic sectors. Technological advances, particularly in applied statistics and computing, have facilitated the development of econometric modelling by simplifying complex computations such as estimating multiple models and performing complex statistical tests. Due to the limitations of linear models in forecasting, especially in the 1970s, alternative models such as autoregressive models with conditional heteroskedasticity were proposed.

1.1 Definition of the GARCH model:

This section focuses on the characteristics of certain economic and financial time series where the volatility or variance changes over time. Conditional heteroskedasticity models have been introduced to address this issue in financial econometrics. These models are relatively recent, with the main contribution coming from Engle (1982) in an article entitled "Autoregressive Conditional Heteroskedasticity for Estimating Inflation Variance in the United Kingdom" published in Econometrica in 1982. Engle proposed the ARCH model, an acronym for Autoregressive Conditional Heteroskedasticity (ARCH).

The ARCH (Autoregressive Conditional Heteroskedasticity) and GARCH (Generalised Autoregressive Conditional Heteroskedasticity) models serve as forecasting mechanisms for volatility modelling. These models are widely used in financial time series modelling because modern investors focus not only on studying and predicting the returns of financial assets in financial markets, but also on assessing risk and uncertainty. In order to analyse uncertainty, specialised models are required to treat asset price volatility as a function of time. These models belong to the family of autoregressive conditional heteroskedasticity (ARCH) models. (ALI , 2015, p. 10)

Time series analysis is an important statistical method used for modelling and forecasting Based on historical data series. One of the most prominent models in this area is ARIMA (Autoregressive Integrated Moving Average), which is used in various fields.

In order to use ARIMA models effectively, certain conditions relating to the stochastic error term of the model must be met.

$$i)E(\varepsilon t) = 0\ ii)E(\varepsilon t2) = 0\ iii)E(\varepsilon t\varepsilon s) = 0\ for\ s \neq t$$

In practice, the second and third conditions are difficult to achieve. Therefore, a model has been developed that takes into account the lack of these conditions and improves the fit of the model to the data. This model is the GARCH (Generalised Autoregressive Conditional Heteroscedasticity) model.

To address the problem of volatility in financial time series, the researcher Engle introduced the ARCH (Autoregressive Conditional Heteroscedasticity) model in 1982. The mathematical formulation of the ARCH model is as follows

$$rt = \mu + at$$
$$at = \sigma t \varepsilon t \; ; \; \varepsilon t \sim iidN\,(0,1)$$
$$\sigma t2 = \alpha 0 + \alpha 1 a2t - 1 + \alpha 2 a2t - 2 + \cdots + \alpha q a2t - q$$

Where:

- αt>0,∀i>0,α0>0 represent the model parameters.
- rt represents the return series.
- μ represents the mean of the return series. (Mohammed , 2009, p. 2)

1.2 Generalised autoregressive conditional heteroskedasticity (GARCH) models:

In 1986, Bollerslev published a paper in the Journal of Econometrics entitled "Generalised Autoregressive Conditional Heteroskedasticity" in which he introduced a generalised version of the GARCH model. Several advanced versions of this model have been developed and added over time. (ALI , 2015, p. 10)

The generalized GARCH model is formulated as follows: (Mohammed , 2009, pp. 2-3)

$$rt = \mu + at$$
$$at = \sigma t \varepsilon t \; ; \; \varepsilon t \sim iidN\,(0,1)$$
$$\sigma t2 = \alpha 0 + \alpha 1 a2t - 1 + \alpha 2 a2t - 2 + \cdots + \alpha q a2t - q + \beta 1 \sigma 2t - 1 + \cdots + \beta p \sigma 2t - p$$

The equation can also be rewritten as:

$$\sigma t2 = \alpha 0 + \sum \alpha i a2t - 1 \; q \; i{=}1 + \sum \beta j \sigma 2t - j \; p \; j{=}1$$

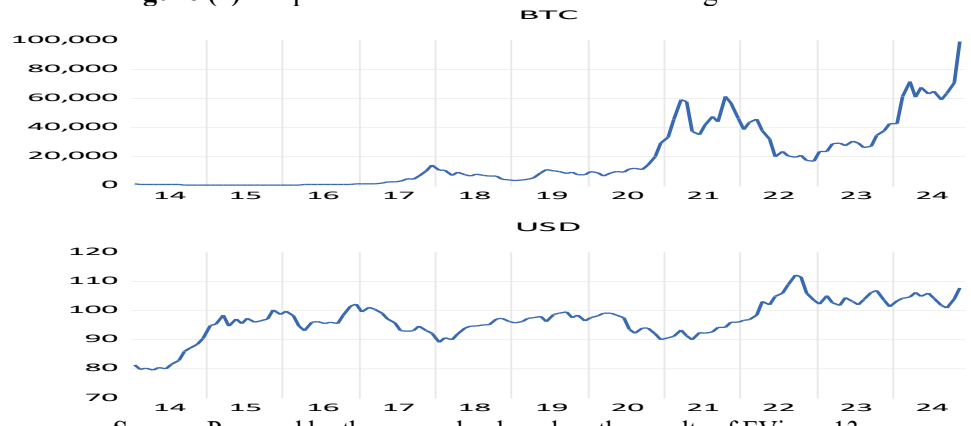
Where:

$$\sum \propto i \; q \; i{=}1 + \sum \beta j < 1 \; p \; j{=}1$$
$$\propto 0 > 0$$
$$\propto t \geq 0 \; i = 1,2,3, \ldots \ldots . , q$$
$$\beta t \geq 0 j = 1,2,3, \ldots \ldots . , p$$

2. Econometric modelling of bitcoin exchange rate volatility against foreign currencies:

2.1 Data: Daily data published on the Investing platform for bitcoin and US dollar exchange rates were used for the period from 01/01/2014 to 01/11/2024. Figure (1) shows a graphical representation of all the exchange rates studied. It can be seen that there is a significant similarity in the fluctuations of these exchange rates, which confirms the strong correlation between these currencies.

Figure (1) Graph of bitcoin and US dollar exchange rate series.



Source: Prepared by the researcher based on the results of EViews 13.

2.2 Descriptive statistics analysis of the time series under consideration

We want to determine whether the bitcoin and US dollar exchange rate series have the characteristics of a normal distribution. For this purpose, table (1) is presented as follows:

Table (1): Descriptive statistics of the bitcoin and US dollar exchange rate series.

	BTC	**USD**
Mean	18075.69	96.68244
Median	8629.900	96.70000
Maximum Value	98686.00	112.1000
Minimum Value	218.5000	79.50000
Standard Deviation	21405.57	6.456654
Skewness	1.305223	-0.502604
Kurtosis	3.944950	3.697835
Jarque-Bera	42.06932	8.173394
Probability	0.000000	0.016795

Source: Prepared by the researcher based on the results of EViews 13.

- **Standard Deviation (Std. Dev):** The table above shows that Bitcoin exchange rate fluctuations are riskier than those of the US dollar exchange rate.
- **Skewness:** The statistical distribution of the US dollar exchange rate is skewed to the left, while the Bitcoin exchange rate distribution is skewed to the right.
- **Kurtosis:** The statistical distribution of both Bitcoin and US dollar exchange rates exhibits excess kurtosis, exceeding the value of three, which corresponds to a normal distribution.
- **Jarque-Bera Statistic:** The Jarque-Bera test confirms that the exchange rate series does not follow a normal distribution, as all p-values are below 5%. This indicates that the exchange rate series for Bitcoin and the US dollar are non-stationary.

- **Economic Interpretation**

During 2024, Bitcoin recorded unprecedented increases, outperforming all traditional currencies. In January 2015, its price was less than $1,000, but it soared sharply, reaching over $98,000 in November 2024.

Some analysts attribute the massive demand for Bitcoin in late 2024 to several factors, including:

- The decline of traditional currencies.
- The slowdown of the global economy.
- Political instability worldwide, including the Russia-Ukraine war and the re-election of Donald Trump as US President.

2.3 Testing the stationarity of exchange rate series:

2.3.1 The stationarity of exchange rate series is tested using the following basic tests:

- **Augmented Dickey-Fuller (ADF) test:**

In 1981, Dickey and Fuller developed a test to analyse the nature and characteristics of time series, known as the Augmented Dickey-Fuller (ADF) test. It is one of the most efficient unit root tests and aims to correct for autocorrelation problems in the residuals by including a certain number of differences in the dependent variable.

The ADF test (1981) is used to detect unit roots and is based on the same principles as the DF test. However, it transforms an AR(1) model into an AR(p) model to eliminate autocorrelation in the error term, thereby improving its ability to detect stationarity and classify the type of non-stationarity (whether it follows a trend stationary (TS) or difference stationary (DS) process). (Dicky & Fuller , 1981)

- **Phillips-Perron (PP) test:**

Phillips developed a test to eliminate the effects of autocorrelation in the residuals of the unit root test equation by making parameter adjustments to the model to account for the dynamic nature of the time series.

In 1988, Phillips and Perron proposed a non-parametric approach to correct for autocorrelation, as opposed to the ADF test which uses a parametric method. (Phillips & Perron, juin 1988, pp. 335-336)

In order to test the stationarity of the time series under consideration, the tests mentioned above were applied, as shown in the following table:

Table (2): Stationarity tests for the exchange rate time series.

THE VARIABLE	**ADF**			**PP**		
	Without Constant & Trend	**With Constant & Trend**	**With Constant**	**Without Constant & Trend**	**With Constant & Trend**	**With Constant**
BTC	0.9866	0.9691	0.9974	0.9758	0.9361	0.9974
USD	0.9274	0.3182	0.2369	0.9205	0.2616	0.2050

Source: Prepared by the researcher based on the results of EViews 13.

Both the ADF and PP unit root tests show that the p-value is greater than the 5% significance level for all time series. Consequently, we cannot reject the null hypothesis, which statistically confirms the existence of unit roots and the non-stationarity of the time series of the variables under study.

This necessitates a further examination of the stationarity of the exchange rate time series at first differences.

2.3.2 Stationarity of exchange rate series at first differences:

Table (3): Stationarity of exchange rate time series at first differences.

THE VARIABLE	ADF			PP		
	Without Constant & Trend	With Constant & Trend	With Constant	Without Constant & Trend	With Constant & Trend	With Constant
BTC	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
USD	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Source: Prepared by the researcher based on the results of EViews 13.

The results of all tests presented in Table (3) indicate that the time series of the bitcoin and US dollar exchange rates are stationary according to the Augmented Dickey-Fuller (ADF) and Phillips-Perron (PP) tests, as the p-value is greater than the 5% significance level.

2.3.3 Stationarity test for the series of exchange rate returns:

The descriptive statistics of the examined series of exchange rate returns are analysed in the following table**:**

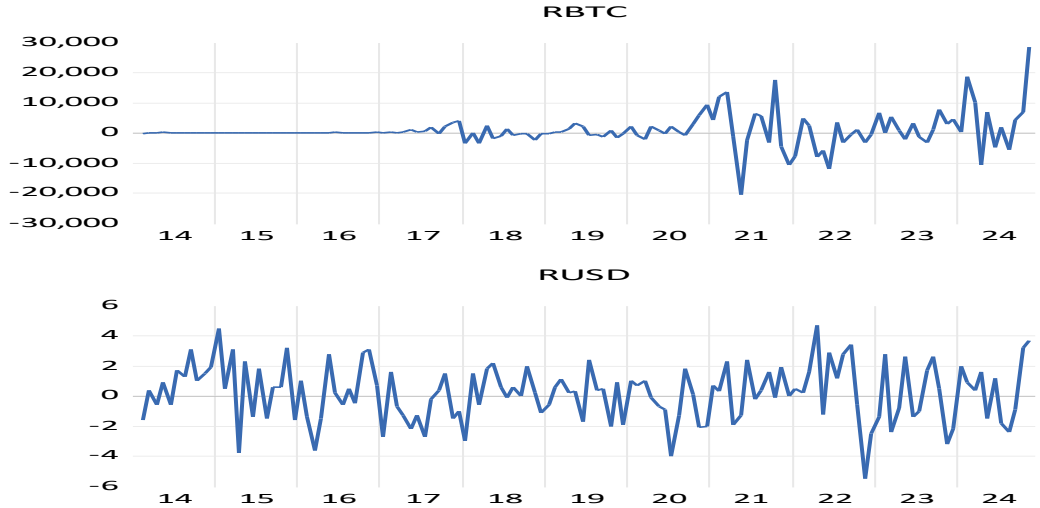
Table (4): Descriptive statistics of the exchange rate return series for bitcoin and the US dollar.

	RBTC	**RUSD**
Mean	751.9015	0.203077
Median	34.25000	0.400000
Maximum Value	28404.20	4.700000
Minimum Value	-20421.30	-5.500000
Standard Deviation	5257.252	1.883710

Skewness	1.135055	-0.144142
Kurtosis	11.34817	2.839751
Jarque-Bera	405.4119	0.589266
Probability	0.000000	0.744805

Source: Prepared by the researcher based on the results of EViews 13.

Figure (02): Graph of the series of exchange rate returns for bitcoin and the US dollar.



Source: Prepared by the researcher based on the results of EViews 13.

Table (5) :Stationarity tests for the series of exchange rate returns.

THE VARIABLE	ADF			PP		
	Without Constant & Trend	With Constant & Trend	With Constant	Without Constant & Trend	With Constant & Trend	With Constant
rBTC	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
rUSD	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Source: Prepared by the researcher based on the results of EViews 13.

The results of all tests presented in Table (5) indicate that the series of exchange rate returns are stationary according to the Augmented Dickey-Fuller (ADF) and Phillips-Perron (PP) tests, as the p-value is greater than the 5% significance level.

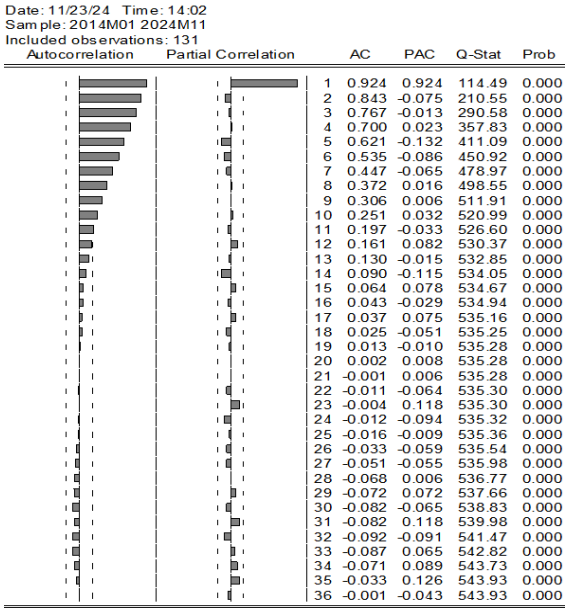
2.3.4 Testing the significance of the autocorrelation coefficients:

Exchange rate time series under study:

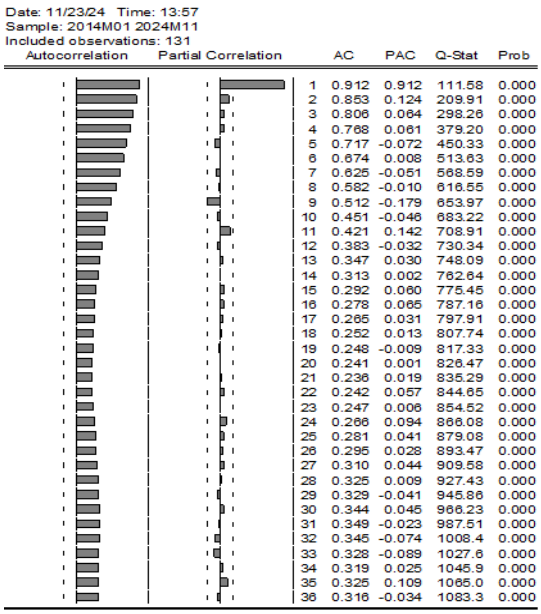
A time series is considered stationary if the coefficients of the autocorrelation function (PK) are insignificant.

The following figure illustrates the simple and partial autocorrelation functions for the time series of the bitcoin and US dollar exchange rates.

Figure (3): Autocorrelation and partial autocorrelation of the exchange rate time series.



US Dollar Exchange Rates



Bitcoin Exchange Rates

Source: Prepared by the researcher based on the results of EViews 13.

From the figure we can see that all partial autocorrelation coefficients are within the confidence interval, except for the first lag of all the series examined, which is outside the confidence interval. From this it can be concluded that the autocorrelation function of the exchange rate time series is not stable.

3. Examining the relationship between bitcoin and US dollar exchange rates:

Table (06):Simple Regression Equation.

Dependent Variable: BTC				
Method: Least Squares				
Date: 04/07/25 Time: 14:42				
Sample: 2014M01 2024M11				
Included observations: 131				
Variable	**Coefficient**	**Std. Error**	**t-Statistic**	**Prob.**
USD	1311.020	268.1007	4.890026	0.0000
C	-108676.9	25977.93	-4.183432	0.0001
R-squared	0.156379	Mean dependent var		18075.69
Adjusted R-squared	0.149840	S.D. dependent var		21405.57
S.E. of regression	19736.82	Akaike info criterion		22.63351
Sum squared resid	5.03E+10	Schwarz criterion		22.67741
Log likelihood	-1480.495	Hannan-Quinn criter.		22.65135
F-statistic	23.91236	Durbin-Watson stat		0.087256
Prob(F-statistic)	0.000003			

Source: Prepared by the researcher based on the results of EViews 13.

From Table (06) it can be seen that the p-value corresponding to the t-statistic is zero, indicating that the estimated parameter of the model is statistically significant and significantly different from zero. In addition, the p-value of the Fisher statistic is also zero, which is less than 0.05, making the model overall acceptable and statistically significant. The coefficient of determination (R^2) is 15.63%, indicating that the ARMA(1,1) model explains 15.63% of the variability in the model parameters, leaving 84.34% unexplained and determined by other factors. The standard deviation was found to be 19,736.82.

4. Results of the ARCH model test:

Table (07):ARCH model test.

Heteroskedasticity Test :ARCH	
TEST	**VALUE**
F-statistic	221,9972
Prob. F(1,128)	0
Obs*R-squared	82,45676
Prob. Chi-Square(1)	0

Source: Prepared by the researcher based on the results of EViews 13.

The test results indicate that the p-value of the (Obs*R-squared) LM test is less than 5%, which leads to the rejection of the null hypothesis that the error variance is constant over time. Instead, the alternative hypothesis is accepted, suggesting that conditional heteroskedasticity is present. As a result, the time series studied is considered suitable for the ARCH model.

5. GARCH model estimation results:

Table (08): Results of GARCH(1,1) model estimation for USD exchange rate series using Student's t distribution**.**

Dependent Variable: USD				
Method: ML ARCH - Student's t distribution (BFGS / Marquardt steps)				
Date: 04/07/25 Time: 15:07				
Sample: 2014M01 2024M11				
Included observations: 131				
Convergence not achieved after 500 iterations				
Coefficient covariance computed using outer product of gradients				
Presample variance: backcast (parameter = 0.7)				
GARCH = C(3) + C(4)*RESID(-1)^2 + C(5)*GARCH(-1)				
Variable	**Coefficient**	**Std. Error**	**z-Statistic**	**Prob.**
BTC	0.000128	1.04E-05	12.27626	0.0000
C	96.15207	0.368283	261.0823	0.0000
	Variance Equation			
C	2.988306	0.868065	3.442493	0.0006
RESID(-1)^2	1.035139	0.355177	2.914430	0.0036
GARCH(-1)	-0.141261	0.103645	-1.362932	0.1729
T-DIST. DOF	111215.4	1.60E+09	6.96E-05	0.9999
R-squared	0.078958	Mean dependent var		96.68244
Adjusted R-squared	0.071818	S.D. dependent var		6.456654
S.E. of regression	6.220483	Akaike info criterion		5.524387
Sum squared resid	4991.579	Schwarz criterion		5.656076
Log likelihood	-355.8474	Hannan-Quinn criter.		5.577898
Durbin-Watson stat	0.103710			

Source: Prepared by the researcher based on the results of EViews 13.

The estimation results indicate that all parameters are statistically significant, as

evidenced by the critical probability value of zero, which is less than 5% for all model parameters. However, the explanatory power of this model is only 7.89%, which is relatively low. In addition, the GARCH coefficient has a probability greater than 5%.

The long-term maximum variance reached 6.220483, while the sum of the ARCH and GARCH coefficients is 0.893878, which is very close to one. This indicates the persistence of volatility shocks in the USD exchange rate.

The table shows that the GARCH coefficient is smaller than the ARCH coefficient, suggesting that recent information has less impact than older information. This implies that market participants do not give more weight to new information than to old information, indicating a lack of weak and semi-strong efficiency in the market.

6. EGARCH test for the time series under consideration:

The EGARCH model is one of the first models not to assume symmetric shocks. It introduces the logarithm of the conditional variance, which allows it to avoid positive constraints on the parameters.

Table (09):Results of the EGARCH model estimation for the USD series.

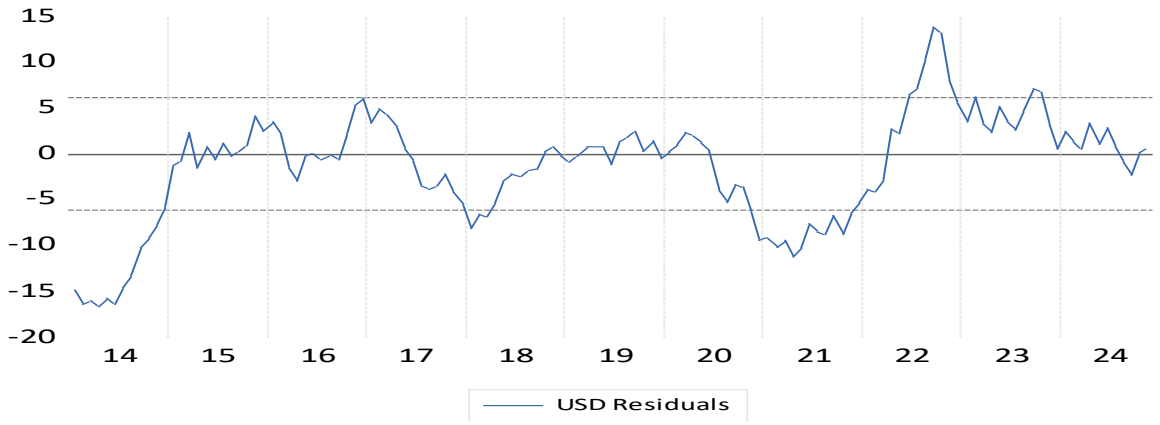
Dependent Variable: USD				
Method: ML ARCH - Student's t distribution (BFGS / Marquardt steps)				
Date: 04/07/25 Time: 15:15				
Sample: 2014M01 2024M11				
Included observations: 131				
Failure to improve likelihood (non-zero gradients) after 35 iterations				
Coefficient covariance computed using outer product of gradients				
Presample variance: backcast (parameter = 0.7)				
LOG(GARCH) = C(3) + C(4)*ABS(RESID(-1)/@SQRT(GARCH(-1))) + C(5)				
*RESID(-1)/@SQRT(GARCH(-1)) + C(6)*LOG(GARCH(-1))				
Variable	Coefficient	Std. Error	z-Statistic	Prob.
BTC	0.000111	1.28E-05	8.730437	0.0000
C	95.99904	0.162363	591.2629	0.0000
	Variance Equation			
C(3)	-0.564098	0.256881	-2.195953	0.0281
C(4)	1.339787	0.374483	3.577696	0.0003
C(5)	0.026374	0.233390	0.113003	0.9100
C(6)	0.767361	0.101099	7.590207	0.0000
T-DIST. DOF	340.7128	0.365106	933.1884	0.0000
R-squared	0.112901	Mean dependent var		96.68244
Adjusted R-squared	0.106024	S.D. dependent var		6.456654
S.E. of regression	6.104786	Akaike info criterion		5.590792
Sum squared resid	4807.625	Schwarz criterion		5.744429
Log likelihood	-359.1969	Hannan-Quinn criter.		5.653222
Durbin-Watson stat	0.104837			

Source: Prepared by the researcher based on the results of EViews 13.

The results in table (09) show that the EGARCH coefficients for the USD exchange rate series are partly positive (c4, c5 and c6) and partly negative (c3). Most of them are less than 1, except for c4, which is greater than 1. This indicates that all the coefficients, except for c4, are statistically significant.

The coefficient of determination (R^2) is 11.29%, which indicates the low explanatory power of the model, meaning that 88.71% of the variability remains unexplained, which is referred to as idiosyncratic risk. In addition, the maximum variance is 6.104786.

Figure (03):Expected variance under the EGARCH (1,1) model for the USD series.



Source: Prepared by the researcher based on the results of EViews 13.

7. Causality test between exchange rates:

Table No. (10): Causality test between bitcoin exchange rates and the US dollar

Pairwise Granger Causality Tests			
Date: 04/07/25 Time: 15:19			
Sample: 2014M01 2024M11			
Lags: 2			
Null Hypothesis:	**Obs**	**F-Statistic**	**Prob.**
BTC does not Granger Cause USD	129	1.54077	0.2183
USD does not Granger Cause BTC		1.54439	0.2175

Source: Prepared by the researcher based on the results of EViews 13.

Based on the one-time lag causality test and the probability values of the tests, the null hypothesis that there is no relationship between the variables is rejected and the alternative hypothesis is accepted at a 5% significance level. The results of the test, as shown in the table above, can be summarised as follows:

- ✓ The bitcoin exchange rate does not affect the US dollar exchange rate.
- ✓ The US dollar exchange rate does not affect the bitcoin exchange rate.

Conclusion :

This study, based on the GARCH model, showed that fluctuations in the exchange rate of bitcoin did not have a significant impact on the stability of the US dollar during the period studied. This supports the study's hypothesis that the US dollar, as a global reserve currency, maintains its resilience and strength despite the volatility generated by the cryptocurrency market. Although the GARCH model successfully captured the temporal volatility dynamics of both markets, the lack of a strong relationship may be due to the dominance of other factors, such as the Federal Reserve's monetary policy or global confidence in the dollar.

Despite the study's limitations, such as the model's assumptions regarding return distributions or the limited data to capture short-term effects, the results provide valuable insights into the stability of the traditional monetary system amid the rise of digital currencies.

We recommend that policymakers continue to monitor the evolution of cryptocurrency markets, with a focus on developing analytical tools to assess potential implications in future contexts where market dynamics may shift. In addition, we suggest that future research should examine the influence of bitcoin on other currencies, such as the euro or the yen, or the role of government regulation in modifying this relationship, in order to deepen our understanding of the interaction between traditional and digital financial systems.

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